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STRATEGIC REGIONAL POLICY

A. STRASZAK AND J.W. OWSIŃSKI
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PART II

WARSAW 1985

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Paradigms, Methods, Issues and Case Studies

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PART II

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V. ENVIRONMENT

AN URBAN SCALE COMPUTER MODEL FOR SHORT-TERM FORECASTING AND CONTROLLING AIR QUALITY IN A CITY

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Abstract. An urban-scale computer model for short term prediction of city air quality is presented. The physical process of pollutant dispersion is described by distributed parameters advection-diffusion equation. The model, utilising meteorological forecast, calculates spatial and temporal distribution of pollutant concentration in complex urban area. Some test computations have been conducted for Warsaw case. Basing on prediction model, a computer program for real-time control of emission redistribution in urban area is constructed.

1. INTRODUCTION

The problem of air pollution forecasting and controlling in urban regions is considered to be important nowadays, because of steady degradation of city air quality effected by power plants, industry, transportation system and other sectors of human activity.

In the paper a computer model for short-term prediction of city air pollution is presented. The model calculates detailed, spatial and temporal distribution of pollutant concentration in a complex urban area for a given time period (approximately 1-3 days).

In Fig.1.1 the block-diagram is presented, where the following basic groups of input data utilized by the model can be distinguished:

a) geometrical and structural description of the domain
   - time period of the forecast - T,
   - geometrical dimensions of the domain,

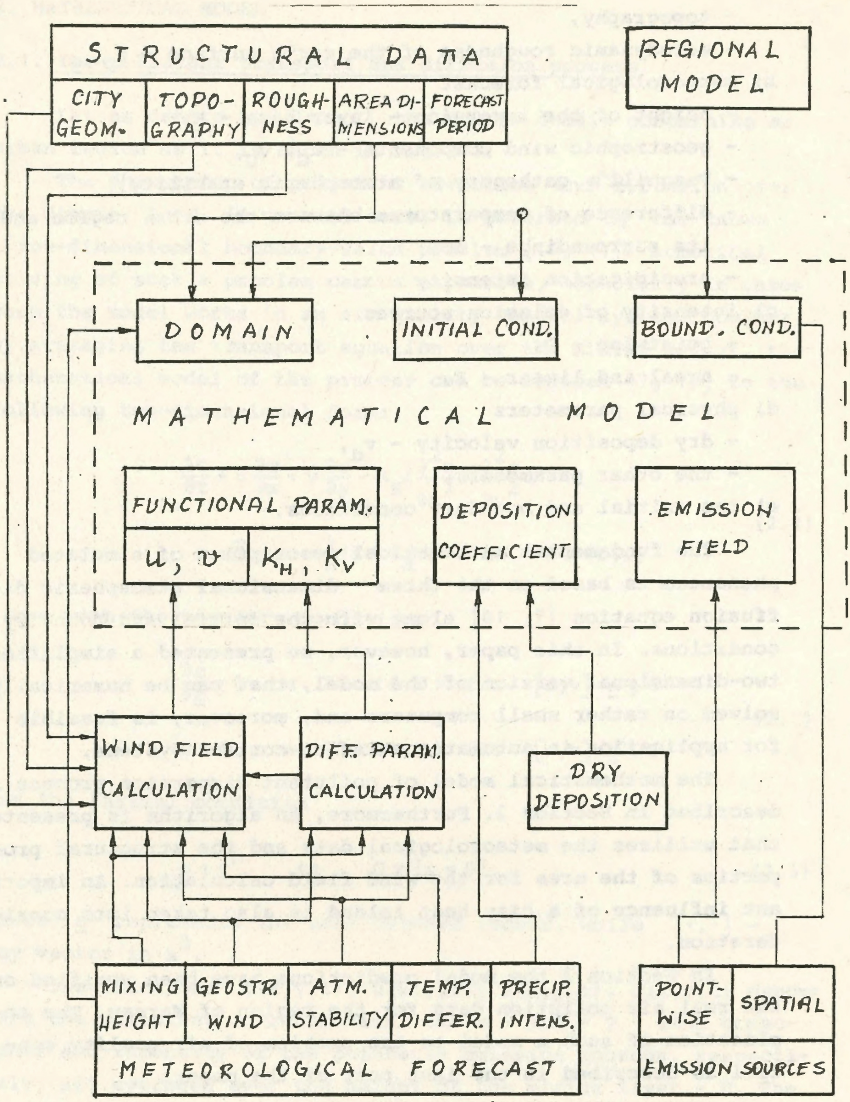


Fig. 1.1. The block diagram of the model

- dimensions of the urbanized area,
- topography,
- aerodynamic roughness of the earth surface - z_o,

b) meteorological forecast
- height of the inversion - layer base - H,
- geostrophic wind components - u_G, v_G,
- Pasquill's cathegory of atmospheric stability,
- difference of temperatures between the urban region and its surroundings - $\Delta\theta$,
- precipitation intensity - α,

c) intensity of emission sources
- pointwise - Q,
- areal and linear - E,

d) physical parameters
- dry deposition velocity - v_d,
- the other parameters,

e) the initial and boundary conditions.

The fundamental mathematical description of simulated phenomena is based on the three - dimensional atmospheric diffusion equation [7, 10] along with the initial and boundary conditions. In this paper, however, we presented a simplified, two-dimensional version of the model, that can be numerically solved on rather small computers and, moreover, is feasible for application in automatic emission-control systems.

