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APPLICATIONS OF INFORMATICS IN ENVIRONMENT ENGINEERING AND MEDICINE

Editors:
Jan Studzinski
Ludosław Drelichowski
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This book consist of the papers describing the applications of informatics in environment and health engineering and protection. Problems presented in the papers concern quality management of the surface waters and the atmosphere, application of the mathematical modeling in environmental engineering, and development of computer systems in health and environmental protection. In several papers results of the research projects financed by the Polish Ministry of Science and Information Society Technologies are presented.

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Systems Research Institute of Polish Academy of Science
Newelska 6, PL 01-447 Warsaw

Section of Scientific Information and Publications
e-mail: biblioteka@ibspan.waw.pl

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CHAPTER 2

Mathematical Modeling in Environment Engineering

BIO-ECONOMICS MODELS TRANSITION TOWARDS SUSTAINABILITY ON THE RURAL AREAS[1]

Joanna MIKLEWSKA
Agricultural University in Szczecin
<miklewsk@erl.edu.pl>

In this paper we focus on the modeling renewable resource (fishery resource in the lakes on the rural areas) under sustainable development with governing dynamics described by two distinct growth functions. We introduce to standard growth model the seasonally varying as disturbance, as an example of extreme event. We employ the Pontryagin's maximum principle to derive the optimal solution (optimal effort policies) for both the growth models. We give solutions in professional software (MATHEMATICA) and in popular program (Excel).

Keywords: Bio-economic model, growth function, optimal control, sustainable harvesting, seasonal variability.

1. Introduction

The paper reveals possibilities of operationalization of sustainable growth of renewable resources achieved with applying the optimal control theory.

The main purpose of this paper is to present the preliminary results of the research project of The Ministry of Science and Information Society Technologies concerned with extreme events in humankind environment. We focus on the modeling renewable resource (fishery resource in the lakes on the rural areas) under sustainable development with governing dynamics described by two distinct growth functions: logistic and Gompertz growth functions. We introduce to standard model the seasonally varying as disturbance, as an example of extreme event.

Among the main target of humankind is a conservation of an environment, e.g. renewable resources. Sustainable exploitation of a fishery resource requires that the sum of the present value of net revenues be maximized. Setting sustainable yield levels for this purpose will depend on: (a) the biological balance between recruitment, somatic growth and mortality rates, (b) dynamic fluctuations in costs and prices in a regional and international context probably reflected in the interest rate, and (c) socio-economic and political conditions. In seminal work (Clark, 1990)

[1] Project of the Ministry of Science and Information Society Technologies, No. PBZ-KBN-086/P04/2003 and Inner University Grant of the Agricultural University in Szczecin No. BW/HE/03/03.

there is excellent introduction to optimal management of renewable resources, wherein he developed optimal harvesting strategies for both single and multiple dimensional deterministic autonomous ecosystems. There has been considerable study in recent years (Ludwig 1997; Carpenter et al, 1999; Janssen et al. 1999; Miklewska 1995, 1996, 2004, 2005a, 2005b) on optimal management of renewable resources from various perspectives. The need for sustaining of the resources for future generations on the rural areas is the motivating factor for these studies.

2. Setting the research problem

Let us consider the following optimal control task for and effort policy, E:

$$\max_{E\in[E_{min},E_{max}]} I(E)=\int_0^\infty e^{-\delta t}\left[p(x,t)\cdot E\cdot g(x,t)-c(x,t)\cdot E\right]dt \tag{1}$$

subject to dynamic equation

$$\dot{x}=f(x,t)-E\cdot g(x,t) \tag{2}$$

where

$c(x,t)$, $f(x,t)$, $g(x,t)$ and $p(x,t)$ are assumed to be C^1 functions of resource x and time t, δ is a social discount rate. The associated current value hamiltonian (Pontriagin *et al.* 1962; Weitzman 2003) is given by

$$H=\lambda\left[f(x,t)-E\cdot g(x,t)\right]+p(x,t)\cdot E\cdot g(x,t)-E\cdot c(x,t). \tag{3}$$

