POLSKA AKADEMIA NAUK
INSTYTUT BADAŃ SYSTEMOWYCH

PROCEEDINGS OF THE 3rd ITALIAN-POLISH CONFERENCE ON APPLICATIONS OF SYSTEMS THEORY TO ECONOMY, MANAGEMENT AND TECHNOLOGY

WARSZAWA 1977

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PREFACE

The present volume comprises the papers from the 3rd Italian-Polish Conference which was held in [illegible] n. Warsaw from May 26 to 31. The conference organized by the Centre of Control Systems and Computation of the Italian National Research Council in Rome and the Institute for Organization, Management and Control Sciences of Polish Academy of Sciences and the Ministry of Science, Higher Education and Technology in Warsaw was the third conference of this type.

The first one took place in Krakow (Poland) in 1972, the second in Pugnochiuso (Italy) in 1974.

This conference, like both the previous ones, gathered the leading representatives of many scientific centres from Italy as well as from Poland, representing also the different aspects of the systems analysis applied to the economic, management, data processing and technological systems.

From the past, such meetings enrich the exchange of views and experiences between the specialists from both countries what is the indicator of its usefulness to both sides. Hence, the natural part, the continuation of the seminars and conferences specially concerned with the essential development of the quantitative methods and the particularly quickly development of large computer systems.

The meaning of such meetings, however, goes far beyond that, what is happening during the conference because the established contacts are also fruitful in form of common scientific research works and — "last but not least" — begin the friendly link between the scientists of both countries which last for years.

The proceedings of the conference were divided into three parts:

1. Optimization and Control Theory;
2. Systems Theory in Economics;
3. Technological, Managerial and Information Systems.

While the first two parts are of rather homogeneous character, the third part contains the papers concerning the different types of models — for the economic, technological, management and data processing systems.

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MODELLING AND IDENTIFICATION OF THE ITALIAN FIBERGLASS INDUSTRY

1. INTRODUCTION

The problem of building models of economic systems has been tackled by many control systems scientists during the past years [1, 5, 6].

From the identification point of view (estimation in econometrics), the main differences between economic and usual engineering systems are the following:

— the dynamical links among the economic model variables are not usualy known as exactly as in engineering systems;

— the great importance of the process and measurement noise, due to the stochastic aspects of the decision-making process, to the modelling approximation, to the stochastic nature of observed data;

— the model has a large number of parameters even for simple economic systems and the strong interdependence among the variables makes model decomposition difficult;

— the presence of concealed feedback links can make the exogenous variables not really independent from endogenous variables;

— the structure of the system and the value of the parameters can change over time, so that only recent data can be used to identif y model parameters so that the size of the sample may turn out to be too small;

— suitably disaggregated data may not be available;

— data sample frequency may not be appropriate;

— it is not possible to select the best inputs from the identification point of view; in addition the inputs are in many cases strongly correlated (multicollinearity);

— the endogenous variables are often characterized by a simultaneous interdependence due to the relatevely long sampling period, so that the system is described by a simultaneous equations model.

A discrete state space linear representation of the system has been utilized to express the economic hypotheses. In this representation some of the parameters have been assumed to be zero or known physical constants, others have been assumed as known functions of unknown parameters. The steady state Kalman filter representation of the model has been used to estimate unknown parameters with a maximum likelihood criterion. Such simultaneous identification method, although very heavy from a computational point of view, has been chosen in order to take into account the strong interaction among variables and to obtain an efficient estimate.

Even though models of industries are acknowledged to be of relevant importance very little attention has been paid to this sector by economic models builders. The main difficulty in tackling the problem of building economic models of industries is that, while taking into account the structure of the technology and of the market, one has to be able to make hypotheses on the interactions among the relevant variables abstracting from the single situations of the firms which make part of the industry.

This philosophy is not easy to apply and the main methodological problems arising in the model construction are the following:

— choice of the time horizon of validity of the model;

— definition of the economic sector in the economy in which it operates both from the geographic point of view and as concerns the interactions with other-economic sectors;

— choice of the aggregation level of the variables involved taking into account that the main concern is to single out variables which characterize the industry as a whole;

— representation of the market structure including the identification of economic operators, their mutual interactions, and the decision-making process;

— imbedding the analysis of the economic sector in a macroeconomic context in order to be able to evaluate the coherence of the hypotheses.

The first attempt has been to build a model for the Italian textile fiberglass industry [8].

The choice has fallen on this sector due to the simplicity of both the production and market structure. As a matter of fact, textile fiberglass production is mainly due to a big corporation and there exist a few commercial branches of foreign producers. Moreover, it is easy to aggregate the whole production in one product only.