The mathematical model of pollutant dispersion process is described in Section 2. Furthermore, an algorithm is presented that utilizes the meteorological data and the structural properties of the area for the wind field calculation. An important influence of a city heat island is also taken into consideration.

In Section 3 the model predictions have been verified on the real air pollution data for the region of Warsaw. The application of such a model to the problem of air quality control is described in the last part of the paper.

2. MATHEMATICAL MODEL

2.1. The pollutant transport and diffusion process

Let us denote by $\Omega = L \times L$ the square domain containing an urban region as it is shown in Fig.2.1.

The phenomenon of pollutant advection and diffusion over the domain $\Omega \times H$, in general case, is governed by the known three-dimensional boundary-value problem [10, 11]. Numerical solving of such a problem can be expensive, especially in cases when the model works in an air quality control system. However, by averaging the transport equation over the mixing height, the mathematical model of the process can be reduced [7, 10] to the following two-dimensional form:

$$\frac{\partial c}{\partial t} + u\frac{\partial c}{\partial x} + v\frac{\partial c}{\partial y} - K_H\left(\frac{\partial^2 c}{\partial x^2} + \frac{\partial^2 c}{\partial y^2}\right) = \bar{Q} - \gamma c + \frac{1}{H}(E - V_d c) \quad \text{in} \quad \Omega \times (0,T) \tag{2.1}$$

along with the boundary conditions

$$\begin{aligned} \frac{\partial c}{\partial \underline{n}} &= 0 \quad \text{on} \quad \partial\Omega \times \langle 0,T\rangle \quad \text{for} \quad [u,v]\cdot\underline{n} \geq 0 \\ c &= 0 \quad \text{on} \quad \partial\Omega \times \langle 0,T\rangle \quad \text{for} \quad [u,v]\cdot\underline{n} < 0 \end{aligned} \tag{2.2}$$

and the initial condition

$$c = c^o \quad \text{in} \quad \Omega \times \{t = 0\}. \tag{2.3}$$

where $\underline{n}$ represents the unit outward vector, while $[\cdot,\cdot]$ - any vector in R^2.

The functions $c(x,y,t)$, $u(x,y,t)$, $v(x,y,t)$, $\bar{Q}(x,y,t)$ denote here the pollutant concentration, the x - and - y - wind components and intensity of the pointwise emission sources, respectively, all averaged over the height of the mixing layer - H. The coefficient K_H represents the horizontal diffusion, while γ - the wet deposition factor, approximated in numerical algorithm by the following formula

$$\gamma(\alpha) = 2 \cdot 10^{-5} \cdot \alpha^{0.6} \quad ,$$

for the precipitation intensity α given in [mm/h].

The expression $E-V_d c$ in the right-hand side of (2.1) represents the ground-level stream of pollutant, where $E(x,y,t)$ is the intensity of the area emission and V_d - dry deposition coefficient.

The atmospheric stability was classified according to the six Pasquill's categories [6, 11] ranging from strong stable to extreme instable. It was assumed [6] that the diffusion coeficient K_H is a function of stability index - s, as listed in Table 2.1.

Table 2.1. The horizontal diffusion versus stability.

Class of stability	Stability index - s	Horizontal diffusion coeficient - K_H
A - extreme instable	-3	250.0
B - moderate instable	-2	100.0
C - weak instable	-1	30.0
D - neutral	0	10.0
E - weak stable	1	3.0
F - strong stable	2	1.0

The atmospheric stability represented by index s influences essentially also the wind field calculation that is described below.

2.2. The wind field modelling

As follows from (2.1), (2.2), the transport of pollutant is caused mainly by advection and depends directly on the air movement. Therefore, the calculation of the wind components u and v is essential problem. The final shape of the wind field is effected by: the change of the wind velocity profile and the twist of its direction versus height, the topographical obstacles, the city heat - island phenomenon. The influence of the aerodynamic roughness has been considered to be comparably less important [1, 14] and is introduced only as a parameter of the vertical flow factor.

In order to make calculations feasible, the considerations are carried out in two dimensions (the wind field is assumed to be flat). The linearization technique is applied in the sense that the listed above factors are treated separately and then their effects are superimposed on each other.

The first step of the procedure consists in finding an initial approximation of the components of the wind velocity vector. These are constant in the whole domain Ω and averaged over the mixing layer height values

$$\bar{u}_o = \frac{1}{H}\int_0^H u_o(z)\,dz\,, \quad \bar{v}_o = \frac{1}{H}\int_0^H v_o(z)\,dz,$$

where $[u_o, v_o]$ is the solution of the set of genaralized Ekman's spiral equations

$$\frac{\partial}{\partial z}\left(K_M \frac{\partial u}{\partial z}\right) + \rho f v = \rho f v_G\,, \tag{2.4}$$

$$\frac{\partial}{\partial z}\left(K_M \frac{\partial v}{\partial z}\right) - \rho f u = -\rho f u_G$$

along with the boundary conditions

$$\begin{aligned} u(\bar{e}) &= v(\bar{e}) = 0, \\ u(H) &= u_G\,, \quad v(H) = v_G\,. \end{aligned} \tag{2.5}$$

Vector $[u_G, v_G]$ represents here the geostrophic wind at the inverse - based height H, $\bar{e}$ - the averaged over Ω value of the ground elevation $e(x,y)$, f - Corriolis coeficient. The vertical flow factor $K_M(z)$ depends on the stability index, the average roughness $\bar{z}_o$ and other parameters

$$K_M(z) = K_M(z; H, u_G, v_G, s, \bar{z}_o)\,.$$

The details of this parametrization can be found in [7, 11].