Hamiltonian H is linear in the control variable E, hence, the optimal solution is a combination of bang-bang and singular controls on the infinite horizon. The associated costate variable $\lambda(t)$ (shadow price for state variable $x(t)$) is given by

$$\dot{\lambda}=\delta\lambda-\frac{\partial H}{\partial x}, \tag{4}$$

that is

$$\dot{\lambda}=\delta\lambda-\lambda(f_x-E\,g_x)-E\left(p_x g+p g_x-c_x\right). \tag{5}$$

The singular control is characterized by the zero of the switching function along an interval, that is, when

$$\frac{\partial H}{\partial E}=0$$

on an interval. Thus along the singular solution we have

$$(p-\lambda)g-c=0. \quad (6)$$

Differentiating (6) we have

$$\left(\frac{\partial p}{\partial t}+\frac{\partial p}{\partial x}\dot{x}-\dot{\lambda}\right)g+(p-\lambda)\left(\frac{\partial g}{\partial t}+\frac{\partial g}{\partial x}\dot{x}\right)=\frac{\partial c}{\partial t}+\frac{\partial c}{\partial x}\dot{x}. \quad (7)$$

Using equations (2), (5) and (6) in (7) we obtain

$$\frac{\partial}{\partial t}(pg)+f\frac{\partial}{\partial x}(pg)-\left(p-\frac{c}{g}\right)\left[\frac{\partial g}{\partial t}+\frac{\partial g}{\partial x}f+g(\delta-f_x)\right]=\frac{\partial c}{\partial t}+\frac{\partial c}{\partial x}f \quad (8)$$

This is an implicit equation for x as a function of t and characterizes the singular solution. (8) with (2) determines the singular control. Clearly, the nature of the singular solution and the singular control depends on the nature of the involved functions $c(x,t)$, $f(x,t)$, $g(x,t)$ and $p(x,t)$ along with their partial derivatives with respect to t and x. Now the optimal control can be constructed by restricting this control to the bounds $[E_{min}, E_{max}]$. Hence, for the considered problem, the optimal control is always a combination of bang-bang and singular controls. The singular path will be a singular optimal path if the singular effort policy $E(t)$ obtained from (8) satisfies $E(t) \in [E_{min}, E_{max}]$ for all t.

3. Two growth models with harvesting and seasonal varying environment

We investigate the nature of solutions of the two population growth models: the logistic (Miklewska 2005a) and Gompertz (Howard 2005) models modified by a harvesting term in a seasonally varying environment. We assume that the growth rate and carrying capacities of the considered population are periodic with the same period T. Our aim is to obtain an optimal harvesting policy $E(t)$ which maximizes the time stream of net revenues $I(E)$ to the owner (harvester) on the infinite horizon when the resource dynamics is governed by either the logistic or the Gompertz equation.

3.1 Problem 1 - the logistic model

Let us consider the first problem

$$\max_{E\in[E_{min},E_{max}]} I(E)=\int_0^{\infty} e^{-\delta t}\left[p\cdot E\cdot x-c\cdot E\right]dt \quad (9)$$

subject to

$$\frac{dx}{dt} = x\left(a(t) - b(t)x\right) - E \cdot x, \tag{10}$$

with

$$x(0) = x_0 \text{ and } E(t) \in \left[E_{min}, E_{max}\right], \tag{11}$$

where

$a(t)$ is a growth rate of the renewable resource and $\frac{a(t)}{b(t)}$ is a carrying capacity of the model (10).

3.2 Problem 2 - the Gompertz model

Let us consider the second problem

$$\max_{E \in [E_{min}, E_{max}]} I(E) = \int_0^{\infty} e^{-\delta t}\left[p \cdot E \cdot y - c \cdot E\right]dt \tag{12}$$

subject to

$$\frac{dy}{dt} = y\left(\alpha(t) - \beta(t)ln(y)\right) - E \cdot y, \tag{13}$$

with

$$y(0) = y_0 \text{ and } E(t) \in \left[E_{min}, E_{max}\right], \tag{14}$$

where

$\alpha(t)$ is a growth rate of the renewable resource and $exp\left(\frac{\alpha(t)}{\beta(t)}\right)$ is a carrying capacity of the model (13).

