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STRATEGIC REGIONAL POLICY

Paradigms,Methods, Issues and Case Studies

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PART I

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III. TECHNOLOGICAL AND SCIENTIFIC ISSUES

A LINEAR-PROGRAMMING APPROACH TO SOLVING INFEASIBLE RAS PROBLEMS **

by

Karen R. Polenske and William H. Crown

The major purpose of this paper is to present a new way in which interregional-trade flows can be adjusted to marginal control totals through the combined use of two estimation techniques: the RAS and linear programming procedures.* In order to present a context for the description of the new adjustment procedure, a critical review of the literature on the application of the RAS procedure to input-output and interregional-trade tables is provided in the first section of this paper. There is a large and growing literature on the use of nonsurvey techniques to estimate input-output tables and interregional trade flows. This literature reflects the delicate compromise that analysts are attempting to make between the cost of data collection and the accuracy of the models with which they work. Initially, most studies of nonsurvey techniques concerned the adjustment of national input-output models to reflect structural changes over time and to obtain approximations of regional input-output tables. More recently, with the rapid growth in the number of models with multiple regions and the resulting need to collect data or to make estimates of interregional linkages, some efforts have

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***The new theoretical method presented in the last part of this paper is based upon the Ph.D. dissertation by Möhr (1975). Selected excerpts are taken from his thesis, with alterations being made to his original text to adjust for the discontinuities introduced by presenting only specific sections of his thesis, as well as to bring the references up to date.**

** revised version, as of February 1985 (eds.).

been made to use nonsurvey techniques for the adjustment or estimation of interregional-trade flows.

Of the numerous nonsurvey techniques that have been developed, the RAS procedure is one of the most widely used for input-output tables. It is so named because it involves the estimation of an unknown matrix from a known matrix, A, subject to row and column control totals R and S, respectively. In the first section of this paper, the literature on the RAS procedure is reviewed in terms of its use for national input-output tables, regional input-output tables, and interregional-trade tables. In the second section, the mathematical properties of the RAS procedure are provided, with emphasis on its use for interregional-trade tables, and a linear-programming procedure for determining critical cells in the tables is set forth. In the third section, applications of the proposed RAS and linear-programming procedures to the multiregional input-output (MRIO) data are discussed. In the final section, major conclusions of the research are presented, with emphasis on the similarities and differences in the use of the RAS procedure for interregional trade tables versus its use for input-output tables. The use of the RAS procedure to make the adjustments to the data in the interregional-trade tables raise a new set of issues (most of which have never been raised before in the literature), which will be presented and discussed thoroughly throughout this paper. In the next section, an extensive review is given of the major issues that have arisen in terms of the use of the RAS procedure for input-output and interregional-trade tables.

LITERATURE REVIEW

This literature review is limited to only one of several nonsurvey techniques: the RAS procedure. In a very good survey paper, Lecomber (1975) provides a detailed description and critique of this and four alternative adjustment procedures used to update and/or project national input-output data. The other four procedures were developed by Almon (1968), Friedlander (1961), Matuszeski, Pitts, and Sawyer (1964), and Theil (1967). The RAS procedure has the virtue of providing estimates that are as accurate as, or in some cases more accurate than, the other procedures. It also requires a minimal amount of data, is one of the simplest procedures to apply, and preserves the signs of the initial elements. Because of these attributes and because, as far as is known, none of the other four adjustment procedures have been used on either regional input-output or interregional trade data, this review will focus primarily on the RAS procedure and the differences and similarities between its application to input-output (national and regional) and interregional-trade data.

The roots of the RAS procedure can be traced from Deming and Stephan (1940), Leontief (1941), and Stone and Brown (1962). While working at the U.S. Bureau of the Census, Deming and Stephan first proposed the simple iterative procedure--later known as the RAS procedure--as a means of adjusting the then upcoming 1940 Census of Population data. It is worthwhile noting that they were the first to use the letters r and s to designate the multipliers. The results obtained from the iterative procedure were almost identical to those obtained from the least-squares method. The 6x4 matrices of artificial population characterisitics they used as illustrations in their article had marginal totals that were less than one percent different from the control totals, had positive entries for all cells in the matrix, and had entries in the matrix cells that ranged from a minimal value of 120 to a maximum value of 10,476, all of which helped them to achieve a final solution after only two complete iteration cycles. Each of these points is relevant for the later discussion.

At about the same time, Leontief (1941, pp. 61-64) also proposed using the method of least squares and discussed two sets of row and column multipliers-- "the productivity coefficient of commodity k" and "the productivity coefficient of industry k", respectively, that were hypothesized to account jointly for input-output coefficient change. The ideas of Leontief and Deming and Stephan were not widely noticed for some twenty years until Stone and Brown (1962, pp. 294-296) proposed the so-called RAS model for use (instead of a least-squares method) on constrained biproportional matrix-estimation problems. Its first important application in the United Kingdom was at the Cambridge Growth Project, by Stone, Bates, and Bacharach (1963), who also were the first to use the term "RAS method."

Work on the constrained biproportional-matrix problem has been conducted by analysts in several disciplines, as summarized in the article by Lecomber (1975). Here, only the research most directly related to the economics and regional-science disciplines will be reviewed. Most of the RAS research has been concentrated on making adjustments to input-output tables; only a few analysts have been concerned with the application of the technique to other types of matrices, usually transportation-flow matrices. Both types of work will be reviewed here.

Adjustment of National Input-Output Tables

In the early 1960s, two adjustment procedures were developed for use in updating national input-output tables: the RAS procedure by Stone and Brown (1962) and a linear-programming procedure by Matuszewski, Pitts, and Sawyer (1963, 1964). Both were viewed as having a decided advantage over previously used single-constraint balancing procedures in that both procedures were used not only to balance demand adjustments for each row, but also to balance supply adjustments for each column--the matrix to be adjusted was therefore subject to two sets of constraints.

Table 1 contains a summary of some of the characteristics of the main national studies presented in this paper. Because analysts have not been systematic in presenting information on their calculations and results, the comparisons are limited to those characterisitics that are presented most consistently in the published work. This review, as noted earlier, will focus primarily on studies of the RAS procedure. However, because the linear programming (LP) procedure illustrates some factors in addition to those relevant to the RAS procedure that are considered when updating and projecting input-output data and because a LP procedure is incorporated into the interregional-trade adjustment procedure explained later in this paper, a brief discussion of the Matuszewski, Pitts, and Sawyer (1963, 1964) work is presented here.

Matuszewski, Pitts, and Sawyer were concerned with updating from 1949 to 1956 the 42-sector Canadian interindustry table. The adjusted table was used to predict 1957 outputs. They developed a linear programming (LP) method of updating to allow for discontinuities in technical change by adjusting only some of the coefficients in the direct-coefficient matrix. The LP procedure was selected by Matuszewski, Pitts, and Sawyer because they did not believe the proportionality assumption was appropriate, and because they wanted a procedure that required minimum amounts of supplemental data and only a few calculations. The intermediate (demand and input) values were used as constraints on the estimated flows. In addition, the estimated flows were not allowed to increase by more than 100 percent nor decrease by more than 50 percent from the base-year flows, an arbitrarily determined contraint. Their

TABLE 1

ADJUSTMENT OF NATIONAL INPUT-OUTPUT TABLES

Authors(s)	Matrix Characteristics: Country	Size	Base Year	Estimated Year	Comments
Stone, Bates, and Bacharach (1963)	United Kingdom	31-sector	1954	1960	Used RAS procedure. Estimated Make and Mix matrices separately. Projected 1960 data to 1966. No actual data were available for error estimation.
Matuszewski, Pitts, and Sawyer (1963)	Canada	42-sector	1949	1956	Used a linear programming procedure. Used the 1956 estimated table to predict 1957 output. Variation between the base and estimated flows was constrained.
Paelinck and Waelbroeck (1963)	Belgian	21-sector	1953	1959	Used the RAS and a modified RAS procedure. The "actual" 1959 table was also partly estimated by industry experts.
Schneider (1965)	United States	6-sector 24-sector	1947	1958	Compared results of the RAS and linear programming procedures.
Tilanus (1966)	Netherlands	27-sector (?)	1948 1951 1954 1957	1951 1954 1957 1960	Compared results of the RAS and a statistical correction method. Number of sectors not clearly indicated.
Lamel, Richter, Teufelsbauer, and Zelle (1974)	Norway	51-sector	1964	1968	Compared results of the RAS and Almon procedures. Developed a new updating procedure called DYMOD.
Davis, Lofting, and Sathaye (1977)	United States	50-sector	1963	1967	Compared results of the RAS and linear programming procedures.
Miernyk (1975)	United States	69-sector 74-sector	1963	1967	Used the RAS procedure. Number of sectors not clearly indicated. Only a brief description of results.

original objective function was set up in terms of an absolute value.