The purpose of this paper is to appropriately complete the model parameter identification scheme only sketched in [8].

2. ECONOMIC HYPOTHESES

Using data from years 1970/1974 a short period dynamical model of the Italian fiberglass industry has been constructed. Given the technological characteristics of several products and the nature of the market, it has been possible to consider just one aggregrated product.

The chosen endogenous variables are the following:

PR - index of the Italian fiberglass production

PI - Italian price index of the aggregated product

VI - National sale index of the Italian producer
VE - export index of the Italian producer
SC - inventory index of national finished product

Data have been collected on a quarter time base. The choice of the variables and the sampling period has been judged appropriate to the short time horizon of validity of the model and to the market fluctuations of the considered economic sector.

The main aspects considered in the model are, firstly the influence of Italian and foreign market on domestic and foreign sales and on production planning.

The economic hypothesis which have been assumed are the following:

— the rigid structure of the production process which makes the horizon of the production planning quite long (1—2 quarters);

— within a wide range demand/supply unbalance does not affect production level; unbalances are firstly met by inventory fluctuations policy and, above a certain level, by price adjustment;

— inventory level affects production policy, Italian price and exports;

— due to the oligopolistic nature of the market, price is mainly determined by the mark-up criterion;

— the most relevant cost items are due to labour and to power consumption, but the labour for unity of product is not available and the labour cost for a worker proved to be not significant;

— the main utilizer of fiberglass production is the building industry through reinforced plastics, whose trend is to substitute traditional building components; the price of fiberglass depend in part on the price of this competing product;

— export is not determined by foreign demand but mainly by an offer policy dependent on internal consumption, inventory level and foreign price;

— import can be assumed as a proxy of internal demand.

Moreover, in addition to usual seasonal dummy variables, three dummy variables have been introduced; the first accounting for governmental price control occurred between the third 1973 quarter and second 1974 quarter, the second for a price fall due to a new competing fir, entering the market in the fourth 1971 quarter, the third for long strike period occured between the third and fourth 1973 quarters.

These hypotheses have been conforted by identification results.

Due to these hypotheses, the exogenous variables are the following:

IM - import index
PE - foreign price index
PC - price index of competing product
CE - power price index
HP, *BP*, *S*2, *S*3, *S*4-dummy variables
CO - reference index level.

3. MODEL FORMULATION

The above-mentioned hypotheses can be expressed by a set of linear difference equations (structural form):

$$\sum_{i=1}^{p} \boldsymbol{A}_i \boldsymbol{y}(t-i) = \sum_{i=0}^{q} \boldsymbol{B}_i \boldsymbol{u}(t-i) + \boldsymbol{e}(t) \tag{1}$$

where $\boldsymbol{A}_i$ and $\boldsymbol{B}_i$ are constant matrices of suitable dimensions whose entries are known constants (possibly zero) or known functions of some unknown parameters: $\boldsymbol{y}(t)$ is the vector of endogenous variables; $\boldsymbol{u}(t)$ is the vector of exogenous variables (input variables); $\boldsymbol{r}(t)$ is the noise vector. On the basis of the economic hypotheses it is possible to assume $\boldsymbol{A}_0 = \boldsymbol{I}$, $\boldsymbol{B}_0 = \boldsymbol{O}$ (with I identity matrix and $\boldsymbol{O}$ zero matrix).

Under very general assumptions, equation (1) can be expressed in a state--space form [5]. For the particular characteristics of the system analyzed in this work, we can choose as state variables all the endogenous variables at time t and a subset of the endogenous variables at time $(t-1)$.

The structural form of the system makes possible this choice under the hypothesis that the state-space form is a minimal representation of the system. This assumption is a necessary condition from the identifiability point of view. Eveen if the endogenous varia bles are not linearly dependent, the introduction of endogenous variables at time $t-1$, $t-2$, ... can produce a non minimal representation for each value of the parameters. In this case instead of these variables we must introduce a lower number of new variables without a clear economic meaning.

In the case examined inthis work some of the state variables (VI, VE, PR, PI) are the output variables. Only the initial value of stock is independently known; the indexed stock level time evolution, arbitrarily assuming the initial value, has been reconstructed by means of the following relation

$$SC(t+1) = SC(t) + \alpha PR(t) - \beta VT(t) \tag{2}$$

where $VT(t)$ is the index of total delivery at time t:

$$VT(t) = \gamma VI(t) + \delta VE(t) \tag{3}$$

and the coefficients (α, β, γ, δ) are introduced (and known "a priori") to take into account the use of indexed variables.

