Developments in Fuzzy Sets, Intuitionistic Fuzzy Sets, Generalized Nets and Related Topics. Volume I: Foundations

Editors

Krassimir T. Atanassov
Michał Baczyński
Józef Drewniak
Janusz Kacprzyk
Maciej Krawczak
Eulalia Szmidt
Maciej Wygralak
Sławomir Zadrożny

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IBS PAN SRI PAS

Systems Research Institute
Polish Academy of Sciences
Newelska 6, 01-447 Warsaw, Poland
www.ibspan.waw.pl

ISBN 9788389475305

Filters with T-norms and B-operations

Bohdan S. Butkiewicz
Warsaw University of Technology,
Nowowiejska 15/19, 00-665 Warsaw, Poland
b.butkiewicz@elka.pw.edu.pl

Abstract

The filters are described by impulse response. The convolution operation is used to establish the relation between input and output signals in the linear filters theory. Conventionally, the product of impulse response of the filter and input signal shifted in time is integrated to calculate output signal. In this paper the product operation in discrete convolution is replaced by triangular norm operation or basic operations, defined by the author some years ago. Such approach is an extension of conventional idea of convolution. It leads to nonlinear filtering. Some interesting properties as identical impulse, step, and frequency responses of digital filters with T-norms and B_{and} operations or conorms (S-norms) and B_{or} operations are shown. Moreover, filters with fuzzy parameters and crisp signals passing by such fuzzy nonlinear filters are investigated.

Keywords: fuzzy filters, convolution, T-norms, B-operations.

1 Introduction

The filters have been used to isolate a signal from another not necessary signals or to recover original signals or images received with noise. In signal processing they are frequently used in multichannel systems to separate particular signal. They have been used also for the feature extraction in image processing, as edge and corner detection, contrasting, smoothing, and so on. The convolution is the most important operation in signal and image processing because it describes the

Developments in Fuzzy Sets, Intuitionistic Fuzzy Sets, Generalized Nets and Related Topics. Volume II: Applications *(K.T. Atanassow, W. Homenda, O. Hryniewicz, J. Kacprzyk, M. Krawczak, Z. Nahorski E. Szmidt, S. Zadrożny, Eds.), IBS PAN - SRI PAS, Warsaw, 2009.*

relation between input and output of the filter. Mathematically, the convolution operations are inherently a linear combination operation. Thus, they have the linearity property.

In this paper, the algebraic product in linear combination is replaced by more general operation, triangular norm or conorm [7] or *B*-operations [3]. Such idea was firstly presented by Lee *et all.* [8]. They considered different types of discrete convolution where product was replaced by: triangular norm or conorm, compensatory operation (see [9] [11]), and order-weighted aggregation (OWA) operators [10]. It leads to many different types of such convolutions. The OWA operations play similar role as weighting in median filter. The idea proposed in [8] was not developed later and properties of such filters are not investigated. Only in [1], published unfortunately in Japanese, such median filters are discussed.

In the paper it was shown that digital filters with T-norms and B_{and} operations or conorms and B_{or} operations have interesting properties. It was proved that such filters have appropriately identical: impulse, step and frequency responses. It does not mean that responses to different input signals are identical because of the nonlinearity property. The properties are investigated also for such filters with fuzzy parameters.

2 Digital Filters with T-norms and conorms

Conventional approach to linear filters are based on convolution operation. Let $h[n]$ be impulse response, and $x[n]$ be input signal of the filter, where n denotes discrete time instant. Generally, discrete convolution has a form of sum

$$y[n] = \sum_{m=-\infty}^{\infty} h[m] \cdot x[n-m]. \tag{1}$$

Suppose now that filter is causal, i.e. $h[n] \equiv 0$ for $n < 0$ and only finite number of values (samples) of $h[n]$ and $x[n]$ are known, $n = 0, 1, 2, ..., N$. Then, the convolution takes simplified form