Because this formulation created a nonlinear procedure, the problem was converted to a linear procedure in two steps. First, the coefficients were transformed into flows through the use of output weights from the projected year, and, second, a set of nonnegative values was introduced, with at most one of the new variables being nonzero for any given interindustry flow. In their procedure, bounds were not imposed until violated, which prevented extreme changes in the coefficients and increased the number of nonzero-valued variables in the basic solution. The procedure will be critiqued later. The final formulation closely approximated the standard transportation linear-programming problem in which there are capacity restrictions on all routes.*

At about the same time, Stone, Bates, and Bacharach (1963) at Cambridge University applied the RAS procedure that had been set forth by Stone and Brown (1962) to update the 32-sector 1954 input-output table for the United Kingdom to 1960. The updated table was then used to estimate a projected 1966 input-output table. The RAS procedure was actually used twice. In the first round, an RAS calculation was done on the Make matrix, which is a sector-by-commodity table showing the composition of output of each sector. An identical type of calculation, referred to as LUL, was done to the Mix matrix, which is a commodity-by-sector table showing the inputs used by each establishment. This step was a bit more complicated by the fact that marginal

*It should be noted that their approach was more advanced in terms of the theoretical formulation and the empirical testing than Ghosh's (1964) LP formulation. He suggested that information from planning authorities, engineers, businesspeople, and so on could be used to place upper and lower boundaries on each of the interindustry flows. When the estimated flows violated the boundaries, he used the LP procedure to adjust the flows.

controls were only available for 1958, rather than for 1960; the r and s multipliers obtained with those controls were then projected to 1960 in order to estimate the final Make matrix. In the second round, the RAS procedure was used to project the 1960 coefficients to 1966.

Some points that will be relevant for the later discussion should be noted. First, the marginal totals were not actual data, but were estimated by a residual method. Second, the projection from 1960 to 1966 was made with the use of the square of the r and s multipliers, because the difference in years between 1960-1966 was the same as between 1954-1960. When the square is used, the assumption is that there is an exponential trend in the values of the individual elements of the matrix, an assumption that is questioned by Lecomber (1975, pp. 10-11) because it causes the column sums of the matrices to augment at an increasing rate, which is not plausible. Third, there was no input-output matrix for 1960; therefore, no errors between the estimated and an actual matrix could be calculated (refer to Table 1). The use of the RAS procedure was therefore justified by Stone, Bates, and Bacharach (1963, pp. 30-32) based upon the testing of the accuracy of the procedure that had been conducted by Paelinck and Waelbroeck (1964) with Belgian input-output data.

One of the important early tests of the RAS procedure was conducted by Paelinck and Waelbroeck (1964), using the 21-sector Belgium input-output tables for 1953 to estimate a 1959 table. The data in the estimated table were then compared with 1959 data compiled by direct observation. In the final set of comparisons, after prespecifying six dominant coefficients, only one of the 270 nonzero elements was "in error" by more than one percent (Paelinck and Waelbroeck, 1964, p. 474). At least four reasons seem to have contributed to this apparent low error: (1) The table presented by the authors (1963, p. 111) shows that 238 of the 270 nonzero coefficients changed by only 0.0-0.5 percent between 1953 and 1959, that only 17 of the coefficients changed by more than 1.0 percent, and that 132 of the coefficients showed no change. (2) Actual vectors of total gross output, total intermediate output, and total intermediate input from the 1959 survey table, rather than estimated vectors, were used as control totals. (3) The values removed from the table and control totals when the six coefficients were deleted were actual 1959 data, rather than estimated figures. (4) The 1959 table was not a survey table, but was itself partially based upon estimates made by industry experts. This latter point leads Barker (1975) to conclude that the RAS procedure with supplemental information does no better than industrial expertise. He goes on to say

> It is misleading to imply that RAS as a method can do almost as well in providing an up-to-date table as taking a census of production. Rather it does almost as badly, judging from British evidence, as using partial information on particular flows. (Barker, 1975, p. 66)

As to the errors that did occur in the Belgian tests, Stone, Bates, and Bacharach (1963, p. 31) indicate that there were three causes: (1) The input-output table was highly aggregated. (2) Variations occurred in the substitution effect along the rows, contrary to an assumption that such variations are not present. (3) A ripple effect resulted in which an erroneous RAS estimate in one of the elements can generate errors throughout the rest of the table, especially if the cells in the matrix are connected (if all, or most of the cells in the matrix contain nonzero entries). Additional discussion of the errors can be found in the Paelinck and Waelbroeck (1963), Barker (1975), and McMenamin and Haring (1973) articles.

In 1965, Schneider tested the RAS and LP procedures by using each method to update the U.S. 1947 input-output table to 1958. One conclusion of his was that the LP procedure was not as good as the RAS procedure in providing accurate input coefficients. He reasoned that

> The linear programming method was formulated to emphasize disproportionality in technical change, but there would seem to be little economic justification for concentrating the adjustments only on the large transactions. The possible growth of small industries is largely ignored. (Schneider, 1965, p. 63)

Schneider found that outputs were predicted more accurately, but the individual coefficients were estimated less accurately, with the LP procedure than with the RAS procedure. He explained this by the fact that if both final demands and total outputs change proportionally between any two years, there is no inherent advantage in altering the input-output coefficients by any method. In this special case, as long as the interindustry flows are consistent with the control totals for the year to be predicted, total output can be predicted accurately. Such proportional changes, however, seem to be exceptions, rather than the rule. If only final demand and total output change proportionally, the RAS method may yield worse results than naive methods, such as the final-demand blow-up method. Schneider concluded that the RAS method provides more accurate results than the LP method if the degree of absorption per unit of output and the degree of fabrication per unit of input change proportionally for each industry.

Actually, the LP and RAS procedures for adjusting input-output tables were developed for different reasons. The Canadians wanted to adjust coefficients for an intermediate year in order to predict total demand (output) in a later year. Their attention was therefore focused on the accuracy of the aggregate results. Stone and his colleagues were interested in developing a technique that could be used not only to project the individual coefficients, but also to predict them accurately for the intermediate year. Their focus therefore was on the accuracy of each coefficient, the hope being that the RAS updating procedure could extend the life of compiled input-output tables by as much as ten years.

When Davis, Lofting, and Sathaye (1977) compared the LP and RAS methods using the 1963 U.S. table as the base to estimate the 1967 table, they also found that the LP procedure was not as accurate as the RAS one. Miernyk reports that tests of the RAS procedure on the same 1963 and 1967 U.S. data at the Regional Research Institute resulted in a mean error of 127 percent for the original RAS procedure and a 126 percent error for a modified RAS procedure (1976, p. 49).

A few other studies of the RAS procedure at the national level are worth mentioning at this point. Lecomber (1969) argues that the RAS method should be used when only one table exists. He maintains that if two or more tables exist, other statistical estimation techniques, such as those employed by Johansen (1968) for Norway or the least-squares method used by Almon et al. (1968) that incorporates exogenous information concerning the statistical confidence placed on each interindustry coefficient, provide more accurate results. In 1966, Tilanus determined that projections made with the RAS procedure were no better than those obtained by a statistical correction method (SCM) applied to an original matrix. He also noted that the "RAS procedure takes more computing time than the SCM due to iteration and inversion" (1966, p. 121). Convergence occurred after 20 iterations.* Lamel et al. extended the RAS method for use on the sum of elements of submatrices

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*It is interesting to note that because of the now-primitive type of computers available in 1966, the iterations required 2.5 hours of computer prime time, and the inversion of the 27-sector table required another 1.0 hours, even though time was saved because 40 percent of the matrix elements were zero (Tilanus, 1966, p. 121).

in the projected-year table.

It should be noted that the analyst is not explicitly adjusting for price changes when the RAS method is used, and price adjustments were not made independently in most of the studies reviewed here. Despite all the caveats associated with the employment of the RAS procedure, it is still being used for national input-output work. The personnel at the Australian Bureau of Statistics, for example, have adopted the RAS procedure as a means of helping update their national input-output tables on an annual basis (Gretton and Cottrell, 1979). They note that the procedure is recommended for use by the United Nations (1973).

There is sufficient variation in the types of data used and the modifications made to the RAS procedure to make any comparisons of the errors in the different studies difficult. (Refer to Table 1.) Some authors, such as Stone, Bates, and Bacharach (1963), could not make error tests, because actual data were not available. Others, such as Schneider (1965), used counts of errors and other means of providing information on the errors. A few, such as Miernyk, used statistical measures, including mean errors, to assess the results. The main point to be stressed here is that the very low errors of measurement obtained from the Belgian study were the results of incorporating independent estimates for specific cells in the table. The RAS procedure was then applied only to the remaining cells. As was mentioned earlier, even the data in their actual 1959 table were partially based upon estimates by industry experts. If the RAS procedure is used without making independent estimates for some of the cells, the errors in specific cells can become very large. The review of the regional input-output studies provided in the next section will show that many more of the regional analysts than national analysts have tested for errors, but once again the methods and results vary widely.