The system of structural relations can be expressed in a state-space form as follows:

$$\begin{aligned} \boldsymbol{x}(t+1) &= \boldsymbol{A}\boldsymbol{x}(t) + \boldsymbol{B}\boldsymbol{u}(t) + \boldsymbol{F}\boldsymbol{w}(t) \\ \boldsymbol{y}(t) &= \boldsymbol{C}\boldsymbol{x}(t) + \boldsymbol{v}(t) \end{aligned} \tag{4}$$

where $\boldsymbol{x}(t)$ is the state-variables vector, $\boldsymbol{y}(t)$ the output variables vector, $\boldsymbol{A}$, $\boldsymbol{B}$, $\boldsymbol{F}$, $\boldsymbol{C}$ matrices with suitable dimensions whose entries are known constants (for example equal to zero or one) or known functions of some unknown parameters, $w(t)$ and $v(t)$ white gaussian noise vectors with non-singular covariance matrices $\boldsymbol{Q}$ and $\boldsymbol{R}$ respectively. The $\boldsymbol{F}$ matrix can be omitted if we drop the condition that Q is non singular. The noise hypotheses are very general and often verified in practice if the mathematical model is a good model of the real system. If we have a colored noise, with a proper rational spectrum, we can introduce some new state-variables and still express the system equations in the form (4) [5].

The use of some canonical representation of the system is not possible if we want to use "a priori" information on the structure of $\boldsymbol{A}, \boldsymbol{B}, \boldsymbol{C}, \boldsymbol{F}$. The short data series, the impossibility of experiments on the system, the interest in the value of some structural parameters, does not make the use of canonical forms possible.

For this reason it is necessary to verify in each case the identifiability of the parametrization [3] (i.e. there do not exist two distinct points of the parameter space with the same probability density function of the random output vector). It is very difficult to verify in practice the identifiability of the parametrization and it is usual to verify, in addition to the identifiability of the transfer function between deterministic and stochastic inputs and output, only the local identifiability of the parametrization in a suitable point α^0 of the parameter space (i.e. to test if there exists an open neighborhood of α^0 containing no other α with the same probability density function of the random output vector).

The conditions for the identifiability of the transfer function concern the persistently exciting character of the input and the stability of the system [16]. A necessary and sufficient condition for the local identifiability of the parametrization is the non singularity of a suitable information matrix of the system.

The information matrix can be defined as follows:

$$\boldsymbol{M}(\alpha) = [m_{ij}(\boldsymbol{\alpha})] = E\left\{\left[\frac{\partial \log f(\boldsymbol{y}|\boldsymbol{\alpha})}{\partial \alpha_i} \frac{\partial \log f(\boldsymbol{y}|\boldsymbol{\alpha})}{\partial \alpha_j}\right]\right\}$$

where α is the unknown parameters vector and $f(\boldsymbol{y}|\boldsymbol{\alpha})$ is a proper probability density function of the output given the parameters vector [2].

In practical cases we obtain an estimate of $M(\alpha)$ from a given sample (see the following equation 23).

Some of the identifiability problems are associated to the stochastic aspects of the system. From a general point of view it is not possible to estimate all the unknown parameters of matrices $\boldsymbol{F}, \boldsymbol{Q}, \boldsymbol{R}$ [1, 9]. In the following we assume a Kalman filter representation of the system.

Then (4) becomes:

$$\begin{aligned} \hat{\boldsymbol{x}}(t+1) &= \boldsymbol{A}\hat{\boldsymbol{x}}(t)+\boldsymbol{B}\boldsymbol{u}(t)+\boldsymbol{A}\boldsymbol{K}(t)\boldsymbol{v}(t) \\ \boldsymbol{y}(t) &= \boldsymbol{C}\boldsymbol{x}(t)+\boldsymbol{v}(t) \end{aligned} \tag{5}$$

with $\boldsymbol{v}(t)$ white gaussian noise vector with nonsingular covariance matrix $\Omega(t)$ and $\hat{\boldsymbol{x}}(t)$ linear optimal one step prediction of the state-vector.

In addition to the (5) we must impose:

$$K(t) = P(t)C^T\Omega(t)^{-1} \tag{6}$$

$$\Omega(t) = CP(t)C^T + R \tag{7}$$

$$P(t+1) = A\big(I - K(t)C\big)P(t)A^T + F\Omega F^T \tag{8}$$

where $P(t)$ is the covariance matrix of $\hat{x}(t)$.