$$y[n] = \sum_{m=0}^{N} h[m] \cdot x[n-m]. \tag{2}$$

The convolution has a symmetry property respectively shift operation

$$y[n] = \sum_{m=0}^{N} h[n-m] \cdot x[m]. \tag{3}$$

Under the symbol Σ, algebraic product of impulse response $h[m]$ and shifted input signal $x[n-m]$ is put. The algebraic product is one of the T-norm operations. Generally, T-norms are binary operations $[0,1] \times [0,1] \rightarrow [0,1]$, which are symmetric, associative, monotonic. Moreover, they fulfil for all $a \in [0,1]$ boundary conditions

$$aT1 = a, \qquad aT0 = 0. \tag{4}$$

Discrete impulse response of a filter is the response to Kronecker delta

$$\delta[n] = \begin{cases} 1 & \text{if} \quad n = 0 \\ 0 & \text{if} \quad n \neq 0. \end{cases} \tag{5}$$

Therefore,

$$y[n] = \sum_{m=0}^{N} h[m]\delta[n-m]. \tag{6}$$

Now, consider a situation where product in (2) is replaced by any T-norm.

Theorem 1 *Let conventional discrete impulse response of the filter be denoted $h[n]$. The impulse, step, and frequency response of the filter with any T-norm are appropriately equal to conventional impulse, step, and frequency response $h[n]$.*

Proof. From boundary conditions (4) it follows that impulse response of the filter with any T-norm is equal to

$$h'[n] = \sum_{m=0}^{N} h[m]\, T\, \delta[n-m] = h[n]\, T\, 1 = h[n]. \tag{7}$$

The step response $k[n]$ of a filter is the response to unit step signal

$$1[n] = \begin{cases} 1 & \text{if} \quad n \geq 0 \\ 0 & \text{if} \quad n < 0. \end{cases} \tag{8}$$

Thus,

$$k'[n] = \sum_{m=0}^{N} h[m]\, T\, 1[n-m] = \sum_{m=0}^{n} h[n]\, T\, 1 = \sum_{m=0}^{n} h[n] = k[n]. \tag{9}$$

The frequency response is Fourier transform of an impulse response. The impulse responses are identical for different T-norms then frequency responses must be also identical. □

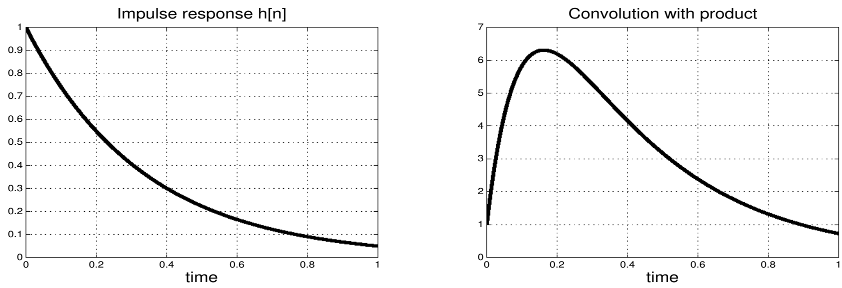

Figure 1: The impulse response (left) and output signal of conventional filter (right).

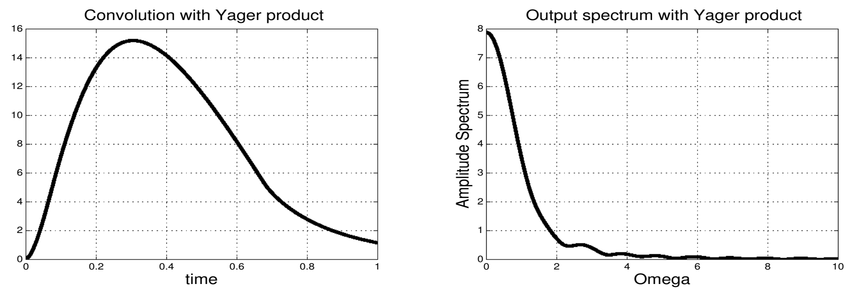

Figure 2: The output signal (left) and its frequency response (right) for filter with Yager product.