Adjustment of Regional Input-Output Tables

The basic issue at the national level has been the accuracy of the RAS procedure as a means of extending the life of input-output tables. While this is also an issue for regional input-output tables, the application of the RAS procedure at the regional level has raised the issue of the distinction that exists between the use of the RAS procedure for national and regional input-output tables. National input-output coefficients reflect technical input relationships relating to the production function for the sector. The production function may vary over time as a result of changes in relative prices, technology, or product composition (sometimes called product-mix), none of which need occur proportionally for each input used by a sector. Regional input-output coefficients, on the other hand, reflect the productive capacity of the region.* Malizia and Bond (1974, p. 358) maintain that

> Basically, regional production capacity can be expected to act on both absorption and fabrication in a manner that will encourage biproportional regional coefficient change.

Such reasoning leads to an a priori assumption that the RAS method may work better for regional than for national input-output tables. The results to date, however, are mixed.

The RAS procedure has been used to construct a number of regional input-output tables, and the results have been tested against alternative

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*Round (1983) has added confusion to the terminology by referring to regional input-output coefficients as regional trade coefficients. In this paper, the term "regional input-output coefficients," will be used to refer to state or city tables of input-output coefficients, and the term "trade coefficients" will be used to refer to region-to-region commodity-flow tables of coefficients, a practice that has been well-established in the literature.

estimation techniques. Table 2 contains a summary of some of the characteristics of the main regional input-output studies reviewed in this paper. As with the national studies, there is a woeful lack of consistency in the information reported. The major published studies are reviewed here.

The first regional studies were by Czamanski and Malizia (1969), Haring and McMenamin (1973), and Morrison and Smith (1974). In each case, the regional input coefficients were estimated based upon national coefficients. Morrison and Smith were the first to use the RAS technique for an urban area. They applied the technique to data from a 1968 national table for the United Kingdom to estimate a 19-sector 1968 table for Peterborough, England. Because the national and regional data were for the same year, no adjustments had to be made for temporal relative price changes; however, adjustments should have been made for differences between regional and national prices. They used five methods to evaluate eight alternative techniques of obtaining the estimated data. The five methods were: mean absolute difference, correlation coefficient, mean similarity index, information index, and chi-square, while the eight alternative techniques were: simple location quotient, purchases-only location quotient, cross-industry location quotient, modified cross-industry location quotient, logarithmic cross-quotient, modified cross-quotient, modified logarithmic cross-quotient, supply-demand pool, and RAS. In each case, the RAS procedure ranked first, ahead of the other seven methods. They incorporated some survey information into the calculations using the RAS method and were therefore not surprised that the RAS procedure ranked first, but they did make special note of the "degree of superiority" of the method.

McMenamin and Haring (1974) used an approach that differed in two

TABLE 2

ADJUSTMENT OF REGIONAL INPUT-OUTPUT TABLES

Author(s)	Matrix Characteristics: Region (Country)	Size	Base Year and Region	Estimated Year	Errors	Comments
Czamanski and Malizia (1969)	Washington State	43-sector 36-sector 28-sector	1958 United States	1963	Mean percentage errors ranged from 39 to 81 percent. Values of the information coefficient ranged from 0.779 to 54.262.	For best results, excluded tertiary sectors and used field-survey information for primary sectors and sectors in which region specialized.
Smith and Morrison (1974)	Peterborough, England	19-sector	1968 United Kingdom	1968	Estimated mean similarity index; mean absolute difference; chi-square; information content; correlation coefficient.	Compared RAS with five other non-survey methods. Concluded that RAS procedure was "overwhelmingly" superior.
Haring and McMenamin (1973)	Southern California	n.a.	1963 United States		n.a.	Referred to in 1974 study by McMenamin and Haring. No details on study provided.
McMenamin and Haring (1974)	Washington State		1963 Washington State	1967	Errors of estimation no better or worse using Haring-McMenamin method than using usual RAS procedure.	Constrained data to sum to total gross output and total gross outlays.
Malizia and Bond (1974)	Washington State	52-sector 43-sector 27-sector 22-sector	1963 Washington State	1967	Mean errors were 124 (52-sector)-regional; 133 (43-sector)-technical; 105 (27-sector)-regional; 118 (22-sector)-technical.	Measured divergence between predicted and survey-based estimates; gross output, intermediate demand, and value added.
Hewings (1977)	Washington State, Kansas State	29-sector	1963 Washington State, 1965 Kansas State	1963, 1965	Obtained absolute and percentage differences in output.	Tried to determine possibility of using survey-based regional tables from one state to estimate coefficients in a second state.
Hinojosa (1978)	Washington State	27-sector	1963,1967, 1972 Washington State	1963,1967, 1972	Compared direct and direct and indirect coefficients; made distribution of frequencies. Lowest mean error was 34 percent.	Took 11 to 21 iterations to converge. Evaluated relative differences between 12 pairs of matrices.
Hewings and Janson (1980)	Hypothetical	5-sector	n.a.	n.a.	Fixing coefficients provided little improvement in multiplier estimation.	Addressed some of the issues raised by Thumann (1978) and investigated effect of predetermining certain coefficients

TABLE 2

ADJUSTMENT OF REGIONAL INPUT-OUTPUT TABLES
(continued)

Author(s)	Region (Country)	Matrix Characteristics Size	Base Year and Region	Estimated Year	Errors	Comments
Hewings and Syverson (1980)	Washington State	49-sector	1963, 1967, 1972	1963 1967 1972	Percent differences between estimated and observed outputs ranged from 0 to plus or minus 50 percent. No statistical tests were performed.	Required 7-10 iterations to converge. Modified RAS procedure used, by identifying a priori inverse-important coefficients.
Horrigan, McGilvray, and McNicoll (1980)	Scotland	46-sector	1973 United Kingdom	1973	Estimated mean absolute difference, Euclidean metric difference, similarity index, sum of chi-square distribution terms, absolute mean relative difference, information index, and, correlation coefficient.	Tested RAS against location quotient, adjusted cross-industry location quotient, logarithmic cross industry location quotient, and commodity balance. Concluded that RAS procedure was best.
Butterfield and Mules (1980)	Western Australia	27-sector	1958-59 Australia	1958-69	Nonparametric test of column sums, regression analysis, chi-square test, mean absolute deviation, standardized mean, absolute deviation. Latter error was 33 percent.	Tested RAS with intermediate sector and total transaction table. Tested against naive estimate.

respects from previous RAS studies.* First, they used the technique to estimate a 1967 table for the state of Washington based upon the full-survey 1963 table for the state, rather than on the national 1963 U.S. table. Second, they constrained the data in the table to sum to total gross output and total gross outlays, rather than to intermediate outputs and inputs. They concluded that the "error comparisons of the technical coefficients obtained by the H-M [Haring-McMenamin] method are no better (or worse) than those from the [usual] RAS method at this level of aggregation" (McMenamin and Haring, 1974, p. 204).

The period 1973 to 1974 resulted in yet another analysis of the use of the RAS method at the regional level, this time by Malizia and Bond (1974). They seem to have used the same RAS method as the nonmodified approach used by McMenamin and Haring for the Washington state 1963 and 1967 tables. Using the 1963 Washington table as a base, they constructed four sets of tables for 1967: 52-sector and 27-sector regional coefficient tables and 43-sector and 22-sector technical coefficient tables. The regional coefficients were based upon intraregional (gross flows) only, while the technical coefficieents were based upon total purchases (gross flows plus domestic imports). They made two major conclusions: First,

> . . . that the RAS method, used without additional exogenous information on interindustry flows for the projection year, is not powerful enough to generate satisfactory forecasts of interindustry coefficients. (Malizia and Bond, 1974, p. 360)

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*McMenamin and Haring (1974) refer to their 1973 unpublished study for which they constructed an input-output table for Southern California, using the RAS procedure to adjust national coefficients to the regional level, but they were unable to test their estimated data because no complete actual table existed for Southern California.

Over the four-year time span, their mean errors were greater than 100 percent for each of the four sets of tables. Second, they concluded that the RAS method was better for regional coefficient tables and for aggregate tables than for technical coefficient tables and disaggregate tables. This latter conclusion, however, must be viewed in the context that the mean errors for the disaggregated tables were 124 (52 sector, regional) and 133 (43 sector, technical) percent compared with 105 (27 sector, regional) and 118 (22 sector, technical) for the aggregated tables. Their explanation of this latter conclusion is that the RAS method is the most viable ". . . when input-output relations are viewed as allocative functions based on market factors such as local capacity rather than as productions functions based on technical relations" (Malizia and Bond, 1974, p. 362).* Given the large size of all of the mean errors, the viability of the technique may be in question regardless of the level of aggregation.