In this work we have assumed a stationary Kalman filter representation, with $P(t) = P$, $K(t) = K$ and $\Omega(t) = \Omega$ constant matrices, and we have identified the system parameters using equations (5) (6) (7). In some cases from the estimated matrices A, B, C, K, Ω, R (8) it is possible to calculate the matrices R (supposing without loss of generality F entirely known). In the general case from the known matrices it is not possible to calculate the P matrix but the PC^T matrix only (6), and then it is not possible to calculate all the entries of FQF^T. In our case from the absence of noise on some state-equations (the identity equations: stock equation and state equations related to endogenous variables at time $t-1$) the F matrix is composed by an identity matrix and a zero matrix:

$$F = \begin{bmatrix} I \\ \cdots \\ 0 \end{bmatrix} \tag{9}$$

In this work the Kalman filter representation of the system is to be identified, hence the stability of the obtained $A(I-KC)$ matrix is not guaranteed "a priori" and must be verified as the stability of the A matrix.

4. PARAMETER IDENTIFICATION

Let (5) be the economic system representation, the unknown parameters are entries of matrices A, B, K, R and Q, the C matrix is entirely known and is composed by on identity matrix and a zero matrix $C = [I \ldots O]$. The maximum likelihood method gives an asymptotically efficient and consistent estimate of the parameters.

Let $p(Y_N|\alpha, U_N, x(1))$ be the likelihood function related to system (4), where:

$$Y_k = \big[y(1)\,\big|y(2)\big|\ldots\big|y(k)\big] \quad 1 \leqslant k \leqslant N \tag{10}$$

$$U_k = \big[u(1)\,\big|u(2)\big|\ldots\big|u(k)\big] \quad 1 \leqslant k \leqslant N \tag{11}$$

we can assume:

$$p\big(Y_N|\alpha, U_N, x(1)\big) = p\big(y(1)|\alpha, x(1)\big)\prod_{j=2}^{N} p\big(y(j)\big|Y_{j-1}, \alpha, U_{j-1}, x(1)\big) \tag{12}$$

From the gaussian noise assumption in (4) it follows that the probability density functions on the right-hand side of (12) are normal with mean $\hat{y}(j)$ and covariance matrix $\Omega(j)$ given from the Kalman filter of system (5, 6, 7, 8)

$$\hat{y}(j) = E\{y(j)|Y_{j-1}, \alpha, U_{j-1}, x(1)\} = C\hat{x}(j) \tag{13}$$

$$\Omega(j) = E\{(y(j)-\hat{y}(j))(y(j)-\hat{y}(j))^T\} = \{v(j)v^T(j)\} \tag{14}$$

From (12, 13, 14) and the characteristics of $v(t)$ (white gaussian noise vector) we obain:

$$\ln p(Y_N|\alpha, U_N, x(1)) = \ln p(y(1)|\alpha, x(1)) +$$

$$+\sum_{j=2}^{N} \ln p(y(j)|Y_{j-1}, \alpha, U_{j-1}, x(1)) = -\frac{rN}{2}\ln 2\pi -$$

$$-\frac{1}{2}\sum_{j=1}^{N}\{\ln(\det \Omega(j)) + v(j)^T\Omega^{-1}(j)v(j)\} \tag{15}$$

With the stationarity hypotheses we can put $\Omega(j)=\Omega$ and $K(j)=K$ and maximize (15) with respect to α subject to (5) (6).

Setting:

$$\Gamma = \Omega^{-1} \tag{16}$$

$$k^T = [k_1^T \vdots k_2^T] \tag{17}$$

with k_1 square matrix and observing that

$$C = [I \vdots 0] \tag{18}$$

$$\Omega^T = \Omega > 0 \quad \text{(positive definite)} \tag{19}$$

$$R^T = R > 0 \tag{20}$$

it follows from (5) (6) (7) and (15) the minimization problem:

$$\min_{a} J = \frac{1}{2}\sum_{t=1}^{N}(v^T(t)\Gamma v(t) - \ln\det\Gamma) \tag{21}$$

subject to:

$$\hat{x}(t+1) = A\hat{x}(t) + Ru(t) + A\begin{bmatrix} I - R\Gamma \\ k_2 \end{bmatrix} v(t) \tag{22}$$

$$y(t) = C\hat{x}(t) + v(t) \tag{23}$$

We can use a minimization algorithm based on gradient calculation: $\triangle J$; and possibly on Hessian calculation: $\boldsymbol{H}$ (or on some approximation of the Hessian matrix).

$$\frac{\partial J}{\partial \alpha_k} = \sum_{t=1}^{N} \left\{ \boldsymbol{v}^T(t) \boldsymbol{\Gamma} \frac{\partial \boldsymbol{v}(t)}{\partial \alpha_k} + \frac{1}{2} \boldsymbol{v}^T(t) \frac{\partial \boldsymbol{\Gamma}}{\partial \alpha_k} \boldsymbol{v}(t) - \frac{1}{2} \operatorname{tr}\left(\frac{\partial \boldsymbol{\Gamma}}{\partial \alpha_k} \boldsymbol{\Gamma}^{-1} \right) \right\} \tag{24}$$

the estimation of parameters of $\boldsymbol{R}$ is equivalent to the estimation of those of Ω, only if "a priori" assumption on parameters of Ω (or of $\boldsymbol{R}$) are not taken into account; this substitution was used in this case to obtain greater computational efficiency.