In the next step the influence of topography and surfface roughness is considered. We want to find the minimal values of

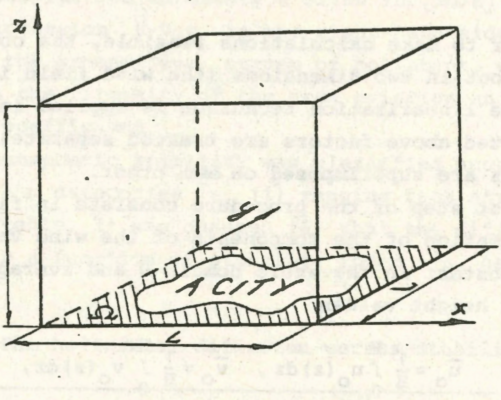
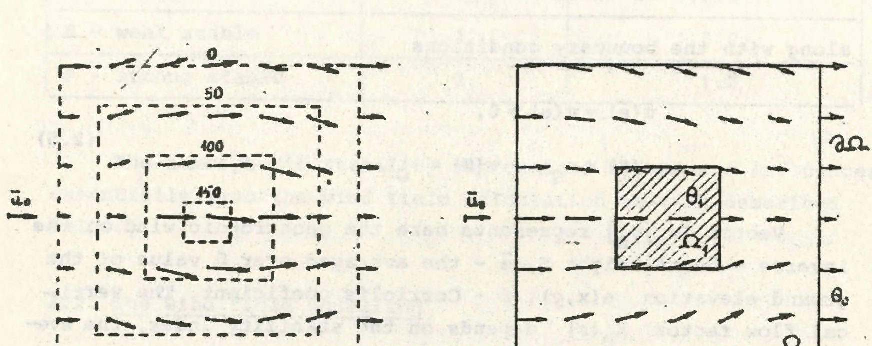


Fig.2.1. Geometry of a simulated area



a) Topographical effect **b) Thermal effect ($\Delta\Theta = 10^{\circ}C$)**

Fig.2.2. The wind field deviations ($H = 200m$, $\Delta L = 200m$)

corrections $\delta_t\bar{u}$, $\delta_t\bar{v}$, such that the modified vector

$$[\bar{u},\bar{v}] = [\bar{u}_o + \delta_t\bar{u}, v_o + \delta_t\bar{v}]$$

satisfies the conservation law [7]. An equivalent formulation [3, 7] is to find minimum of the functional

$$J(\bar{u},\bar{v}) = \int_\Omega [h^2(\bar{u}-\bar{u}_o)^2 + h^2(\bar{v}-\bar{v}_o)^2 + \lambda(\frac{\partial(h\bar{u})}{\partial x} + \frac{\partial(h\bar{v})}{\partial y})]d\Omega , \tag{2.6}$$

where $h(x,y)=H-e(x,y)$, while λ is a Lagrange multiplier corresponding to the conservation equation constraint.

It can be shown [7] that (2.6) is minimized for

$$\delta_z\bar{u} = \frac{1}{2h}\frac{\partial\lambda^*}{\partial x}, \quad \delta_t\bar{v} = \frac{1}{2h}\frac{\partial\lambda^*}{\partial y}, \tag{2.7}$$

where the optimal value of Lagrange multiplier λ^* is the solution of the following Dirichlet problem

$$\Delta\lambda = -\bar{u}_o\frac{\partial h}{\partial x} - \bar{v}_o\frac{\partial h}{\partial y} \quad \text{in} \quad \Omega, \tag{2.8}$$

$$\lambda = 0 \quad \text{on} \quad \partial\Omega.$$

Since $h(x,y)$ is independent of time, the solution λ^* can be expressed as a linear combination of functions $\lambda(1,0)$ and $\lambda(0,1)$. These are the solutions to (2.8) for $[\bar{u}_o,\bar{v}_o]=[1,0]$ and $[\bar{u}_o,\bar{v}_o]=[0,1]$, respectively. Thus, using the superposition principle, we can express the topographic corrections of the wind vector components as

$$\delta_t\bar{u} = \frac{1}{2h}(\bar{u}_o\frac{\partial}{\partial x}\lambda(1,0) + \bar{v}_o\frac{\partial}{\partial x}\lambda(0,1)),$$

$$\delta_t\bar{v} = \frac{1}{2h}(\bar{u}_o\frac{\partial}{\partial y}\lambda(1,0) + \bar{v}_o\frac{\partial}{\partial y}\lambda(0,1)). \tag{2.9}$$

Note, that the most time consuming part of the problem - the computation of $\lambda(1,0)$ and $\lambda(0,1)$ - can be made off-line.