Since these early applications of the RAS technique at the regional level by Czamanski and Malizia (1969), Haring and McMenamin (1973), McMenamin and Haring (1974), Malizia and Bond (1974), and Morrison and Smith (1974), many regional analysts have written papers expanding upon the tests. Hewings (1977), in an article severely critiqued by Thumann (1978), discussed the issue of the accuracy of the RAS procedure in estimating individual coefficients compared with the accuracy of the marginal vector. Thumann's issues concerning the Hewing's study included the need to use alternative statistics (such as the Chi-square) to measure the errors, the use of sectoral gross outputs as the basis for comparison, and whether or not the LP technique

*The reader is also referred to Ghosh (1964, pp. 111-113) for his discussion on the question of technical factors being expressed by the production function, while market factors are expressed as allocations.

should have been tested as a possible alternative to the RAS procedure. Hewings and Janson (1980) responded to many of Thumann's criticisms and extended the RAS procedure by fixing selected coefficients a priori. They reached two major conclusions: (1) Their results seemed to substantiate Jensen's (1979) proposal that the largest 10 percent of the coefficients are the most important to consider when updating regional input-output tables; therefore, it is useful to make prior estimates of them before applying the RAS procedure. (2) The two sets of constraints (intermediate demand and supply) must be accurate.

In another study in 1980, Hewings tested Jensen's (1980) statement that it is important to focus attention in input-output analyses on major aggregates rather than individual coefficients. Hewings applied the RAS procedure to the 49-sector Washington state tables for 1963, 1967, and 1972, first identifying the inverse-important coefficients. Although no statistical measures of errors were obtained, he concluded that "the contribution of the inverse important parameters to vector accuracy in cases where the RAS algorithm is applied provides minimal additional benefits" (Hewings, 1980, p. 5).

Hinojosa (1978) used the RAS procedure to estimate two 1972 matrices for Washington state, the first with the 1963 tables as the base and the second with the 1967 tables as the base. When he compared the differences in the actual versus estimated direct and direct and indirect coefficients, the mean error of estimation was at best 34 percent. Hinojosa is one of the few authors to indicate the number of iterations required to obtain convergence--a point that will become important in the later discussion of the use of the RAS procedure for interregional trade data. Convergence occurred with 11 to 21 iterations.

The RAS procedure was overwhelmingly the most accurate of six methods tested by Harrigan, McGilvray, and McNicoll (1980). They estimated a table for Scotland, based upon the 1973 input-output table for the United Kingdom. The other five methods tested were the location quotient, adjusted cross-industry location quotient, logarithmic coss-industry location quotient, and commodity-balance techniques. Although the size of the tables is not provided, they are exceptionally thorough in their analysis. They used seven different measures to test the errors: mean absolute difference, Isard/Romanoff Euclidean metric difference, similarity index, sum of Chi-Square distribution terms, absolute mean relative difference, information index, and correlation coefficient. The RAS procedure in each case outranked the other five estimation techniques. Even so, the errors were not inconsequential.

Butterfield and Mules (1980) compared a 1958-59 input-output table for Western Australia with tables estimated from the 1958-1959 Australian national table. They compared both the RAS procedure (including the Haring-McMenamin method of adjusting the entire transactions matrix) and a naive estimate. The tables had 27 sectors. They made a nonparametric test of column sums to detemine whether there was consistent overestimation or underestimation. They also did a regression analysis, chi-square test, mean absolute deviation, and standardized mean absolute deviation. In the latter case, the error was 33 percent (1980, p. 306), which is approximately the same as the lowest levels of errors obtained by other regional analysts. Once again, the level of accuracy of the RAS procedure is not very high, although it is, in general, better than for the naive method and the Haring-Mcmenamin method of adjusting the entire transactions table.

As is noted by Hewings and Janson (1980, p. 847), one of the largest uses of input-output models is for the analysis of economic impacts. The use of error measures such as the mean percentage error for individual coefficients may not, therefore, be as relevant as estimations of errors for the outputs, income, or employment calculated from the coefficients. Even so, the errors from an unadjusted RAS procedure are sufficiently large to cause any analyst to use information obtained with the procedure only with extreme care.

Some general conclusions can be made from the studies reviewed here. In the cases where the RAS procedure was compared with other techniques, the use of the RAS procedure was determined to result in more accurate estimates than other nonsurvey techniques. Such a statement, however, does not imply that the use of the RAS procedure has solved the problem of constructing up-to-date input-output tables or that the procedure will produce similar results when applied to noninput-output data. First, the most accurate estimates have generally been made by taking a regional (national) input-output table for one year and applying the RAS procedure to estimate a table for the same region (nation) for a later (4-10 year) period. The estimates are less accurate when a regional input-output table is estimated from a national one or where the years of the base and the estimated table are far apart. A relatively recent base-year table for the same region is therefore still preferred. Second, the column sums derived from the inverse of an RAS-estimated table are usually more accurately estimated than the individual coefficients in the table.

Third, individual coefficients have mean errors that, on the average, are 30 percent or more even under the best of conditions. These errors while less than those obtained from other nonsurvey techniques are still very large. An unambiguous, objective means has yet to be determined for evaluating the difference between the flows (coefficients) in the matrices. Methods employed have included frequency distributions of the errors in individual coefficients (classified by size), mean similarity index, information content, Chi-square statistic, mean absolute difference, and regression.

Fourth, in measuring the errors, the analysts vary as to whether or not they account for the number of cells for which little or no change occurred between the base and the estimated year in the actual data. They also vary as to whether it is the accuracy of individual coefficients or accuracy of estimates made with the coefficients that is important. In discussing the results, the analysts also generally fail to mention other factors such as the size of the matrix, the span of years over which the estimates are being made, and possible structural changes in the economy, such as recessions or inflation, that may affect the results.

Fifth, and of greatest importance for the current paper, the input-output tables on which the tests have been conducted have very unique structures, such as dominant diagonals and only a few nonzero entries. This has meant that the RAS procedure not only converges, but converges rather quickly (say, after 5-20 iterations). Matrices with data for other economic activites, such as interregional trade flows, do not always have these properties; therefore, the questions of existence, uniqueness, convergence, and speed of convergence become important issues for the use of the RAS procedure with these other data.

Adjustment of Interregional-Trade Tables

The final set of literature to be reviewed concerns the use of the RAS procedure to adjust and/or project interregional-trade data. Table 3 contains a summary of the characteristics of the four international/interregional

TABLE 3

ADJUSTMENT OF INTERREGIONAL-TRADE TABLES

Author(s)	Matrix Characteristics: Country	Size	Base Year	Estimated Year	Error	Comments
Bénard (1964)	Capitalist Industrialized Countries	5-region	1953 1957	1957 & 1960	Mean square relative error was 17 percent for four-year period and 20 percent for seven-year period.	Convergence achieved in six to seven iterations.
Waelbroeck (1964)	Entire World	9-region	1938	1948, 1951-52, & 1959-60	n.a.	No information on convergence provided. Only 4 of 81 cells were zero.
Grandville, Fontela, and Gabus (1968)	No Test	--	--	--	--	Proposed use of Dutch method of Virtual Exports and a gravity model as potential alternatives.
Möhr (1975)	United States	51-region	1963 unadjusted trade data	1963 adjusted	n.a.	Used a linear-programming procedure to help identify critical zero cells in the trade-flow tables.

n.a. = not available
-- = not applicable

studies that have used the procedure. The first three studies related to adjusting international-trade figures; adjustments of interregional-trade figures with the RAS procedure has been the focus only of the research conducted by Möhr (1975).

Two European studies were published in 1964, one by Bénard and the other by Waelbroeck. As noted in both articles, they conferred on their work. Bénard's research was conducted at the Centre d'Etude de la Prospection Economique a Moyen et Long Termes (Ceprel). Export and import data for the capitalist industrialized countries were combined into five groups for each of three years: 1953, 1957, 1960. Using these data and the RAS procedure, Benard updates a 5x5 matrix of international trade flows from 1953 to 1957 and from 1953 to 1960. For the four-year period, the mean square relative error was 17 percent; for the seven-year period, it was 20 percent. He states that convergence was achieved in six to seven iterations.

Waelbroeck (1964) tested a modified RAS procedure on trade flows for the entire world (including socialist countries), with the flows grouped into nine regions. Using 1938 trade flows as a base, he estimated 1948, 1951-52, and 1959-1960 flows, using the RAS procedure. The analysis was conducted as part of an attempt to measure the effects of the Common Market. No errors of estimation were calculated, and no information is given on the number of iterations required for convergence to occur. It should be noted, however, that because of the highly aggregated nature of the data, only 4 of the 81 cells contained zeros, and the nonzero cells for 1938 only varied in value fom 6 to 1534; consequently, convergence probably occurred very rapidly.