Computation of $\boldsymbol{H}$ is necessary to utilize Newton-Raphson minimization method and to verify local properties of the point to which the algorithm converges.

This computation is heavy and the solution obtained can be affected by large rounding errors.

We have then utilized the Gauss-Newton minimization method (quasilinearization method) based on approximated Hessian [7, 12]:

$$\boldsymbol{H} = E\left\{ \left(\frac{\partial J}{\partial \alpha} \right) \left(\frac{\partial J}{\partial \alpha} \right)^T \right\} \tag{25}$$

$$H_{ki} \approx \sum_{t=1}^{N} \left\{ \left(\frac{\partial \boldsymbol{v}(t)}{\partial \alpha_k} \right)^T \bar{\boldsymbol{\Gamma}} \left(\frac{\partial \boldsymbol{v}(t)}{\partial \alpha_i} \right) + \frac{1}{2} \operatorname{tr}\left(\frac{\partial \boldsymbol{\Gamma}}{\partial \alpha_k} \boldsymbol{\Gamma}^{-1} \frac{\partial \boldsymbol{\Gamma}}{\partial \alpha_i} \boldsymbol{\Gamma}^{-1} \right) + \right.$$

$$\left. + \frac{1}{4} \operatorname{tr}\left(\frac{\partial \boldsymbol{\Gamma}}{\partial \alpha_k} \boldsymbol{\Gamma}^{-1} \right) \operatorname{tr}\left(\frac{\partial \boldsymbol{\Gamma}}{\partial \alpha_i} \boldsymbol{\Gamma}^{-1} \right) \right\} = \underline{M}_{ki} \tag{26}$$

The matrix $\underline{M}$ can be also be interpeted as an approximation of the information matrix $\underline{M}$ [12].

Hence we can estimate the parameters covariance matrix V as follows [14]:

$$V = \underline{M}^{-1} \tag{27}$$

5. NUMERICAL RESULTS

Shortness of data sample (which is near to the minimum size for the use of simultaneous methods, in the case of dynamic systems, as given in [15]), and peculiar economic behaviour of the modeled industry during the observation period (a steady, rapid growth, followed by a sharp crisis at the very end of the sample) have made simultaneous estimation computationally difficult. Fresh data are needed to draw more precise conclusions about formulated economic hypothesis and adequacy of the obtained model; however obtained results can be considered satisfactory and several economic hypothesis have been verified.

Model structure determination has been performed, on the basis of economic hypothesis, by means of single equation regression analysis. Several structures so determined have been tested and used to initialize the multiequation minimization algorithm; in several cases this second stage of identification procedure has moved initial values of parameters towards regions in which matrix A and/or $A(I\text{-}KC)$ become unstable, which makes the results meaningless both from the identification and the economic point of view (fig. 2).

This fact can be explained on the one hand with the shortness of data sample, and on the other hand with the higher sensitivity of simultaneous estimation methods to errors on model specification. This result can be taken as a further proof of the necessity to take into account the strong interaction among variables in econometric models and then to utilize a simultaneous estimation method.

State space has been expanded to take into account time correlation of the noise in the export structural equation. This equation resulted to be inadequate to represent the export behaviour of the system. This fact has been explained by the multinational character of italian firms and by the lack of a suitable index of the international demand.

Using an output error method it is possible to obtain a good fitting also for this equation, but the statistical properties of the estimate in presence of a process noise are not well known [8].

A good result, both from the identification and the economic point of view, has been obtained as far as production, italian price and national sale equations are concerned.