In the next step of the procedure the calculation of thermal effect is done. Let $\Omega_1 \subset \Omega$ be the area occupied by the city $(\partial\Omega_1 \cap \partial\Omega=\Phi)$, and let Θ_1, Θ_o denote the ground temperatures in Ω_1 and on $\partial\Omega$, respectively (compare Fig.2.2b). Then the ground temperature field in Ω can be expressed as

$$\Theta_g = \begin{cases} \tilde{\Theta}_g & \text{in} \quad \Omega \setminus \Omega_1, \\ \Theta & \text{in} \quad \Omega_1, \end{cases} \tag{2.10}$$

where $\tilde{\Theta}_g$ is the solution of the boundary-value problem

$$\begin{aligned} \Delta\tilde{\Theta}_g &= 0 \quad \text{in} \quad \Omega \setminus \Omega_1, \\ \tilde{\Theta}_g &= \Theta_o \quad \text{on} \quad \partial\Omega, \\ \tilde{\Theta}_g &= \Theta_1 \quad \text{on} \quad \partial\Omega_1. \end{aligned} \tag{2.11}$$

Let us denote by $\Theta_g(1)$ the temperature field (2.10) corresponding to the boundary conditions $\Theta_o=0$, $\Theta_1=1$. Then, using the superposition technique applied in the former step, we can easily show that, in general,

$$\Theta_g = \Theta_o + (\Theta_1 - \Theta_o) \cdot \Theta_g(1) \quad \text{in} \quad \Omega. \tag{2.12}$$

Taking advantage of parametrization (2.12) the function $\Theta_g(x,y)$ can be obtained by a single solving of (2.11) for a given geometry of the city.

In order to find the components of the thermal wind velocity, existence of such a potential ϕ is assumed [1], that

$$\delta_\Theta \overline{u} = \frac{A}{H} \cdot \frac{\partial\phi}{\partial x}, \quad \delta_\Theta \overline{v} = \frac{A}{H} \frac{\partial\phi}{\partial y},$$

where ϕ is the solution of the following Dirichlet problem

$$\begin{aligned} \Delta\phi &= (\Theta_g - \overline{\Theta}_g) \quad \text{in} \quad \Omega, \\ \phi &= 0 \quad \text{on} \quad \partial\Omega. \end{aligned} \tag{2.13}$$

Here A is an empiric coeficient depending on architectural structure of the city. It has been determined for some cities, see e.g. [5]. The averaged over Ω value of Θ_g is denoted by $\overline{\Theta}_g$. Since formula (2.12) holds also for average value, we have

$$\overline{\Theta}_g = \Theta_o + (\Theta_1 - \Theta_o) \cdot \overline{\Theta}_g(1).$$

The homogeneous boundary conditions in (2.13) allow us to apply parametrization again. Let $\phi(1)$ be the solution for $\Theta_g = \Theta_g(1)$ and $\overline{\Theta}_g = \overline{\Theta}_g(1)$. Then, any solution to (2.13) can be written as

$$\phi = (\Theta_1 - \Theta_o) \cdot \phi(1)$$

and the final form of thermal corrections is

$$\delta_\Theta \overline{u} = \frac{A}{H}(\Theta_1 - \Theta_o) \frac{\partial \phi(1)}{\partial x} ; \qquad (2.14)$$

$$\delta_\Theta \overline{v} = \frac{A}{H}(\Theta_1 - \Theta_o) \frac{\partial \phi(1)}{\partial y} .$$

Hence, the presented above procedure allows us to solve all the partial differential equations off-line. Computation of the wind field in each time interval of the model has been, therefore, reduced to some matrice summations.

The final form of the wind components is

$$\overline{u} = \overline{u}_o + \delta_t \overline{u} + \delta_\Theta \overline{u} ,$$

$$\overline{v} = \overline{v}_o + \delta_t \overline{v} + \delta_\Theta \overline{v} ,$$

where the corrections are expressed by (2.9) and (2.14).

The algorithm has been tested on a set of data representing various topographical configurations, geometric contours of a city heat island and meteorological conditions. A simple, illustrative case is presented in Fig.2.2.

3. THE REAL DATA EXPERIMENT

The URFOR model has been tested on the case of Warsaw Metropolitan Area for the real emission data and selected meteorological episodes of January and August 1978. The test calculation consisted in generating a number of short - term forecasts of sulfur dioxide concentration. The region of interest containing the city itself and its surroundings (a square 40 km x 40 km) is shown in Fig.3.1. It was discretized by a square mesh with the spacing $h=1$ km. The same discretization was used in all space - depending input data.

The available emission sources consisted of 5 big power plants or heavy industry factories and 83 intermediate or small heating plants and industrial sources. Furthermore, there were distinguished 65 area sources given in aggregated form and representing 7 city quarters and 58 suburban districts. The intensities of all the sources were assumed to be independent of time. The spacial distribution of a global emission field is shown in Fig.3.2 for discretization step $h=2$ km.

The wind field was calculated according to the algorithm presented in section 2.2. Since the terrain elevation was introduced with accuracy 5 m, the surface rougness was neglected with the exception to the city center where $z_o=5$ m. As the temperature difference between the city and its surroundings we assumed the average literature value $\Delta\Theta=2°C$.

The simulation was performed for a number of 48 - hours meteorological episodes, both for the winter and summer seasons of 1978. Here we have selected for illustration the episode described in Table 3.1

Table 3.1. The meteorological forecast

DATE	HOUR	H [m]	u_G [m/s]	u_G [m/s]	s -	α [mm/h]	v_d [cm/s]
7.01.78	00	700	0.69	3.94	-1	0.0	0.8
7.01.78	06	500	-3.86	-1.04	-1	0.0	0.8
7.01.78	12	1250	-1.93	-0.52	-1	0.0	0.8
7.01.78	18	550	-4.23	-4.23	-1	0.0	0.8
8.01.78	00	680	-5.36	-4.50	-1	0.0	0.8

The time interval $DT=6$ h was determined by the available frequency of meteorological forecast. The sufficient accuracy of computation was obtained for the time step of numerical integration procedure $\tau=DT/18=0.3$ h.