Grandville, Fontela, and Gabus (1968) proposed a modified RAS procedure, which they called RAST, that had three multipliers instead of two. It is specified as

$$a^{gh}_{i(t)} = a^{gh}_{i(o)} * R^{g}_{(t-o)} * S^{h}_{(t-o)} * T_{i(t-o)}$$

where

a_i^{gh} = trade flow from country g to country h of commodity i

R^g = row multiplier for row g

S^h = column multiplier for column h

T^i = total trade multiplier for commodity i

(t-o) = year t minus year o.

No empirical results are provided. They state that a "retrospective verification" of the procedure was made by CEPREL (1968, p.596). This probably refers to the research Bénard discusses in the article just reviewed. Their RAST method appears to be identical to the Kouvei (1965) method briefly discussed in Lecomber's article (1975, p. 17). Kouvei published his study in a CEPREL bulletin. Grandville, Fontela, and Gabus also presented two other methods that could be used to estimate the trade flows: the Dutch Method of Virtual Exports and a gravity model.

As noted earlier, there are several similarities and differences in the use of the RAS procedure for international/interregional-trade tables versus its use for input-output tables. Only a few of these have been noted previously in the literature. Special problems were encountered when the interregional trade data for the multiregional input-output (MRIO) model were being estimated (Polenske, 1980, pp.189-207). The most important are discussed here.

The sources of data for the two sets of data, of course, differ. Regional input-output and interregional trade tables are both assembled primarily from census data; however, in the United States, the census data sources of the two sets of data differ. The data in the interregional trade tables are obtained from the Census of Transportation and other transportation publications, while the data for the regional input-output tables are obtained mainly from the various Censuses of Agriculture, Construction, Manufactures and, Trade. The regional production and consumption totals for each sector in the input-output tables, therefore, differ from those in the interregional trade tables. In order for the multiregional input-output (MRIO) model to operate, the two sets of control totals must be identical. For the 1963 model, it was decided to adjust the data in the interregional trade tables to make the controls from those tables match the ones from the regional input-output tables, because the interregional trade data assembled seemed to be less reliable than the input-output data assembled for each state.

The 1963 MRIO interregional-trade tables appear to be the first case in which the RAS procedure sometimes failed to achieve convergence or in which the convergence occurred extremely slowly. As will be discussed later in the Mathematical Issues section, Bacharach (1970) proved only the necessary, but not the sufficient, conditions for the iterations of a biproportional matrix problem to converge. Previous to the present study, it appears that analysts have not encountered cases of nonconvergence because of the special mathematical properties of the input-output matrices and the small size of the interregional-trade matrices. In the literature, therefore, almost no reference is made to the convergence problem, or if a reference is made, the assumption is made that convergence will always occur. For similar reasons, only a few analysts (such as Tilanus, 1963; Stone, Bates, and Bacharach, 1963; Bénard, 1964; and Hinojosa (1968)) have noted the number of iterations required for convergence to occur. The number they cite varies from about six to about twenty. Both convergence and the speed of convergence are issues that emerge as important ones for the adjustment of interregional-trade data in the MRIO model.

MATHEMATICAL ISSUES

Early applications of the RAS procedure revealed that it sometimes failed to converge or converged very slowly. To investigate the convergence properties of the RAS procedure, several analysts began intensive studies of the closely related model of biproportional-matrix adjustment in the mid-1960s and early 1970s. The results of this research were a series of theorems concerning the existence, uniqueness, and convergence properties of the biproportional-matrix model. Underlying each of these theorems was the basic finding that the cases in which the RAS procedure failed to converge were due to the existence of certain zero partitions in the matrix to be adjusted. Because zero elements are unaffected when multiplied by a scalar, any adjustments to a matrix necessary to make it conform to a prespecified set of row and column controls must be made to the nonzero elements of the matrix. If the pattern of zeros in the original matrix is such that the positive elements cannot be adjusted to sum to the given row and column constraints, the RAS procedure will not converge.

Despite the enormous progress made in the 1960s and 1970s regarding the nature of convergence failures of the RAS procedure, at least two major issues remained unanswered. First, the theorems offered little guidance as to how the problem-causing zero partitions of the matix could be determined. Not all patterns of zeros in the matrix would create convergence failures--only certain ones. Thus, how were the theorems of existence, uniqueness, and convergence to be applied if the critical zero partitions were not known? Second, even if the critical partitions were known, the theorems offered no guidance as to how the matrix could be adjusted to enable successful application of the RAS procedure.

In the remaining sections of this paper, the major theorems regarding the existence, uniqueness, and convergence properties of the RAS model are summarized, and two previously unpublished theorems developed by Möhr (1975) are presented. The theorems by Möhr provide a means for making the theorems developed during the 1960s and 1970s operational. The first of these shows how the existence conditions for the RAS model can be reformulated for evaluation in a linear-programming model. In the event that a particular RAS adjustment is infeasible, Möhr (1975) has shown that a linear-programming formulation can also be used to identify the critical zero partitions. His second theorem shows how optimal augmentations to these critical zero partitions can be made to ensure a solution to the RAS procedure. Finally, the theorems developed by Möhr are applied to a problem of estimating consistent interregional trade flows previously thought to be infeasible using the RAS procedure.

Previous Theoretical Research

As noted above, researchers sought to address the problem of convergence failures in application of the RAS procedure by utilizing developments in the theory of biproportional-matrix estimation. To see why they did so, consider the following definition of the existence problem in the model of biproportional-matrix change:

> Given a nonnegative m x n matrix A with positive row and column sums, an m x m diagonal matrix u > 0, and an n x n matrix v > 0, such that the sums of u and v are equal; does there exist a nonempty set of nonnegative mxn matrices {E} such that

$$a_{ij} = 0 \Longleftrightarrow e_{ij} = 0 \qquad i \varepsilon I,\ j \varepsilon J$$

$$\sum_{i \varepsilon I} e_{ij} = v_j, \qquad j \varepsilon J$$

$$\sum_{j \varepsilon J} e_{ij} = u_i, \qquad i \varepsilon I$$

where I and J are sets of rows and columns, respectively, containing one or more zeros.

The similarity of the RAS and the biproportional-matrix estimation problems can be seen by comparing the above definition with that of Bacharach's description of the RAS procedure:

> Starting with the given matrix A, each row is first multiplied by a scalar that will make the row sum equal to the row constraint; each column of the resulting matrix, A-, is next multiplied by a scalar that will make its sum equal its constraint. This gives a matrix A- that serves as a starting point for the next iteration (1970, p. 46).

It is apparent from comparing the two definitions that the RAS procedure is a way of iteratively approximating a solution to the more general problem of biproportional-matrix change. Thus, if a solution to the RAS model exists, it is in the set of solutions {E} of the biproportional-matrix problem. Alternatively, any conditions that are necessary and sufficient for the solution of the model of biproportional-matrix change are also necessary and sufficient for the solution of the RAS procedure.

The literature tracing the theoretical development of the existence, uniqueness, and convergence properties of biproportional-matrix problems has been summarized by Macgill (1977, 1979) and will not be reviewed again here. However, to provide the basis for the subsequent discussion, the major theoretical results from the literature will be outlined.

Interestingly, identical necessary and sufficient conditions are required for the existence, uniqueness, and convergence of solutions to the biproportional-matrix problem. This is because the conditions for convergence of an iterative biproportional-matrix procedure are the same as those for the existence of a solution to a biproportional-matrix problem regardless of its method of solution. Moreover, if such a solution exists, it will be unique

(Bacharach, 1965; 1970).

As discussed by Macgill, the above conditions were derived by many analysts--often using completely different methodological approaches. According to Macgill (1977), the earliest known general convergence proof was developed by Gorman (1963). This unpublished proof uses a mathematical programming approach to develop convergence conditions for biproportional-matrix problems. Bacharach (1965) was the first to publish a proof of convergence. His approach was based on establishing global convergence of the iterative biproportional-matrix procedure. Herrman (1973) arrived at the same existence, uniqueness, and convergence properties by analyzing the problem from a topological perspective. More recently, Macgill (1977) provided a more streamlined alternative to the earlier proof by Bacharach (1965, 1970) and also developed existence, uniqueness, and convergence properties for problems where only some of the row and column control totals are prespecified.

<u>Existence, Uniqueness, and Convergence Conditions</u>

Given a vector of row control totals, U, and column control totals, V, the existence, uniqueness, and convergence of a solution to the biproportional-matrix problem (and thus the RAS procedure) requires that:

$$\sum_i U_i = \sum_j V_j$$

and also that for every critical zero element a_{ij} in the base matrix A:

$$\sum_{i \in I} U_i \leq \sum_{j \in J'} V_j$$

$$\sum_{j \in J} V_j \leq \sum_{i \in I'} U_i$$

where I' and J' are the complements of I and J, respectively. The first conditions merely reflect the fact that the sum over all column and row totals must be equal for any matrix. The latter conditions are designed to allow the pattern of zero elements in {E} to be different from the pattern in A. They are necessary because in some instances entire sets of elements in the A matrix may be driven to zero, as explained, for example, in Bacharach's (1970, pp. 56-57) corollaries 4 and 5. To see this, consider the examples of feasible, just-feasible, and infeasible RAS problems depicted in Figures 1 through 3, respectively.