Example 1 *Let impulse response of the filter* $h(t) = \exp(-a\,t)\,1(t)$ *and input signal* $x(t) = \exp(-b\,t)\,1(t)$*. Sampling interval is equal* Υ*=0.01s, number of samples* $N = 100$*. Conventional, discrete, impulse and output responses are shown in Fig. 1. The output signal and its amplitude spectrum for filter with Yager operations are shown in Fig. 2.*

Algebraic product in discrete convolution sum can be replaced also by triangular conorm, not quite correctly called S-norms. The S-norms are also binary operations $[0,1] \times [0,1] \rightarrow [0,1]$, which are symmetric, associative, and monotonic as T-norms. However, boundary conditions are different

$$aS1 = 1, \quad aS0 = a \tag{10}$$

for all $a \in [0,1]$.

Theorem 2 *Filters with any conorm (*S*-norm) operation have appropriately identical: impulse, step, and frequency responses.*

Proof. The impulse response of the filter equals

$$h'[n] = \sum_{m=0}^{N} h[m] S \delta[n-m] = h[n] S 1 + \sum_{m=0, m \neq n}^{N} h[m] S 0 = 1 + \sum_{m=0, m \neq n}^{N} h[m]. \tag{11}$$

Step response

$$\begin{aligned} k'[n] = \sum_{m=0}^{N} h[m]\, S\, 1[n-m] = \sum_{m=0}^{n} h[m]\, S\, 1 + \sum_{m=n+1}^{N} h[m]\, S\, 0 = \\ = \sum_{m=0}^{n} 1 + \sum_{m=n+1}^{N} h[m] = n + 1 + \sum_{m=n+1}^{N} h[m]. \end{aligned} \tag{12}$$

Frequency response must be identical because impulse responses are identical irrespectively of S-norm used. □

If someone treats $h[h]$ and $1[n]$ as infinite sequences, $h[n]$ and $1[n]$ completed by zeroes for $n < 0$ then $k'[n]$ as the sum is disconvergent for all values of n, because the terms $0S1 = 1$ for $m = ... - 2, -1$ must be added. It is not true in the case of $h'[n]$ because $0S0 = 0$.

3 Digital Filters with B-operations

First, recall the definition of B-operations [3].

Definition 1 *(Basic operation B_{and}) Basic operation B_{and} is a binary operation $[0,1] \times [0,1] \to [0,1]$ satisfying for all $a, b, c \in [0,1]$ conditions:*

$aB_{and}b = bB_{and}a$	*commutativity*;
$(aB_{and}b)B_{and}c = aB_{and}(bB_{and}c)$	*associativity*;
$aB_{and}1 = a, \quad aB_{and}0 = 0$	*boundary*;
$aB_{and}b > 0$ *if* $a, b > 0$	*positively defined*.

Now, the algebraic product in discrete convolution sum is replaced by B_{and} operation.

$$y[n] = \sum_{m=0}^{N} h[m]\, B_{and}\, x[n-m]. \tag{13}$$

Theorem 3 *The impulse, step, and frequency response of the filter with any B_{and} operation are appropriately equal to conventional (with algebraic product) impulse, step, and frequency response.*

Proof. Proof for T-norms was based only on boundary conditions. For B_{and} operations the boundary conditions, symmetry and associativity conditions are exactly the same. Thus, the conclusion is similar. The impulse, step, and frequency responses are identical. □

Now, consider B_{or}-operations [3].

Definition 2 *(Basic operation B_{or}) Basic operation B_{or} is a binary operation $[0,1] \times [0,1] \to [0,1]$ satisfying for all $a, b, c \in [0,1]$ conditions:*

$aB_{or}b = bB_{or}a$	*commutativity*;
$(aB_{or}b)B_{or}c = aB_{or}(bB_{or}c)$	*associativity*;
$aB_{or}1 = 1, \quad aB_{or}0 = a$	*boundary*;
$aB_{or}b < 1$ *if* $a, b < 1$	*negatively defined*.