4. Optimal solutions

Both in the two problems, δ is the instantaneous annual rate of discount, p is the price per unit harvest, c is the cost per unit effort and $E_{max}(E_{min})$ represents the maximum (minimum) allowable effort in the harvesting activity by the harvesting enterprise. In this paper δ, p and c are assumed to be constants. The function $E(t) \in [E_{min}, E_{max}]$ which solves Problem 1 (Problem 2) is the optimal harvest policy and the corresponding solution $x(t)$ of Problem 1 ($y(t)$ of Problem 2) with $E = E(t)$ is the optimal path. We apply the observations made in section 2 to the problems 1 and 2 and obtain optimal harvesting policies i.e., the values of $E(t)$ such that $I(E)$ is maximized for the respective problems. We obtain the path traced by the solutions $x(t)$ and $y(t)$ with the optimal harvesting policy so that if the populations under study are kept along this path, we are assured of achieving the objective of the harvesting enterprise.

4.1 Optimal solution (optimal harvest policy) for problem 1

Following the optimal control problem discussed in section 2, we have $p(x, t) = p$, $c(x, t) = c$, $g(x, t) = x$ and $f(x, t) = x\,(a(t) - b(t)\cdot x)$. The singular trajectory of the population is given by equation (8) which is nothing but the positive solution, denoted by $x_s(t)$, $t > 0$, of the quadratic equation

$$2b(t)px^2 - x\big(b(t)c + p\big(a(t)-\delta\big)\big) - \delta c = 0\,. \tag{15}$$

Using the equation (10) we obtain the singular effort policy to be

$$E_s(t) = a(t) - b(t)x_s(t) + \\ -\frac{pa'(t) + b'(t)c - 2pb'(t)x_s(t)}{4pb(t)x_s(t) - cb(t) - p\big(a(t)-\delta\big)}\,. \tag{16}$$

We emphasize that $E_s(t)$ is a function of t only. It can be easily verified that the singular effort policy reduces to the policy developed by Clark (1990) in case the coefficients $a(t)$ and $b(t)$ are constants. Note that the functions $x_s(t)$ and $E_s(t)$ are defined in terms of an implicit equation (15). We can establish that this implicit equation admits a unique physically meaningful solution. We can also show that this solution is periodic and that the denominator term in the equation (16) can never tend to zero. This proves that the functions $x_s(t)$ and $E_s(t)$ are well defined.

In first step we compute numerical solutions for the optimal harvesting problems considered in our paper. We assume the following explicit periodic forms for the coefficient functions. $a(t) = a_1(t) + a_2(t)\cos^2(t)$, $b(t) = b_1(t) + b_2(t)\sin(t)$, $\alpha(t) = \alpha_1(t) + \alpha_2(t)\cos^2(t)$, $\beta(t) = \beta_1(t) + \beta_2(t)\sin(t)$.

One of solutions for Problem 1 is presented on Fig. 1 and Fig. 2. We assume the values of the parameters and initial stock levels to be: $a(t) = 1 + 0.1\cos^2(t)$, $b(t) = 1 + 0.1\sin(t)$, $p = 5$, $c = 1$, $\delta = 0.01$, $E_{min} = 0$, $E_{max} = 1$, $x(0) = 1$. Solutions were achieved in MATHEMATICA and Excel independently.

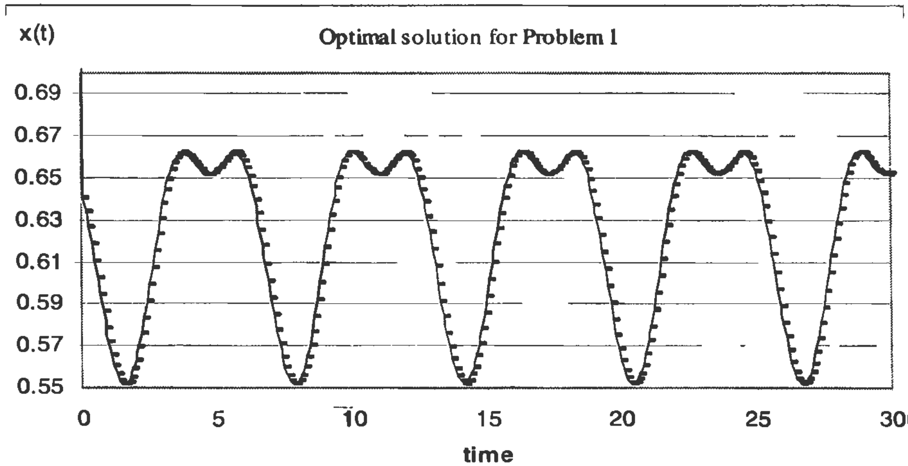

Figure 1. Optimal *x(t)* for Problem 1. (Source: Own investigations in MATHEMATICA and Excel)

