# New Trends in Fuzzy Sets, Intuitionistic Fuzzy Sets, Generalized Nets and Related Topics
# Volume I: Foundations


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# On the intuitionistic fuzzy implications $\rightarrow'_{@}$ and $\rightarrow''_{@}$

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**Abstract**

Some basic properties of two new intuitionistic fuzzy implications are studied. They are modifications of the intuitionistic fuzzy implication $\rightarrow_{@}$ introduced by the author and extended by P. Dworniczak.

**Keywords:** Implication, Intuitionistic fuzzy logic

## 1 Introduction

In [5], two modifications of the intuitionistic fuzzy implication $\rightarrow_{@}$ were introduced by the author and some of their basic properties were discussed. This implication $\rightarrow_{@}$ was introduced by the author in [4] and it was extended by Piotr Dworniczak in [6, 7, 8, 9]. Here, some of the basic properties of the two new implications, namely, $\rightarrow'_{@}$ and $\rightarrow''_{@}$ are discussed.

In the beginning, we should note that the concept of Intuitionistic Fuzzy Propositional Calculus (IFPC) was introduced about 25 years ago (see, e.g., [1, 2]). In IFPC, if $x$ is a variable, then its truth-value is represented by the ordered couple

$$V(x) = \langle a, b \rangle,$$

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so that $a, b, a+b \in [0,1]$, where $a$ and $b$ are degrees of validity and of non-validity of $x$, respectively.

Below, we shall assume that for the three variables $x, y$ and $z$ the equalities: $V(x) = \langle a, b \rangle, V(y) = \langle c, d \rangle, V(z) = \langle e, f \rangle$ $(a, b, c, d, e, f, a+b, c+d, e+f \in [0,1])$ hold.

For the needs of the discussion below, following the definition from [2], we shall define the notion of Intuitionistic Fuzzy Tautology (IFT) by:

$$x \text{ is an IFT, if and only if for } V(x) = \langle a, b \rangle \text{ holds: } a \geq b,$$

while $x$ will be a tautology iff $a = 1$ and $b = 0$. As in the case of ordinary logics, $x$ is a tautology, if $V(x) = \langle 1, 0 \rangle$.

For two variables $x$ and $y$ the operations "conjunction" (&) and "disjunction" ($\vee$) are defined (see [1]) by:

$$V(x \& y) = \langle \min(a, c), \max(b, d) \rangle,$$

$$V(x \vee y) = \langle \max(a, c), \min(b, d) \rangle.$$

For two variables $x$ and $y$ the relation $\leq$ is defined (see [1]) by:

$$V(x) \leq V(y) \text{ if and only if } a \leq c \text{ and } b \geq d.$$

## 2 Previous results

In [3], we introduced the implication $\rightarrow_@$ by

$$V(x \rightarrow_@ y) = \langle \frac{b+c}{2}, \frac{a+d}{2} \rangle.$$

It generates the negation

$$V(\neg_@ x) = \langle \frac{b}{2}, \frac{a+1}{2} \rangle.$$

In [5], we first introduced the implications $\rightarrow'_@$ and $\rightarrow''_@$ by

$$V(x \rightarrow'_@ y) = \langle \frac{b+c+\min(b,c)}{3}, \frac{a+d+\max(a,d)}{3} \rangle, \tag{1}$$

$$V(x \rightarrow''_@ y) = \langle \frac{b+c+\max(b,c)}{3}, \frac{a+d+\min(a,d)}{3} \rangle. \tag{2}$$

and there we showed that the values of both implications are intuitionistic fuzzy pairs and

$$V(x \to'_@ y) \leq V(x \to_@ y) \leq V(x \to''_@ y).$$

Second, using the well-known formula (see, e.g. [11])

$$\neg x = x \to 0,$$

in the present case we can construct the following two negations:

$$V(\neg'_@ x) = \langle a, b\rangle \to \langle 0, 1\rangle = \langle \frac{b}{3}, \frac{a+2}{3}\rangle, \tag{3}$$

$$V(\neg''_@ x) = \langle a, b\rangle \to \langle 0, 1\rangle = \langle \frac{2b}{3}, \frac{a}{3}\rangle. \tag{4}$$

Obviously, (3) and (4) are intuitionistic fuzzy pairs and for them

$$V(\neg'_@ x) \leq V(\neg_@ x) \leq V(\neg''_@ x).$$

In [5], it is proved that implication $\to'_@$ does not satisfy Modus Ponens in the case of tautology and it satisfies Modus Ponens in the IFT-case, while mplication $\to''_@$ does not satisfy Modus Ponens as in the case of tautology, as well as in the IFT-case.