Some of the parameters associated with the stochastic aspects of the model have given a low significance estimate. In these cases minimization algorithm convergence has been poor, typically because of near-singularity of the information matrix. In order to obtain a suitable representation of the economic

$$
\begin{bmatrix} VI(t+1) \\ VE(t+1) \\ PR(t+1) \\ PI(t+1) \\ SC(t+1) \\ VI(t) \\ VE(t) \\ NO(t+1) \end{bmatrix}
=
\begin{bmatrix}
a_{11} & & & a_{14} & & & & \\
a_{21} & 1 & & & a_{25} & & & 1 \\
 & & & & a_{35} & a_{36} & a_{37} & \\
a_{41} & & & & a_{45} & a_{46} & & \\
a_{51} & a_{52} & a_{53} & & 1 & & & \\
1 & & & & & & & \\
 & 1 & & & & & & \\
 & & & & & & & a_{88}
\end{bmatrix}
\begin{bmatrix} VI(t) \\ VE(t) \\ PR(t) \\ PI(t) \\ SC(t) \\ VI(t-1) \\ VE(t-1) \\ NO(t-1) \end{bmatrix}
+
\begin{bmatrix}
b_{11} & & & & & & b_{17} & b_{18} & b_{19} & b_{110} \\
 & b_{22} & & & & & b_{27} & b_{28} & b_{29} & b_{210} \\
 & & & & b_{35} & & b_{37} & & & b_{310} \\
 & & b_{43} & b_{44} & & b_{46} & & & & b_{410} \\
 & & & & & & & & & \\
 & & & & & & & & & \\
 & & & & & & & & & \\
 & & & & & & & & &
\end{bmatrix}
\begin{bmatrix} IM(t) \\ PB(t) \\ PC(t) \\ CE(t) \\ HP(t) \\ BP(t) \\ S2(t) \\ S3(t) \\ S4(t) \\ CO \end{bmatrix}
+ Ak\nu(t)
$$

$a_{51} = -.9969$

$a_{52} = -.7542$

$a_{53} = 1.681$

$a_{41} = -a_{46}$

$a_{36} = \frac{.5693}{.4307} a_{37}$

Fig. 1

		VI(t)	VI(t-1)	VI(t)- VI(t-1)	VT(t)	PI(t)	SC(t)	IM(t)	IM(t)- IM(t-1)	PE(t)	PC(t)	CE(t)	BP(t)	HP(t)	S2(t)	S3(t)	S4(t)	CO	R^2	F
1a	VI(t+1)	.669 (4.89)				-1.185 (5.44)		.054 (2.75)							9.381 (1.94)	-13.353 (2.89)	31.946 (5.97)	-21.034	.964	70.4
1b	VI(t+1- VI(t)					-.846 (5.88)			.047 (3.82)						12.705 (3.29)	-6.896 (1.64)	42.019 (8.77)	-.952	.937	52.3
2a	VE(t+1) -VE(t)	-.490 (2.20)					.184 (2.24)			.930 (2.35)						-10.049 (1.43)	-60.348 (6.01)	.439	.815	15.5
2b	VE(t+1) -VE(t)	-.464 (1.38)		-.305 (.51)		-.385 (.60)	.191 (1.89)			1.012 (1.74)					-4.603 (.26)	-13.221 (1.02)	-69.306 (2.93)	.358	.822	6.6
3a	PR(t+1)				.551 (4.37)		-.142 (3.85)							8.224 (3.68)	7.937 (2.50)			-58.745	.903	43.2
3b	PR(t+1)		.274 (5.68)				-.150 (5.06)							8.791 (4.58)				-31.700	.911	76.5
3c	PR(t+1)	.260 (5.39)		-.347 (4.85)			-.145 (5.01)							9.187 (4.87)					.921	54.9
4a	PI(t+1)			.104 (1.43)			-.052 (1.55)				.408 (5.50)	.179 (1.53)	-7.172 (2.64)					-67.553	.942	52.8
4b	PI(t+1)	.101 (1.12)					-.059 (1.75)				.319 (2.58)	.182 (1.41)	-9.547 (2.50)					-56.963	.939	53.6

( ) Student's t test

Fig. 2

$$VI(t+1) = \underset{(5.89)}{.7322}\, VI(t) \;\underset{(4.62)}{-1.044}\, P(t) \;\underset{(2.09)}{+.0402}\, IM(t) \;\underset{(2.14)}{+10.240}\, S2 \;\underset{(3.45)}{-14.21}\, S3 \;\underset{(6.63)}{+30.960}\, S4 \;+121.08$$

$$VE(t+1) = VE(t) \;\underset{(2.84)}{-.5775}\, VI(t) \;\underset{(2.84)}{+.2139}\, SC(t) \;\underset{(3.08)}{+1.143}\, PE(t) \;\underset{(1.10)}{-7.281}\, S3 \;\underset{(7.83)}{-67.728}\, S4 \;+ NO(t) \;-71.48$$