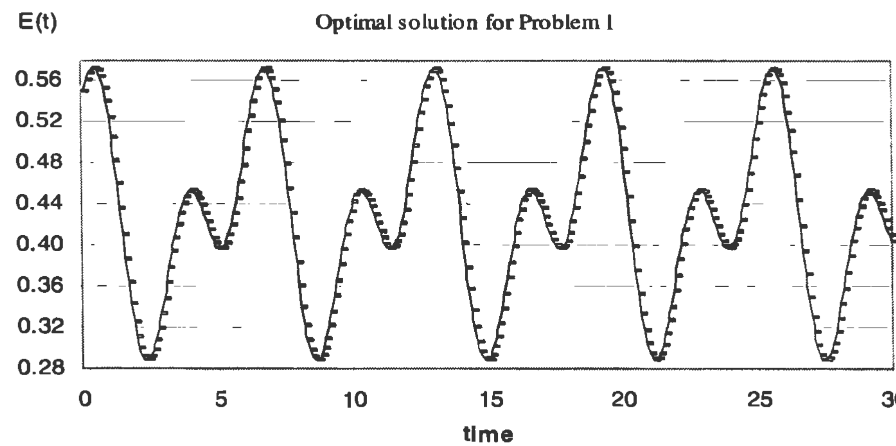

Figure 2. Optimal *E(t)* for Problem 1. (Source: Own investigations in MATHEMATICA and Excel)

4.2 Optimal solution (optimal harvest policy) for problem 2

The procedure to obtain the optimal harvest policy in this case is similar to the previous one. We list the important equations with explanation wherever it is necessary. We observe that the function $f = y\,(\alpha(t) - \beta(t)\cdot ln(y))$ is a C^1 function on the set $y > 0$. The current value hamiltonian for this problem is given by

$$H(y,\lambda,t)=\left[pE(t)y(t)-cE(t)\right]+\lambda y\left(\alpha(t)-\beta(t)ln(y)-E\right) \tag{17}$$

which is linear in control variable E. Dynamics of the costate variable $\lambda(t)$ for this problem is described by

$$\frac{d\lambda}{dt}=\delta\lambda-\left[pE+\lambda\left(\alpha(t)-\beta(t)ln(y)-\beta(t)-E\right)\right]. \tag{18}$$

The singular trajectory for the optimal control problem (8), is given by the positive solution $y_s(t)$ of the equation

$$\beta(t)y\left(ln(y)+1\right)+\left(\delta-\alpha(t)\right)y-(c/p)\left(\delta+\beta(t)\right)=0 \tag{19}$$

and the corresponding singular effort is given by

$$E_s(t)=\alpha(t)-\beta(t)ln\left(y_s(t)\right)+ \\ -\frac{(c/p)\beta'(t)-y_s(t)\left(\beta'(t)ln\left(y_s(t)\right)-\alpha'(t)+\beta'(t)\right)}{y_s(t)\left[\beta(t)\left(ln\left(y_s(t)\right)+2\right)-\alpha(t)+\delta\right]}. \tag{20}$$

The functions $y_s(t)$ and $E_s(t)$ are well defined and they are periodic of period T.

Solutions were achieved in MATHEMATICA and Excel independently.

5. Conclusions

The operationalization of the sustainable development requires providing decision makers and managers of renewable resources harvesting with effective tools to solve rather complex problems of optimal control. Incorporating into classical growth models, namely, the logistic and Gompertz equations, seasonality by assuming periodic growth rates and periodic carrying capacities allowed for obtaining optimal harvest policies and optimal solutions for both the models. The optimal harvest policy is meant to maximize the net revenue to the harvester on the infinite horizon.