The SO_2 concentration field generated by the model (for space discretization h=2 km) at the succesive hours of simulation is shown in Fig.3.3.

The results of simulation were compared with the measured values of SO_2 concentration at the selected points of the city [7]. Although the observations were fragmentary and related only to maximal day-average values - it is possible to indicate a correlation between the prediction and the observations. It is concerned, however, with extremal concentration values rather then its spatial distribution (compare [7]). For this reason, the future precise evaluation of the model quality requires the complex spatial and temporal observations of SO_2 concentration covering the forecasting period.

The model admits more disagregated description of emmission field than it was applied in the presented scenario. Treating a city quarter as an elementar area source causes that all the city represents nearly homogeneous area source. It is also possible to introduce the tranportation network as a set of linear sources.

The two-dimensional averaging of transport - diffusion process requires a special treatment of high emission sources. It is known that the maximal pollution caused by such a source appears at some distance, depending on the source parameters and the meteorological conditions. This effect was obtained by a special procedure that respectively modifies a current emissior field (compare the position of "9" s in Fig.2.2 and Fig.3.3).

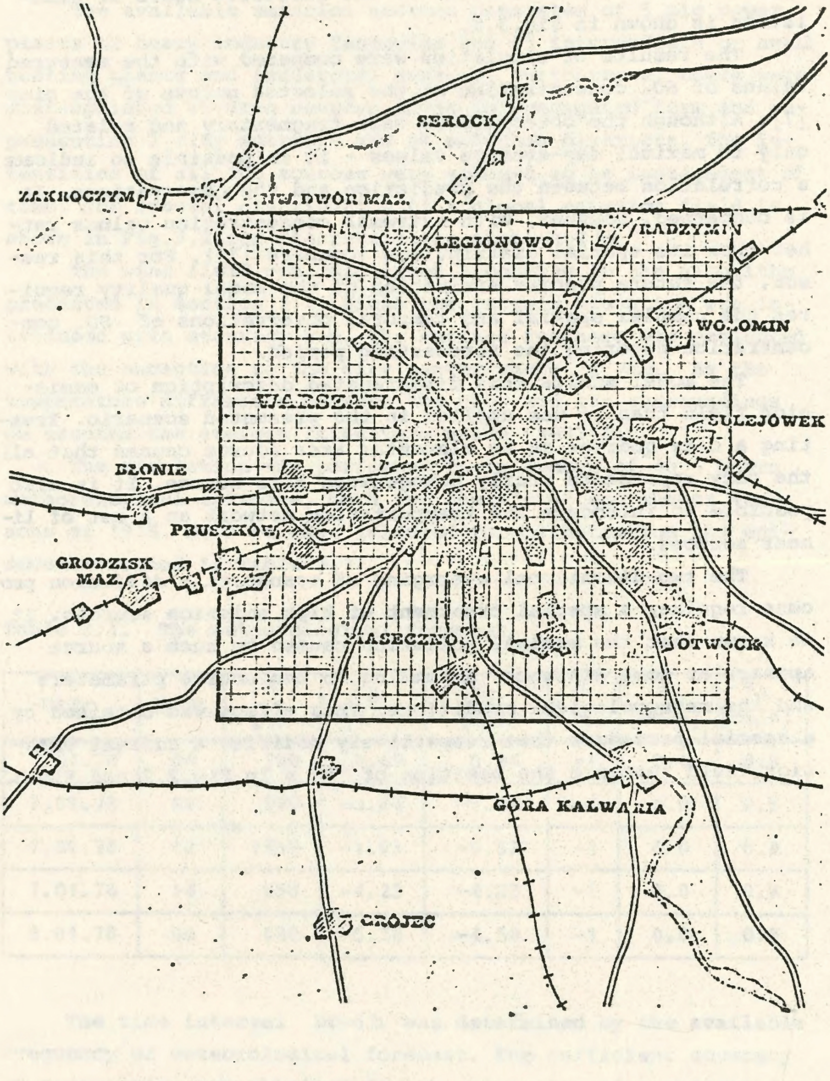


Fig.3.1. Region of simulation. Discretization

0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0
0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	6	0	0	9	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	4	3	1	0	1	0	0	0	0	0
0	0	0	0	0	4	1	2	3	1	6	0	3	1	0	0	0	0	0	0	0
0	0	0	0	0	0	7	0	0	1	0	1	0	0	0	0	0	0	0	0	0
0	0	1	0	0	0	0	1	1	0	0	1	0	0	0	4	0	0	0	0	0
0	0	0	0	0	0	0	1	0	3	0	0	9	0	0	0	0	0	0	0	0
0	0	0	0	2	2	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1
0	0	0	5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	5	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	5	0	0	5	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	2	0	0	0	0	0	0	0	3	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

LEGEND [$\mu g/m^2 s$]

0: 0.000 - 2.663
1: 2.663 - 4.527
2: 4.527 - 7.696
3: 7.696 - 13.08
4: 13.08 - 22.24
5: 22.24 - 37.81
6: 37.81 - 64.27
7: 64.27 - 109.2
8: 109.2 - 185.7
9: 185.7 - 315.8