There, it is proved also, that for the two new intuitionistic fuzzy implications and negations none of the following three properties:
**Property P1**: $A \to'_@ \neg'_@\neg'_@ A$ and $A \to''_@ \neg''_@\neg''_@ A$,
**Property P2**: $\neg'_@\neg'_@ A \to'_@ A$ and $\neg''_@\neg''_@ A \to''_@ A$,
**Property P3**: $\neg'_@\neg'_@\neg'_@ A = \neg'_@ A$ and $\neg''_@\neg''_@\neg''_@ A = \neg''_@ A$
is valid.

# 3 Main results

First, from (1) and (2) we see that

$$\langle 0, 1\rangle \to'_@ \langle 0, 1\rangle = \langle \frac{1}{3}, \frac{2}{3}\rangle,$$

$$\langle 0, 1\rangle \to'_@ \langle 1, 0\rangle = \langle 1, 0\rangle,$$

$$\langle 1, 0\rangle \to'_@ \langle 0, 1\rangle = \langle 0, 1\rangle,$$

$$\langle 1, 0\rangle \to'_@ \langle 1, 0\rangle = \langle \frac{1}{3}, \frac{2}{3}\rangle,$$

$$\langle 0,1\rangle \rightarrow''_{@} \langle 0,1\rangle = \langle \frac{2}{3}, \frac{1}{3}\rangle,$$

$$\langle 0,1\rangle \rightarrow''_{@} \langle 1,0\rangle = \langle 1,0\rangle,$$

$$\langle 1,0\rangle \rightarrow''_{@} \langle 0,1\rangle = \langle 0,1\rangle,$$

$$\langle 1,0\rangle \rightarrow''_{@} \langle 1,0\rangle = \langle \frac{2}{3}, \frac{1}{3}\rangle.$$

Some variants of fuzzy implications (marked by $I(x, y)$) are described in book [10] by George Klir and Bo Yuan and the following nine axioms are discussed, where

$$I(x, y) \equiv x \rightarrow y$$

and

$$N(x) \equiv I(x, 0).$$

**Axiom 1:** $(\forall x, y)(x \leq y \rightarrow (\forall z)(I(x, z) \geq I(y, z))$.
**Axiom 2:** $(\forall x, y)(x \leq y \rightarrow (\forall z)(I(z, x) \leq I(z, y))$.
**Axiom 3:** $(\forall y)(I(\overline{0}, y) = \overline{1})$.
**Axiom 4:** $(\forall y)(I(\overline{1}, y) = y)$.
**Axiom 5:** $(\forall x)(I(x, x) = \overline{1})$.
**Axiom 6:** $(\forall x, y, z)(I(x, I(y, z)) = I(y, I(x, z)))$.
**Axiom 7:** $(\forall x, y)(I(x, y) = \overline{1}$ iff $x \leq y)$.
**Axiom 8:** $(\forall x, y)(I(x, y) = I(N(y), N(x)))$.
**Axiom 9:** $I$ is a continuous function,
where

$$V(\overline{0}) = \langle 0,1\rangle,$$

$$V(\overline{1}) = \langle 1,0\rangle.$$

**Theorem 1:** Implication $\rightarrow'_{@}$ satisfies axioms 1, 2, 8 and 9.
**Proof:** Let $x \leq y$, i.e. $a \leq c$ and $b \geq d$. Then, for Axiom 1 we obtain that

$$V(I(x, z)) = \langle \frac{b + e + \min(b, e)}{2}, \frac{a + f + \max(a, f)}{2}\rangle,$$

$$V(I(y, z)) = \langle \frac{d + e + \min(d, e)}{2}, \frac{c + f + \max(c, f)}{2}\rangle.$$

From

$$\frac{b + e + \min(b, e)}{2} \geq \frac{d + e + \min(d, e)}{2}$$

and

$$\frac{a + f + \max(a, f)}{2} \leq \frac{c + f + \max(c, f)}{2}$$

we obtain that $V(I(x,z)) \geq V(I(y,z))$.

All other assertions are proved in a similar way.