Pontryagin's maximum principle was applied to derive the optimal effort policies for both the growth models. These policies are derived as an application to a general optimal harvesting problem considered on the infinite horizon where the control variable, i.e., effort, enters linearly into the hamiltonian. This procedure can be easily extended to diverse situations where price, cost are also seasonally dependant along with stock and effort. This method can also be applied to the cases where the growth dynamics of the resource is perturbed seasonally varying forcing functions provided the dynamic constraint in the optimal harvesting problem admits a unique globally stable periodic solution for a reasonable periodic harvesting effort i.e., whenever the harvesting function belongs to the control set $[E_{min}, E_{max}]$.

We used the first solvers which can be run in widely available computer software environment, e.g. MATHEMATICA and MATLAB. Moreover, on the whish of the reader, solver done in Excel can be provided, what broadens a circle of receivers.

The presented approach may be useful as a link of rather sophisticated problem of optimal growth under sustainability conditions with rather simple practical solutions. It is an example of bridging between science and action.

References

Carpenter S., Brock W. Hanson, P. (1999) Ecological and social dynamics in simple models of ecosystem management. *Conservation Ecology*, **3**(2), 4. [online] URL: http://www.consecol.org/vol3/iss2/art4/.

Clark C.W. (1990) *Mathematical Bio-economics: The optimal management of renewable resources.* Wiley and Sons, New York, 2nd edition.

Howard P. (2005) *Modeling with ODE.* [online]: URL: www.math.tamu.edu/~yalchin.efendiev/ math647_spring05/model_ode.pdf.

Janssen M.A., Carpenter S.R. (1999) Managing the Resilience of Lakes: A multi-agent modeling approach. *Conservation Ecology*, **3**(2), 15. [online] URL: http://www.consecol.org/vol3/iss2/art15.

Ludwig D., Walker B., Holling C.S. (1997) Sustainability, stability, and resilience. *Conservation Ecology*, **1**(1), 7. [online] URL: http://www.consecol.org/vol1/iss1/art7.

Miklewska J. (2005a) Modelowanie Bio-ekonomiczne w warunkach rozwoju zrównoważonego – Zastosowanie modeli wzrostu. SGGW Warszawa, in: *Metody ilościowe w ekonomii – V.* In press.

Miklewska J. (2005b) Modelowanie wzrostu ekonomicznego na obszarach wiejskich. SGGW Warszawa, in: *Metody ilościowe w ekonomii – VI.* In press.

Miklewska J. (2004) Rethinking rural sustainable development – landscape, main driving forces and new challenges after EU accession of Poland, in: J. Studziński, L. Drelichowski, O. Hryniewicz (2004). Wspomaganie Informatyczne Rozwoju Społeczno-Gospodarczego i Ochrony Środowiska. Seria: *Badania Systemowe*, **37**. IBS PAN, 93-122.

Miklewska J. (1996) Niepewność i ryzyko w modelowaniu strategii zachowania bioróżnorodności, w: *Ryzyko i niepewność w modelach ekonomiczno-ekologicznych*, Szczecin, 139-148.

Miklewska J. (1995) The need for sustainable agriculture in view of water quality, The Miedwie Lake case study. Department of System Ecology, Stockholm University 106 91 Stockholm, Sweden, w: *Ecological Economic Aspects of the agricultural Sector in the Baltic Drainage Basin*, ISSN 1104-8298.

Pontryagin L., Boltyanskii V., Gamkrelidze R.V., Mishchenko E. (1962) *The Mathematical Theory of Optimal Processes.* New York, Wiley-Interscience.

Weitzman M.L. (2003) *Income, Wealth, and the Maximum Principle.* Harvard University Press. Cambridge, Massachusetts. London, England.

Jan Studzinski, Ludosław Drelichowski, Olgierd Hryniewicz
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The purpose of the present publication is to popularize applications of informatics in environment and health engineering and protection. Runned papers are thematically chosen from the works presented during the conference *Multiaccessible Computer Systems* (*Komputerowe Systemy Wielodostępne*) that has been organized by the Systems Research Institute and University of Technology and Agriculture of Bydgoszcz for several years in Ciechocinek. Problems described in the papers concern quality management of the surface waters and the atmosphere, application of the mathematical modelling in environmental engineering, and development of computer systems in health and environmental protection. In several papers results of the research projects financed by the Polish Ministry of Science and Information Society Technologies are presented.

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