Theorem 4 *The impulse, step, and frequency response of the filter with any S and B_{or} operation do not depend on the type of S and B_{or} operation used in the filter equation.*

Proof. Proof for S-norms was based only on boundary conditions. For B_{or} operations the boundary conditions, symmetry, and associativity conditions are exactly the same. Thus, impulse, step, and frequency response are appropriately identical for different S and B_{or} operations. □

4 Fuzzy Filters with T-norms and B-operations

A question arises, can the result obtained for T and B operations be generalized for filters with fuzzy parameters? Let only one filter parameter be fuzzy, more precisely be fuzzy number α, described by membership function $\mu_\alpha(a)$. The concept of fuzzy convolution was discussed by the author in [2] [4]. It is based on the notion of α-level curves [6]. The concept of fuzzy convolution for discrete case, based on discrete α-level curves $h_\alpha^-[m,a]$ $h_\alpha^+[m,a]$, is used here.

Theorem 5 *Fuzzy filter with any T-norm and B_{and} operation have appropriately identical: impulse, step, and frequency response.*

Proof. Replacing product by T-norm it obtains for impulse response

$$h_\alpha'^{\mp}[n,a] = \sum_{m=0}^{N} h_\alpha^-[m,a] T \delta[n-m] = h_\alpha^-[n,a] \tag{14}$$

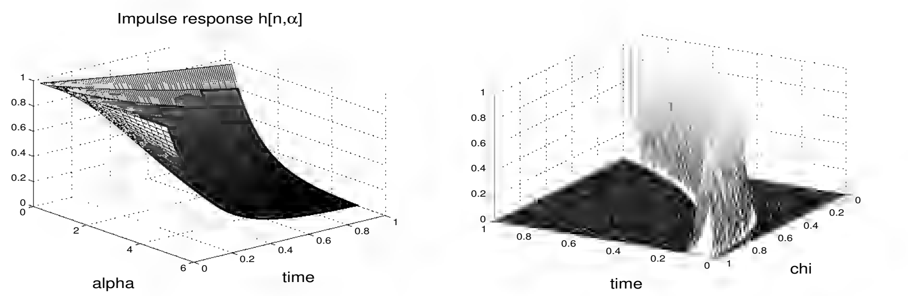

Figure 3: The impulse response (left) of the filter and its membership function (right).

$$h'^{\pm}_{\alpha}[n, a] = \sum_{m=0}^{N} h^{+}_{\alpha}[m, a] T \delta[n - m] = h^{+}_{\alpha}[n, a] \tag{15}$$

for any $a \in supp(\alpha)$. Thus, following notation is justified

$$\mathbf{h}'[n, \alpha] = \sum_{m=0}^{N} \mathbf{h}[m, \alpha] T \delta[n - m] = \mathbf{h}[n, \alpha]. \tag{16}$$

Similar proof can be done for step response. For frequency response, the concept of discrete Fourier transform of fuzzy function [2] [5], based on α-level curves was applied

$$H^{\mp}_{\alpha}[m, a] = \sum_{n=0}^{N} h^{-}_{\alpha}[n, a] \exp(-2\pi i n m/N) \tag{17}$$

and similarly for $H^{\pm}_{\alpha}[m, a]$ and $\mathbf{H}[m, \alpha]$, $m = 0, 1, .., N$. It can be shown that $\mathbf{H}[m, a]$ is fuzzy function. The operation B_{and} fulfils the same boundary conditions thus the results are identical as for T-norms.

Example 2 *Let $h[n, \alpha] = \exp[-\alpha\, n/N] 1[n]$ be impulse response of the filter and $x[n] = [bn/N] \exp[1 - bn/N]\, 1[n]$ be input signal. Sampling interval is equal Υ=0.01s, number of samples $N = 100$. Fuzzy impulse response for T and B_{and} operations and its membership function are shown in Fig. 3. The output signal for minimum and drastic product are shown in Fig. 4. The amplitude spectrum for logic and drastic product are shown in Fig. 5.*