Now, we shall modify two of the above axioms:

**Axiom 3':** $(\forall y)(I(0,y)$ is an IFT).

**Axiom 5':** $(\forall x)(I(x,x)$ is an IFT).

**Theorem 2:** Implication $\rightarrow'_{@}$ does not satisfy any one of the above axioms (1,2, ..., 9, 3', 5').

**Theorem 3:** Implication $\rightarrow''_{@}$ satisfies axioms 1, 2, 8 and 9.

**Theorem 4:** Implication $\rightarrow'_{@}$ satisfies axioms 3' and 5'.

Following [12], we introduced the list of axioms for propositional intuitionistic logic:

(a) $A \rightarrow A$,

(b) $A \rightarrow (B \rightarrow A)$,

(c) $A \rightarrow (B \rightarrow (A \& B))$,

(d) $(A \rightarrow (B \rightarrow C)) \rightarrow (B \rightarrow (A \rightarrow C))$,

(e) $(A \rightarrow (B \rightarrow C)) \rightarrow ((A \rightarrow B) \rightarrow (A \rightarrow C))$,

(f) $A \rightarrow \neg\neg A$,

(g) $\neg(A \& \neg A)$,

(h) $(\neg A \vee B) \rightarrow (A \rightarrow B)$,

(i) $\neg(A \vee B) \rightarrow (\neg A \& \neg B)$,

(j) $(\neg A \& \neg B) \rightarrow \neg(A \vee B)$,

(k) $(\neg A \vee \neg B) \rightarrow \neg(A \& B)$,

(l) $(A \rightarrow B) \rightarrow (\neg B \rightarrow \neg A)$,

(m) $(A \rightarrow \neg B) \rightarrow (B \rightarrow \neg A)$,

(n) $\neg\neg\neg A \rightarrow \neg A$,

(o) $\neg A \rightarrow \neg\neg\neg A$,

(p) $\neg\neg(A \rightarrow B) \rightarrow (A \rightarrow \neg\neg B)$,

(q) $(C \rightarrow A) \rightarrow ((C \rightarrow (A \rightarrow B)) \rightarrow (C \rightarrow B))$

and proved the following two theorems.

**Theorem 5**: (a) No axiom is a tautology for $\rightarrow''_{@}$

(b) No axiom is an IFT for $\rightarrow''_{@}$.

**Theorem 6**: (a) No axiom is a tautology for $\rightarrow''_{@}$.

(b) Axioms (a), (i), (j), (k) and (n) of the propositional intuitionistic logic are IFTs for $\rightarrow''_{@}$.

# 4 Conclusion

The above research shows that the behaviour of the first implications ($\rightarrow'_{@}$) is very unsuitable for use, while the behaviour of the implications $\rightarrow''_{@}$ is more suitable. So, in future, the author proposes the use only of the second implication and search of its real applications.

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The papers presented in this Volume 1 constitute a collection of contributions, both of a foundational and applied type, by both well-known experts and young researchers in various fields of broadly perceived intelligent systems.
It may be viewed as a result of fruitful discussions held during the Eleventh International Workshop on Intuitionistic Fuzzy Sets and Generalized Nets (IWIFSGN-2012) organized in Warsaw on October 12, 2012 by the Systems Research Institute, Polish Academy of Sciences, in Warsaw, Poland, Institute of Biophysics and Biomedical Engineering, Bulgarian Academy of Sciences in Sofia, Bulgaria, and WIT - Warsaw School of Information Technology in Warsaw, Poland, and co-organized by: the Matej Bel University, Banska Bystrica, Slovakia, Universidad Publica de Navarra, Pamplona, Spain, Universidade de Tras-Os-Montes e Alto Douro, Vila Real, Portugal, Prof. Asen Zlatarov University, Burgas, Bulgaria, and the University of Westminster, Harrow, UK:

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The consecutive International Workshops on Intuitionistic Fuzzy Sets and Generalized Nets (IWIFSGNs) have been meant to provide a forum for the presentation of new results and for scientific discussion on new developments in foundations and applications of intuitionistic fuzzy sets and generalized nets pioneered by Professor Krassimir T. Atanassov. Other topics related to broadly perceived representation and processing of uncertain and imprecise information and intelligent systems have also been included. The Eleventh International Workshop on Intuitionistic Fuzzy Sets and Generalized Nets (IWIFSGN-2012) is a continuation of this undertaking, and provides many new ideas and results in the areas concerned.

We hope that a collection of main contributions presented at the Workshop, completed with many papers by leading experts who have not been able to participate, will provide a source of much needed information on recent trends in the topics considered.

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