Fig. 3.2. A map of the averaged emission field

SIMULATION HOUR 06; AVERAGE WIND [m/s] : u_o=-2.26, v_o=-1.24

0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	3	1	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	1	2	1	0	0	0	0	0	0	0	0	0	0	0	0	0
0	1	1	1	1	1	1	2	1	0	0	0	0	0	0	0	0	0	0	0	0
1	0	1	1	0	1	0	0	2	1	1	0	1	0	0	0	0	1	0	0	0
0	0	1	1	2	3	5	7	1	2	2	1	1	1	0	0	0	1	0	0	0
3	3	4	5	6	7	7	9	4	2	3	1	1	1	1	0	0	0	0	0	0
5	5	6	7	7	8	7	9	6	1	2	3	2	1	0	0	0	0	0	0	0
4	5	5	6	5	6	2	0	4	3	4	4	4	2	1	2	0	0	0	0	0
4	4	4	5	5	6	6	5	5	5	6	5	5	3	2	0	0	0	0	0	0
5	5	5	6	6	6	7	5	6	5	6	0	4	3	2	3	0	0	0	0	0
4	4	5	5	6	6	7	5	5	5	3	8	2	3	2	4	0	0	0	0	0
4	4	4	5	5	5	5	6	6	7	7	9	3	3	3	4	2	0	0	0	0
3	3	3	5	5	5	6	6	7	8	9	9	7	1	4	2	2	0	0	2	1
3	3	4	6	5	5	6	6	7	8	8	5	3	3	2	1	0	1	1	3	0
3	4	4	5	3	3	5	6	6	6	4	4	2	2	1	0	0	1	2	1	0
4	3	3	1	1	2	3	4	4	3	4	1	2	0	0	0	1	1	1	0	0
2	1	1	1	1	1	2	2	2	3	6	1	0	3	0	0	0	0	0	0	0
1	0	1	1	2	2	3	3	4	5	6	3	0	5	0	0	0	0	1	0	0
0	0	1	2	2	3	3	4	5	5	5	4	3	5	2	0	0	0	3	0	0
1	1	2	3	4	4	4	4	5	4	4	4	2	0	0	0	2	2	3	0	0

LEGEND [μg/m^3]

0: 0.000 - 2.646
1: 2.646 - 4.498
2: 4.498 - 7.648
3: 7.648 - 13.00
4: 13.00 - 22.10
5: 22.10 - 37.57
6: 37.57 - 63.88
7: 63.88 - 108.6
8: 108.6 - 184.6
9: 184.6 - 313.8

SIMULATION HOUR 12; AVERAGE WIND [m/s] : u_o=-1.61, v_o=-0.81

0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	1	3	1	0	0	0	0	0	0	0	0	0	0	0	0
0	0	1	1	1	1	2	1	0	0	0	0	0	0	0	0	0	0	0	0	0
0	1	1	1	1	1	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0
1	1	1	1	1	1	0	3	2	1	1	0	1	0	0	0	1	1	0	0	0
1	1	1	1	2	2	4	0	2	2	2	1	1	1	1	1	1	1	0	0	0
1	2	3	4	5	6	7	7	2	2	2	1	1	1	1	0	0	0	0	0	0
4	4	5	6	7	7	8	9	5	2	2	3	2	1	1	0	0	0	0	0	0
5	6	6	7	7	8	8	8	6	3	5	5	4	3	1	2	0	0	0	0	0
6	6	6	7	7	7	7	4	5	6	6	5	5	3	2	1	0	0	0	0	0
6	6	6	6	7	7	7	6	6	6	6	5	4	3	2	3	0	0	0	0	0
6	6	6	6	6	7	7	6	5	5	4	0	3	3	3	4	1	0	0	0	0
5	5	6	6	6	6	5	5	5	6	6	7	3	3	4	4	2	0	0	0	0
5	5	5	6	5	5	5	6	6	7	8	9	5	3	4	2	0	0	1	2	1
5	5	6	6	5	5	6	6	7	8	9	9	6	1	3	1	1	1	2	2	0
5	5	6	6	5	6	6	7	8	8	8	5	3	2	1	1	1	2	2	0	0
5	5	5	5	5	6	7	7	7	7	5	3	2	0	1	1	1	1	1	0	0
5	4	5	5	6	6	6	6	6	5	6	2	1	3	1	1	1	1	0	0	0
4	4	5	5	5	6	5	5	5	6	6	3	3	5	1	0	0	0	1	0	0
4	4	5	5	5	5	5	5	5	5	5	4	4	4	2	0	1	2	3	0	0
4	4	4	4	4	4	5	5	5	4	4	3	3	1	1	1	2	3	2	0	0

LEGEND [μg/m^3]

0: 0.000 - 1.431
1: 1.431 - 2.433
2: 2.433 - 4.136
3: 4.136 - 7.031
4: 7.031 - 11.95
5: 11.95 - 20.32
6: 20.32 - 34.55
7: 34.55 - 58.73
8: 58.73 - 99.83
9: 99.83 - 169.7