In Figure 1, an adjustment is sought to the prior matrix A with row totals, X, and column totals, Y, such that the new row and column totals will be U and V, respectively. After 500 iterations the RAS procedure results in the matrix A*. As discussed earlier, although the proper measure to be used for comparing A and A* is arguable, it does appear from visual inspection that A* is substantially different from A. By inspecting A*, it may be concluded that the RAS procedure tends to augment larger flows while driving smaller flows toward zero. However, this is not necessarily true. Rather, the tendency to augment some flows while decreasing others is a function of the relative sizes of the row and column multipliers. For example, the desired total for the first row of 299 is larger than the original row total of 280. Thus, the first-round multiplier for this row will be 299/280--which will clearly augment the size of the entries in the first row. Moreover, since the original and desired column totals are the same, the column multipliers will serve to dampen, but not reverse, the row adjustments. In a similar fashion the desired total for the last row of 10 will tend to decrease all of the entries in that row because it is smaller than the original total of 20.

Figure 1

FEASIBLE RAS-ADJUSTMENT PROBLEM

Unadjusted Matrix A				Original Row Totals X	Desired Row Totals U
90	0	95	95	280	299
5	101	2	2	110	105
5	101	2	2	110	106
0	18	1	1	20	10

Original Column Totals Y	100	220	100	100	520
Desired Column Totals V	100	220	100	100	520

Adjusted Matrix A^*

99.45	0	99.76	99.76
.2724	104.53	.1036	.1036
.2750	105.53	.1045	.1045
0	9.95	.0276	.0276

Note that in this example, all of the conditions for the existence and convergence of a solution to the RAS problem were satisfied. That is, the sum over all of the desired row totals equaled that of the sum over all of the desired column totals, and the sum of the rows and columns with at least one zero cell were in each case less than the sum of the remaining columns and rows, respectively.

In the example shown in Figure 1, these inequality constraints were as follows:

$$299 < 100 + 100 + 100$$

$$10 < 220 + 110 + 110$$

$$220 < 105 + 106 + 10$$

$$100 < 299 + 105 + 106$$

The need for the sum of the column and row totals to be equal in the adjusted matrix is apparent, because the grand total of any matrix must be equal to the sum of the marginal totals by definition. However, the need for the inequality conditions is less obvious. As Bacharach (1975) points out, their value becomes apparent in so-called boundary value problems. An example of such a problem is given in Figure 2.

The problem illustrated in Figure 2 is very similar to that shown in Figure 1 except that two of the inequality constraints on the row and column sums of the zero partitions are binding. These are:

$$300 = 100 + 100 + 100$$

$$221 = 105 + 106 + 10$$

The tendency for such binding constraints to drive certain elements to zero is indicated in the A* matrix in Figure 2. After 1,000 iterations, the six

Figure 2

JUST FEASIBLE RAS-ADJUSTMENT PROBLEM

	Unadjusted Matrix A				Original Row Totals X	Desired Row Totals U
	90	0	95	95	280	300
	5	101	2	2	110	105
	5	101	2	2	110	106
	0	18	1	1	20	10
Original Column Totals Y	100	220	100	100	520	
Desired Column Totals V	100	221	100	100		521

Adjusted Matrix A^*

99.7632	0	99.8982	99.8982
0.1178	105.002	.0447	.0447
.1189	106.002	.0451	.0451
0	9.996	.0119	.0119

elements in the lower right-hand corner of the matrix show a definite pattern of progressing towards zero. Thus, when the inequality constraints are binding, they imply that the only way to achieve convergence is to force a subset of the elements in A* to zero.

This tendency to drive certain elements of the A* matrix to zero is shown even more graphically in Figure 3. In this case, the RAS procedure is infeasible because:

$$301 \nless 100 + 100 + 100$$

After 1,000 iterations, it is clear that the problem will never fully converge. This is because there is no way to adjust the first row of A* so that the sum of its elements is equal to 301 without upsetting the column totals. Nevertheless, it clearly might be possible to apply the RAS procedure successfully to such a problem if the critical zero partitions could be augmented by arbitrarily small amounts. However, because not all zero cells affect the convergence of the RAS procedure, to make such augmentations efficiently would require being able to identify the critical zero elements. In the following section, two theorems developed by Möhr (1975) are presented that identify infeasible RAS problems and provide a means of transforming them into a form feasible for solution. These theorems are then successfully applied to an infeasible interregional trade-flow estimation problem.

EXISTENCE RESULTS FOR 11 TRADE-ESTIMATION PROBLEMS

As discussed in Polenske (1980), the multiregional input-output (MRIO) model requires that several data-consistency conditions be met--most notably the industry-specific consumption and production totals in the state input-output tables must be consistent with the interstate-shipment totals

Figure 3

INFEASIBLE RAS-ADJUSTMENT PROBLEM

	Unadjusted Matrix A				Original Row Totals X	Desired Row Totals U
	90	0	95	95	280	301
	5	101	2	2	110	104
	5	101	2	2	110	105
	0	18	1	1	20	10
Original Column Totals	100	220	100	100	520	
Desired Column Totals	100	220	100	100		520

⇓

Adjusted Matrix A^*			
100	0	100	100
0	104	0	0
0	105	0	0
0	10	0	0

from the corresponding interregional-trade tables for each commodity. Because the input-output and interstate-trade tables were estimated from different data sources, the RAS procedure was applied to achieve the necessary consistency. However, as reported by Polenske (1973, pp. 31-35, 180) and Rodgers (1973, p. 68) the RAS procedure failed initially to converge to a solution for 11 of the 61 interstate-trade tables.

The 11 troublesome commodities and the results of applying the existence conditions outlined above after 100 iterations of the RAS procedure are shown in Table 4.* Of the 11 trade tables thought to be infeasible initially, three were found to converge slowly. However, the remaining eight tables were all identified as having failed the necessary and sufficient existence conditions put forth above.

Having verified that the trade-flow tables for these eight commodities did indeed fail the existence conditions for the RAS procedure, the problem became one of finding a successful completion that would satisfy these conditions. In other words, how could a small number of augmentations to the original trade matrix be determined such that a solution to the RAS procedure would exist?

Completion of Infeasible Biproportional-Matrix Problems

A particular RAS problem has a solution if and only if it is possible to alter the set of nonzero entries in the prior matrix, A, such that the row

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*Before all of these existence tests were run, small numbers were added to any zero elements on the main diagonal of the prior matrices, because in the MRIO model the row sums and the column sums of the trade tables have to be strictly positive. In other words, the mathematics of the MRIO model require that every commodity is produced and consumed in every region at some positive level; this level can, of course, be small [Bon, 1975].

Table 4

APPARENT CONVERGENCE FAILURES OF THE RAS PROCEDURE

	Commodity			Failure to Meet Existence Conditions
No.	Traded Commodity	Input-Output Number	Title	
1	5	IO-6	Nonferrous metal ores mining	Yes
2	6	IO-7	Coal Mining	Yes
3	8	IO-9	Stone & clay mining & quarrying	Yes
4	9	IO-10	Chemical & fertilizer mineral mining	Yes
5	10	IO-13	Ordnance & accessories	Yes
6	18	IO-21	Wooden containers	Yes
7	22	IO-25	Paperboard containers & boxes	No
8	25	IO-28	Plastics & synthetic materials	Yes
9	30	IO-33	Leather tanning & industrial leather products	No
10	36	IO-39	Metal containers	No
11	49	IO-52	Service industry machines	Yes

Source: John M. Rodgers, 1973. State Estimates of Interregional Commodity Trade, 1963. Lexington, MA: D. C. Heath and Company, Lexington Books, p. 68.

and column sums equal the given column totals. This can be expressed by the following "closed" linear-programming formulation.

THEOREM. An RAS problem has a solution iff and only if a solution X exists to the following linear program:

$$\text{Maximize} \quad \sum_{i \in S} \sum_{j \in S} X_{ij} \tag{1}$$

$$\text{subject to} \quad \sum_{i \in S} X_{ij} = V_j \quad \text{for } j = 1,2,\ldots,n \tag{2}$$

$$\sum_{j \in S} X_{ij} = U_i \quad \text{for } i = 1,2,\ldots,(m-1) \tag{3}$$

$$X_{ij} \geq \varepsilon \tag{4}$$

where $S = [(i,j) \mid a_{ij} > 0]$

ε = a small positive number.

PROOF. When a solution, X, to a linear program exists, the simplex algorithm is guaranteed to find it. By definition of the linear program and the RAS problem, any solution to the linear program is in the solution set of the RAS problem.