$$PR(t+1) = \underset{(4.72)}{.531}\, VT(t) \;\underset{(4.43)}{-.140}\, SC(t) \;\underset{(4.38)}{-8.19}\, HP \;\underset{(3.03)}{+8.05}\, S2 \;+77.46$$

$$PI(t) = \underset{(1.62)}{.106}\left[VI(t) - VI(t-1)\right] \;\underset{(1.81)}{-.0526}\, SC(t) \;\underset{(6.27)}{+.405}\, PC(t) \;\underset{(1.80)}{+.183}\, CE(t) \;\underset{(3.06)}{-7.226}\, BP \;+46.61$$

$$NO(t+1) = \underset{(6.27)}{-.847}\, NO(t) \;\underset{(1.44)}{+.085}\left[VE(t)_{OBS} - VE(t)\right]$$

Fig. 3

system, with the given data sample, rather crude hypothesis on the noises covariance have resulted necessary.

High correlation of some of the inputs, has made difficult to have equation structure matching with economic hypotheses: firstly price index of competing product could not be introduced in the national sale index equation because

$$r = \begin{bmatrix} .049 & -.010 & 0 & 0 \\ -.010 & .0166 & 0 & 0 \\ 0 & 0 & .0521 & 0 \\ 0 & 0 & 0 & .0869 \end{bmatrix} \qquad \Omega = \begin{bmatrix} 18.20 & 11.07 & 0 & 0 \\ 11.07 & 53.51 & 0 & 0 \\ 0 & 0 & 19.19 & 0 \\ 0 & 0 & 0 & 11.51 \end{bmatrix}$$

$$R = \begin{bmatrix} 2.28 & 0 & 0 & 0 \\ 0 & 2.28 & 0 & 0 \\ 0 & 0 & 2.28 & 0 \\ 0 & 0 & 0 & 2.28 \end{bmatrix} \qquad K = \begin{bmatrix} .888 & .023 & 0 & 0 \\ .023 & .963 & 0 & 0 \\ 0 & 0 & .882 & 0 \\ 0 & 0 & 0 & .802 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & .085 & 0 & 0 \end{bmatrix}$$

Fig. 3b

Fig. 4

of the high correlation with the italian price; secondly it was not possible to introduce all the relevant cost and price indexes in the Italian price index equation.

In order to obtain a reliable result, the choice of a suitable descent algorithm and of parameters initial point is very important. This is essentially due to the presence of several local minima of the highly nonlinear objective function and to the presence of a possibly redundant parametrization.