Fig. 3.3. The forecasted SO_2 concentration field

SIMULATION HOUR 18;   AVERAGE WIND [m/s] : u_o=-1.98, v_o=-3.02

```
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 3 1 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 2 1 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 2 2 2 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 1 0 0 1 0 0 0 0 0 0 0 0 1 0 0 0
0 0 0 0 1 2 4 3 1 2 1 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 1 6 0 2 1 2 0 0 0 0 0 0 0 0 0 0
0 0 1 1 4 2 5 7 2 2 0 3 2 1 0 1 0 1 0 0 0
0 1 0 2 5 5 5 9 2 3 4 4 4 2 1 2 0 0 0 0 0
1 1 3 3 4 7 6 7 4 3 6 3 4 3 2 0 0 0 0 0 0
1 2 4 5 5 8 8 8 7 2 5 5 4 3 2 3 1 0 0 0 0
2 3 4 6 6 7 7 5 5 5 5 0 4 3 2 5 1 0 0 0 0
3 4 5 6 7 8 8 6 4 5 5 8 3 3 2 3 1 0 0 1 0
4 5 6 6 7 6 5 5 5 5 3 9 3 3 3 3 2 0 0 2 1
4 5 6 7 7 6 5 4 5 5 7 8 3 2 4 4 2 0 0 1 1
5 5 6 6 6 5 5 4 5 3 9 8 6 1 3 1 0 0 1 2 0
5 5 6 6 6 4 4 4 5 5 8 8 6 1 3 1 0 0 2 2 0
5 5 6 6 5 4 4 4 4 7 8 6 3 4 2 0 0 1 1 0 0
6 5 5 5 4 4 4 4 5 7 8 5 2 5 1 0 0 1 3 0 0
5 5 5 4 3 3 4 5 7 7 7 4 2 3 0 0 0 1 4 0 0
6 5 4 4 4 4 5 6 7 7 6 4 4 4 2 0 2 1 0 0 0
```

LEGEND [μg/m³]

0: 0.000 - 2.181
1: 2.181 - 3.708
2: 3.708 - 6.304
3: 6.304 - 10.72
4: 10.72 - 18.22
5: 18.22 - 30.97
6: 30.97 - 52.65
7: 52.65 - 89.50
8: 89.50 - 152.2
9: 152.2 - 258.7

SIMULATION HOUR 24;   AVERAGE WIND [m/s] : u_o=-2.61, v_o=-3.37

```
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 3 1 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 2 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 1 1 1 2 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 2 0 1 1 0 0 0 0 0 0 0 0 1 0 0 0
0 0 0 0 1 2 4 1 3 1 1 0 0 0 0 0 0 0 0 0 0
0 0 0 1 0 2 6 1 0 1 2 0 0 0 0 0 0 0 0 0 0
0 0 1 2 3 1 5 2 7 2 0 3 2 1 0 1 0 1 0 0 0
0 1 0 3 4 4 4 4 9 3 4 4 4 2 1 2 0 0 0 0 0
0 2 2 3 4 6 7 4 7 3 6 3 4 3 2 0 0 0 0 0 0
1 3 4 4 4 7 8 6 7 6 4 5 3 2 2 3 0 0 0 0 0
2 3 4 5 6 6 7 6 8 6 4 0 4 3 2 5 1 0 0 0 0
3 3 4 6 7 6 7 6 5 5 5 8 3 3 2 3 0 0 0 1 0
4 4 5 6 7 6 7 6 4 4 3 9 3 3 2 1 1 0 0 2 1
4 5 6 7 7 6 6 5 5 6 5 8 3 4 3 4 2 0 0 1 1
4 5 6 6 6 6 5 4 5 7 8 7 6 2 2 0 0 0 1 2 0
5 6 6 6 6 5 5 4 5 7 8 8 6 0 3 0 0 1 2 2 0
5 6 6 6 5 4 4 5 6 6 7 4 3 4 2 0 0 1 1 0 0
6 5 6 5 4 4 4 6 7 7 7 4 2 5 0 0 1 1 2 0 0
6 5 5 4 4 4 4 6 7 7 7 4 1 3 0 0 1 1 4 0 0
6 5 5 4 4 5 6 6 7 6 6 5 2 3 2 0 1 0 0 0 0
```

LEGEND [μg/m³]

0: 0.000 - 1.789
1: 1.789 - 3.042
2: 3.042 - 5.171
3: 5.171 - 8.790
4: 8.790 - 14.94
5: 14.94 - 25.40
6: 25.40 - 43.19
7: 43.19 - 73.42
8: 73.42 - 124.8
9: 124.8 - 212.2

for Warsaw Metropolitan Area (Jan. 7th 1978)

4. THE EMISSION CONTROL

Basing on the forecasting model URFOR, the real time emission control problem for the system of sources covering the area was formulated. A general idea of controling consists in minimizing an environmental damage by redistributing the production (emission) among the set of selected sources, according to the meteorological situation.

Let us introduce the functionals depending on concentration of pollutant and production levels respectively:

$$J(\underline{u}) = \int_{\Omega} \int_{0}^{T_h} r \max^2(0, c(u) - c_d) d\Omega dt$$

$$J_p(\underline{u}) = \int_{0}^{T_h} \sum_{i=1}^{N} \beta_i (u_i - u_i^o)^2 dt$$

$$\underline{u} = (u_1, \ldots, u_N)^T$$

where $c(u)$ is the solution of the state equation

$$\frac{\partial c}{\partial t} + w \cdot \nabla c - K_H \cdot \Delta c + \sigma c = Q + \sum_{i=1}^{N} \chi_i F_i(u_i) \quad \text{in} \quad \Omega \times [0, T_h] \tag{4.1}$$

with the boundary-initial conditions (2.1)-(2.2).