The value of the objective function of the linear program is constant for all feasible solutions: $\sum_j V_j = \sum_i U_i$. As a result, every feasible solution is also an optimal solution. The set of origin-destination pairs s = $[(i,j) \mid a_{ij} > 0]$ indicates that positive trade flows, X_{ij}, are defined if and only if elements a_{ij} of the prior matrix A are positive. Equations (2) and

(3) describe the (n + m - 1) equality constraints for the desired row and column totals. The (n + m)th constraint for U_m is left out because it is redundant. In terms of implementing the linear program, it is important not to include this last constraint because doing so can lead to infeasibility as a result of rounding errors. Without the constraint U_m, the rounding errors are collected in the elements that must sum to U_m, but may not do so precisely. Finally, all estimated trade flows, X_{ij}, corresponding to the prior trade flows, $a_{ij}>0$, are required to be positive at a level greater than or equal to ε, so that boundary solutions are ruled out.

The linear-programming formulation comprised of equations (1) through (4) provides an alternative procedure to the one given earlier and establishes the RAS existence conditions in a much more straightforward fashion than the traditional approach used by Bacharach (1965; 1970) and Macgill (1977), because the proof is based upon the well-established properties of linear programs. It is debatable whether the linear-programming existence tests are more efficient than those developed by the Bacharach and Macgill; however, if a particular RAS problem is found to be infeasible, it is possible to formulate a closely-related "open" linear program that will identify the critical zero partitions creating the convergence failtures. The form of this linear program is as follows:

$$\text{maximize} \quad \sum_{\text{all } i} \sum_{\text{all } j} c_{ij} x_{ij} \tag{5}$$

$$\text{subject to} \quad \sum_{\text{all } i} x_{ij} = v_j \quad \text{for } j = 1,\ldots,n \tag{6}$$

$$\sum_{\text{all } j} x_{ij} = u_i \quad \text{for } i = 1,\ldots,(m-1) \tag{7}$$

$$x_{ij} \geq c_{ij}\varepsilon \quad \text{for } i \in I,\ j \in J \tag{8}$$

where $a_{ij} > 0 \Longrightarrow c_{ij} = 1$

$a_{ij} = 0 \Longrightarrow c_{ij} = 0$

ε is a small positive number.

The coefficients, c_{ij}, in the objective function (5) have the effect of maximizing the objective function in terms of activities that are in positions of previously positive flows $a_{ij} > 0$. In other words, the constant coefficients, $c_{ij} = 1$, for all previous positive flows will tend to augment existing flows and hence introduce fewer new flows than a variable coefficient would do in its place. Some previously zero partitions, however, are allowed to enter the objective function by setting the zero a_{ij} values equal to an arbitrarily small value ε in the row and column constraint equations. Nevertheless, it should be noted that while the above formulation dampens the effect of the introduction of new trade flows into the estimated matrix, the number of these new activities is not minimized. To do so would require an integer programming problem of unmanageable size. Note also that for a feasible matrix problem, the set of new trade flows is empty because the value of the objective function is maximized only when all flows are accommodated in activities that correspond to positive flows in A. Using the set of new trade-flow positions identified by the open linear program, described by equations (5) through (8), it is possible to identify a sufficient completion to the original infeasible matrix-estimation problem.

<u>THEOREM</u>. Let P be the set of new trade-flow positions, identified by the open linear program, such that:

$$P = [(i,j)\ x_{ij} > 0 \text{ and } a_{ij} = 0] \tag{9}$$

then the application of the RAS procedure to the prior matrix A, with augmentations T at positions P has a solution in the closed linear program.*

PROOF. The RAS procedure with augmentations T at positions P has a solution in the closed linear program, because the RAS procedure only fails to converge when there are certain zero cells in the original matrix A. The augmentations ensure the convergence of the RAS procedure by transforming these zero cells into small positive cells.

At this point, it is important to realize that the two linear programs presented above can be combined conceptually in order to restrict the set of activities from which a sufficient completion is formed by the algorithm. The open linear program can be closed partially by excluding some of the activities as possible flows. In other words, it is possible to designate a subset of zeros in matrix A on a priori grounds as potential trade flows, to let the simplex algorithm attempt to find a feasible (and optimal) flow, and, thereby, to determine a set of positions, P, for augmentation. This half-open, or half-closed, linear program is, however, not necessarily feasible.

Application of the Linear-Programming Procedure

The open linear program was applied to the 1963 MRIO interregional-trade data for commodity 10-6, Nonferrous metal-ores mining, to demonstrate a successul completion to an infeasible trade-estimation problem. As is evident from Figure 4, 10-6 presents a particularly difficult completion problem because of the large number of zero cells (shown as white in the

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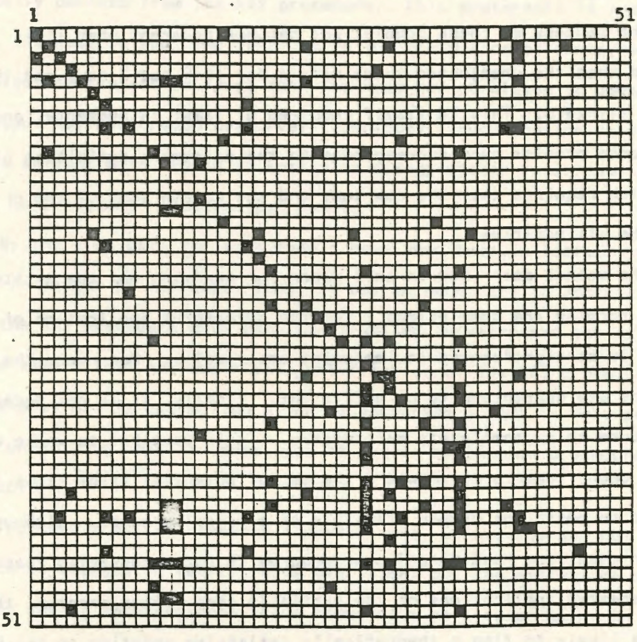
*Augmentations T take the form of a nonnegative m x n matrix; T is positive at positions P.

Figure 4

POSITIVE ELEMENTS IN IO-6, NONFERROUS METAL ORES MINING, PRIOR TO AUGMENTATION



zero cells

positive cells

figure). To solve this problem, the open linear program was run first for the insufficiently completed matrix-estimation problem (that is, some a priori insertions were made in the matrix) to determine an additional set of trade-flow positions. Then, the open linear program was run once more, using the original matrix without any completions. In the first run, 46 augmentations were required. In the second run, 34 new trade-flow positions were determined.

The fewness of these completions becomes apparent when it is considered that the unadjusted trade matrix for 10-6 had 8 rows and 15 columns that were all-zero. This in itself required at least 15 augmentations before the existence problem even corresponded to the existence definition given above, which requires that the row sums and the column sums of the prior matrix A be all positive.

The method described in this paper for carrying out the existence tests and forming the completions T for the successful application of the RAS procedure is straightforward to implement empirically. Moreover, the completions are guaranteed to be sufficient. Finally, it is not necessary for the procedure to be completely mechanistic. It is possible to write a partially open linear program where the set of potential trade flows is restricted to positions deemed plausible on a priori grounds. Although, in the latter case, the resulting linear program is not necessarily feasible, through repeated applications of the partially open linear program, the analyst is likely to find a theoretically satisfying solution to the RAS problem in an expeditious fashion. Regardless of the linear programming formulation used, it is important to recognize that the resulting trade flows are just estimates, and the amount by which they differ from the "true" flows can only be approximated by comparing these mechanically derived estimates with trade flows based upon survey research.

CONCLUSIONS

Numerous analysts, such as Miernyk (1975), have argued against the use of the RAS procedure for the estimation of changes input-output coefficients over time--preferring direct empirical estimates to estimates that are mechanically derived from the RAS procedure. This preference is indisputable. However, the expense and time involved in the production of survey-based input-output tables are so large that the need has persisted to update input-output tables through nonsurvey techniques. Such considerations are even more salient with regard to spatially disaggregated interindustry models, like the multiregional input-output model.

By the time data for such models have even been collected, in fact, they may no longer be current. Thus, it may be necessary to apply a technique such as the RAS procedure even when constructing survey-based tables. The extensive theoretical and empirical justification of the RAS procedure for biproportional-matrix estimation given by Bacharach (1970) is very persuasive when the underlying need for some type of matrix-estimation procedure is recognized.

The thrust of the research reported here, of course, was not the estimation of changes in coefficients over time, but the estimation of a consistent set of interregional-trade flows at a particular point in time. From the standpoint of the RAS procedure, these two types of problems are mathematically and conceptually identical. Necessary conditions for the successful completion of the RAS procedures were developed within the framework of a closed linear program. For those infeasible RAS problems that fail to meet these conditions, an open linear program was developed to determine the positions of all new trade-flow insertions that would be mathematically necessary and sufficient to complete the infeasible problem.