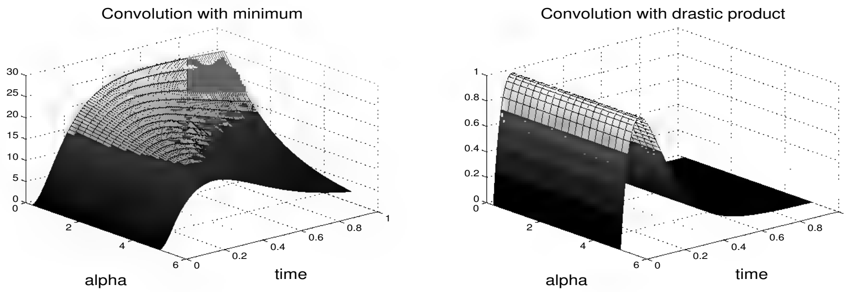

Figure 4: The output signal for filter with minimum (left) and drastic product (right).

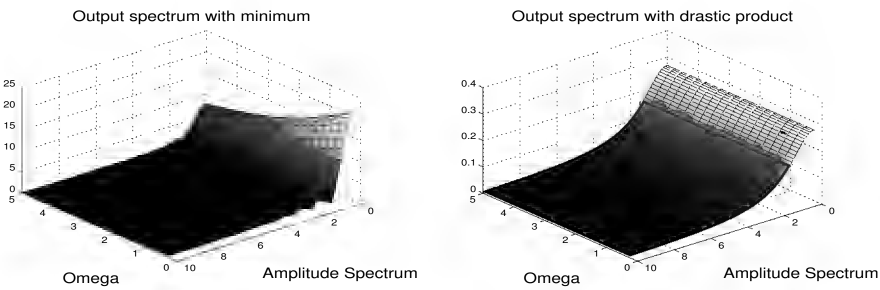

Figure 5: The output signal for filter with minimum (left) and drastic product (right).

5 Conclusions

The convolution with T-norms and B_{and} operations can be considered as generalizations of conventional convolution operation. Such concept leads to nonlinear filtering, excluding the case of algebraic product. Analyzing several examples of T-norms filters, the conclusion can be drawn - general properties as output signal spectrum are rather similar. There are no big changes of the spectrum nature. If conventional filter is low pass then other T-norm filters are also low pass. Taking in consideration that drastic product is minimal T-norm and minimum is maximal T-norm and both cases were analyzed such conclusion seems correct. However, any proof is not shown here. The number of filters with B_{and} operations was not large then similar conclusion must be treated as not sufficiently verified. Filters with triangular conorms and B_{or} operations have different nature. Impulse and step responses are quite different, i.e. are not similar to responses of filters with T-norms and B_{and} operations. They cannot be considered as generalizations of conventional linear filter.

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The papers presented in this Volume 2 constitute a collection of contributions, both of a foundational and applied type, by both well-known experts and young researchers in various fields of broadly perceived intelligent systems.
It may be viewed as a result of fruitful discussions held during the Eighth International Workshop on Intuitionistic Fuzzy Sets and Generalized Nets (IWIFSGN-2009) organized in Warsaw on October 16, 2009 by the Systems Research Institute, Polish Academy of Sciences, in Warsaw, Poland, Centre for Biomedical Engineering, Bulgarian Academy of Sciences in Sofia, Bulgaria, and WIT – Warsaw School of Information Technology in Warsaw, Poland, and co-organized by: the Matej Bel University, Banska Bistrica, Slovakia, Universidad Publica de Navarra, Pamplona, Spain, Universidade de Tras-Os-Montes e Alto Douro, Vila Real, Portugal, and the University of Westminster, Harrow, UK:

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The Eighth International Workshop on Intuitionistic Fuzzy Sets and Generalized Nets (IWIFSGN-2009) has been meant to commence a new series of scientific events primarily focused on new developments in foundations and applications of intuitionistic fuzzy sets and generalized nets pioneered by Professor Krassimir T. Atanassov. Moreover, other topics related to broadly perceived representation and processing of uncertain and imprecise information and intelligent systems are discussed.

We hope that a collection of main contributions presented at the Workshop, completed with many papers by leading experts who have not been able to participate, will provide a source of much needed information on recent trends in the topics considered.

ISBN-13 9788389475305
ISBN 838947530-8
9 788389 475305