The optimization problem is then formally stated as follows: Minimize the cost functional

$$I(\underline{u}) = \alpha_{11} J(\underline{u}) + \alpha_{12} J_p(\underline{u}) \tag{4.2}$$

under the constraints

$$g(\underline{u}) = \alpha_{21} J(\underline{u}) + \alpha_{22} g(\underline{u}) \leqslant K \tag{4.3}$$

$$\underline{u}_i \leqslant u_i \leqslant \overline{u}^i \qquad i=1,\ldots,N \tag{4.4}$$
$$-D_i \leqslant \frac{du_i}{dt} \leqslant D_i$$

$$\sum_{i \in N_j} a_{ij} \; u_i \geqslant b_j, \quad j=1,\ldots,M \qquad \bigcup_{j=1}^{M} N_j = N \tag{4.5}$$

The functions $\chi_i(x,y)$ in (4.1) describe the locations of controlled sources; $F_i(u_i,t)$ relate emissions to productions levels. The factor $r(x,y)$ in functionals is a region weight function, $c_d(x,y)$ - the admissible level of pollutant. The functional J_p constitutes the cost of deviation of production intensities from the economical values u_i^o. The inequalities (4.4) represent technological constraints, while (4.5) reflects demand requirements b_j imposed on homogeneous groups of plants - N_j.

The parameters α_{ij} in (4.2) make it possible to formulate a variety of optimization problems, ranging from minimization of environmental damage under limited resources ($\alpha_{ii}=1, \alpha_{ij}=0$) to minimizations of outlays with environmental constraint ($\alpha_{ii}=0, \alpha_{ij}=1$).

The problem is solved by a quickly convergent version [15] of the linearisation algorithm [16]. The numerical experiments involved a case of five controlled sources, based on modified data for Warsaw. An ilustrative result $\alpha_{ii}=1$, $\alpha_{ij}=0$ is shown in Fig. 4.1. Since the exact solutions are unknown, the result must confirm physical intuition. In our case the transitions of productions from sources {1,5} to {2,3,4} in the first 6 hours, and then from {5} to {1} satisfy this condition.

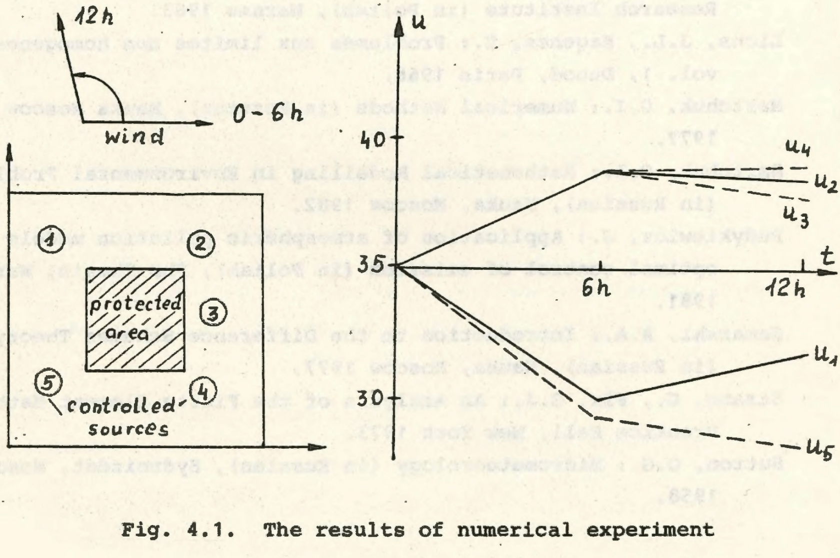


Fig. 4.1. The results of numerical experiment

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DISCUSSIONS

Paper by L. Kairiukštis

Discussion participants: A. Straszak, J. Owsiński, K. Polenske, R. Espejo, L. Kairiukštis.

Most of the questions were asked in order to obtain extra explanation as to the notions and structures used throughout the paper. These questions, related e.g. to the place of regions in the systemic structure outlined or to implementation and application of the comprehensive view presented, are satisfactorily answered in the text at hand.

Apart from that a question was asked in what way it is intended to influence policy makers - i.e. national and local governments - in order to increase the understanding of problems at hand. The answer pointed out that the only practicable means was to provide information to these who are resposible for activities influencing the biosphere, to make them aware of reasonable constraints connected with such activities.

Paper by S. Ikeda

Discussion participants: A. Kochetkov, K. Polenske, L. Kairiukstis, S. Ikeda.

Several explanations were asked for. First: the empirical basis for the input/output model - at the national level according to the international standard breakdown, i.e. approx. 200 categories, and at the regional level more aggregate tables synthesized from otherwise available information. Some activities, like fish processing industry, were left out because of lock of adequate data. Some other ones, like private sector investments, are treated through aggregates.

As to the formal side of the model development, it is provided by the three-year contract from the Ministry of Science and Education, and the model developers hope for an extension of another three year period in order to complete the work.

Paper by P. Holnicki and A. Żochowski

Discussion participants: T. Vasko, K.P. Moeller, D. Boekemenn, A. Straszak, R. Bolton, P. Holnicki.

In response to questions one of the co-authors explained that: the control variable of the model was production level of a given factory, the sources located outside of the area considered had not been accounted for because of lack of appropriate data, and: location and time variables had not been used as control ones in the model, although this could be done within the same model structure. Many of the model features resulted directly from specifications made when accepting the contract.

On the policy side, in view of the preliminary nature of the model no experience could as of then be gained on the enterprises' reactions. In fact, the model builders were responsible solely for model development.

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