Two difficulties arise in terms of calling these the necessary and sufficient conditions for convergence to occur. First, the application of the open linear program only determines the critical cells, leaving the value to be inserted up to the discretion of the analyst. Convergence will occur if the required insertions are made, but it may occur very, very slowly if appropriate values are not selected for the insertions. The linear-programming results provide no guidance as to what the values should be. When there are many, very large matrices, as is the case with the 1963 MRIO data, the costs of the iterations require that the convergences should occur quickly. More research is therefore required to determine whether or not additional ways could be developed to help assess the value of the flows that will permit a fast convergence to occur. Second, some of the critical cells may represent regional flows for which the analyst knows no transaction has occurred. It was therefore suggested that such infeasible problems might be solved with the use of a partially-open linear program. This formulation allows the analyst to insert trade flows in the positions thought to be most theoretically plausible. While there can be no guarantee that these insertions will alone be sufficient for successful completion of the problem, the analyst can be certain of an expeditious solution by incrementally inserting values in the remaining cells of the completion set.

The "partially open" approach is preferable to the purely mechanical approach on both theoretical and empirical grounds. Clearly, if the analyst has reason to believe that a certain cell should contain a trade flow, it is important to override the workings of a mathematical procedure that may not introduce such a flow. Such judgments on the part of analysts have strong empirical support as well. Virtually every survey has found that the incorporation of additional information improves the accuracy of the results obtained (for example, Harrigan, McGilvray, and McNicoll, 1981; Round, 1984). When information can be gained by a calculated judgment on the part of the analyst, there is little justification for not overriding a mechanical procedure.

When the incorporation of additional information involves increased time and expense, the decision to include or omit it must be made within the context of the objectives and constraints of the study at hand. The choice between alternative techniques is dictated by the particular circumstances of each study--bounded, on the one hand, by simple mechanistic adjustment techniques and, on the other hand, by full surveys of regional economic activity.

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ADITIONAL EXHIBITS

EXHIBIT 1

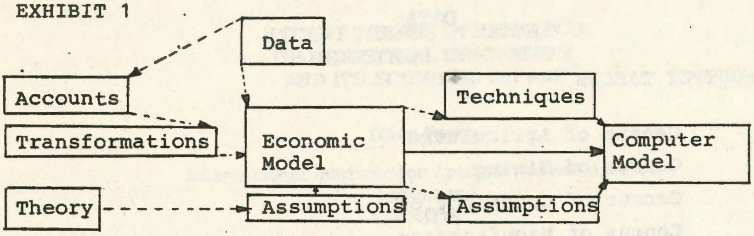
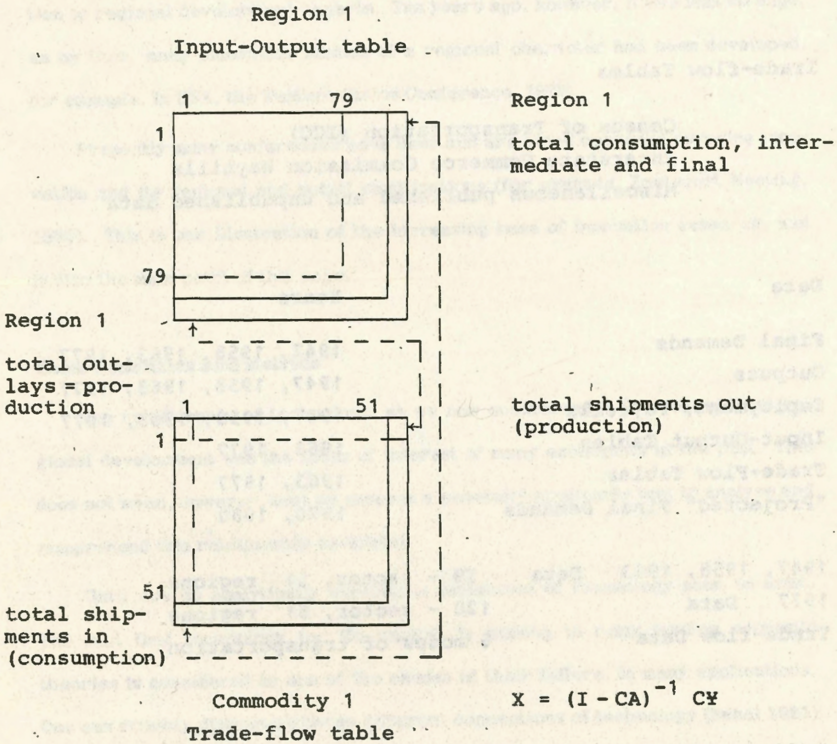


EXHIBIT 2



$X = (I - CA)^{-1} CY$

A - input-output coefficients

C - trade-flow coefficients

Y - final demand

X - outputs

EXHIBIT 3

DATA

INPUT-OUTPUT TABLES

Census of Agriculture
Census of Mining
Census of Construction
Census of Manufactures
Census of Wholesale and Retail Trade
Miscellaneous published and unpublished data

Trade-flow Tables

Census of Transportation (ICC)
Interstate Commerce Commission Waybills
Miscellaneous published and unpublished data

Data	Years
Final Demands	1947, 1958, 1963, 1977
Outputs	1947, 1958, 1963, 1977
Employment, Payrolls	1947, 1958, 1963, 1977
Input-Output Tables	1963, 1977
Trade-Flow Tables	1963, 1977
"Projected" Final Demands	1970, 1980

1947, 1958, 1963 Data	79 - sector, 51 regions
1977 Data	120 - sector, 51 regions
Trade-flow Data	6 modes of transportation

DISCUSSIONS

Paper by S. Dresch

Discussion participants: R. Bolton, P. Joynt, A. Straszak, U. Loeser, L. Kajriukstis, S. Dresch.

Levely discussion centered around two issues:
How are regional problems and decisions delimited and formulated - are they substantially based or "merely" political?, and: What is the link between science, education system etc. and technological and economic change?
With regard to the first question instances were quoted where regional problems arise in a natural way out of geographical and economic circumstances, waiting only for proper solutions, engaging also political structures. The cases quoted referred to riversheds and to geographico-economic East-West situation in South America, where large areas along the Western coast have much greater development capacity than is presently released, due to economic, but also political conditions.
As to the second question it was stated that the relations in question are of the necessary, but not sufficient condition type, so that simple reasoning can fail both ways. The situation is further made even more vague by the lack of clear definitions in the domain.

Paper by A. Mouwen and P. Nijkamp

Discussion participants: A. Straszak, R. Kulikowski, L. Lacko, S. Ikeda, A. Kochetkov, A. Mouwen.

This discussion, which to a large extent continued the themes of the paper itself and of discussion to the previous paper, focussed mainly on conditions and mechanisms of knowledge and technology transfer from science to production practice. Within this context social and spatial mobility of scientists, research centers and knowledge-intensive firms was assessed. Instances were quoted of large, scientifically self-sufficient firms moving out of bigger urban centers, with the small ones moving in, for instance, to get closer to the research resources. On the other hand the example of Tsukuba was shown to indicate the real possibility of speeding up the regional development around a large scientific compound - by attracting businesses which could profit from cooperation. This development occurred over 15 years, and there is another one, chip-oriented, underway in Japan in the Kyushu region. Thus, while it was deemed important to secure the link between science and actual promotion, other conditions may play an important role, e.g. communication infrastructure or competitiveness. Experience from one place may not be fully transferable to another, and hence differences between the Dutch and the Swedich case. Knowledge-based development requires special orientation of investments - it was said that in the case of the Netherlands approx. 4% of GNP would be devoted R and D.

Paper by K. Polenske and Wm. Crown

Discussion participants: G. Bianchi, P. Joynt, K. Polenske.

The main question raised concerned the way in which the interregional coefficients can be obtained, since this was deemed to be far more difficult than for the technical coefficients. The procedure taken in the work presented started with trade tables, on which a balancing is performed. Then goals transportation data come in. Both these steps, however, do in fact still leave out some cells in the matrix. Hence, an expert-based range estimation is applied and final row and column balancing is performed. The whole procedure is implemented with two main computer programs MATHER and PASSION.

Paper by T. Vasko

Discussion participants: M. Steiner, A. Straszak, J. Owsiński, T. Vasko.

First, a clarification was asked for as to the meaning of information space. The answer consisted in statement that a general innovation is composed of simple innovations such as market innovation, product improvement etc., and that any simple innovation can hardly have an economic effect. Thus, innovations appear as compounds in the simple innovation space.

Then, a portion of discussion was devoted to identification of the logistic curves involved. Besides the very identification question, where the starting time-point was deemed of special importance, the problem of interplay of product values: exchange value, use value and production cost, was emphasized. Answering another question the speaker said that by looking at the innovations side he gets the idea that the new general economic upswing has had began by then, but that other analysts, e.g. C. Marchetti, see it coming in only about a decade.

Paper by R. Funck and J. Kowalski was not discussed since it was presented after the workshop.

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