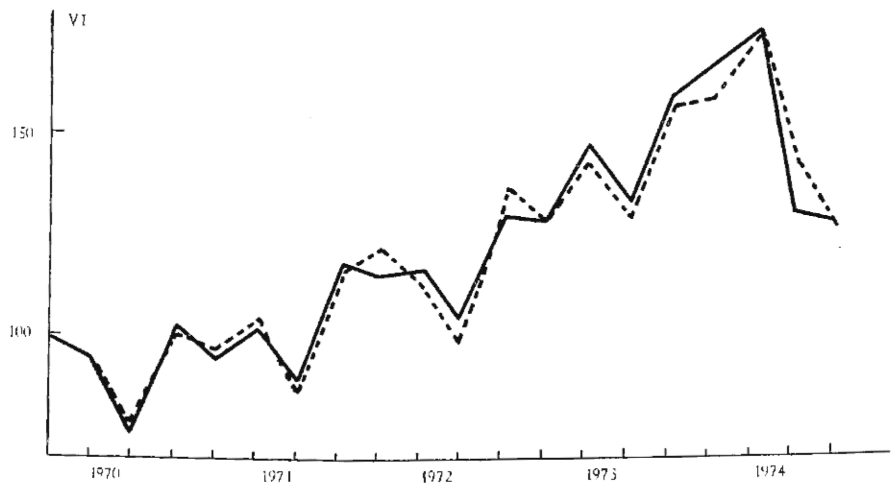

Fig. 5

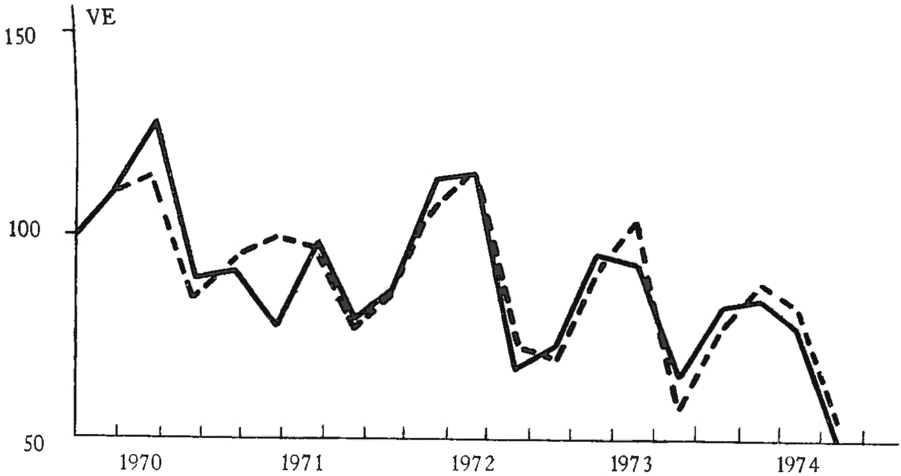

Fig. 6

Conjugate gradient and Davidon-Fletcher-Powell methods have a not very fast convergence in terms of total computation time, but these methods are more reliable when the information matrix is ill-conditioned. Gauss-Newton method has a higher convergence speed, although a single iteration is more time consuming; particular procedures must be used in this case if the information matrix is ill conditioned [7]. To avoid information matrix inversion at each iteraction, Davidon-Fletcher-Powell method using the inverse of the information matrix as initial weight matrix of the method has been used;

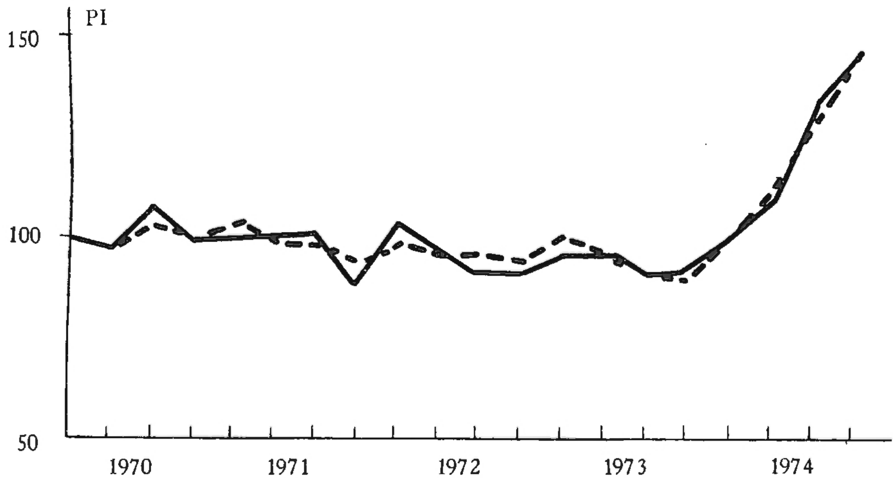

Fig. 7

in this case it was not possible to complete a cycle of as many iterations as the number of unknown parameters, without reinitialize the algorithm.

In fig. 1, 3 the results of identification procedure are reported, in fig. 2 the initializations utilized and in fig. 4, 5, 6, 7 are plotted the results of one step estimated state prediction.

APPENDIX

If we drop, as a further approximation, the costraints (6, 7), the following simplifications can be made.

We can maximize (15) with respect to the Ω parameters only and, imposing that the gradient of the likelihood function subject to (5) with respect to these parameters to be zero, we can obtain the parameters of Ω as a function of the vector Θ of the parameters of $\boldsymbol{A}$, $\boldsymbol{B}$ and K (i.e. the so called concentrated likelihood function).

Let $\otimes$ be the Kronecker product and L (nm$\times$1) the vector obtained from $L(n\times m)$ as follows:

$$= [\gamma_1^T \vdots \gamma_2^T \vdots \dots \vdots \gamma_n^T] \quad \text{with} \quad \boldsymbol{\Gamma}^T[\gamma_1 \vdots \gamma_2 \vdots \dots \vdots \gamma_n]$$

Deleting the constant term we obtain:

$$\hat{J} = -\frac{1}{2}\sum_{j=1}^{N} \{\ln(\det \boldsymbol{\Omega}) + \boldsymbol{v}(j)^T \boldsymbol{\Omega}^{-1} \boldsymbol{v}(j)\} =$$

$$= -\frac{1}{2}\sum_{j=1}^{N} \{\ln(\det \boldsymbol{\Omega}) + (\boldsymbol{v}(j)^T \otimes \boldsymbol{v}(j)^T)\boldsymbol{\Omega}^{-1}\} \tag{28}$$

and then:

$$\frac{\partial J}{\partial \boldsymbol{\Omega}} = -\frac{1}{2}\left(N\boldsymbol{\Omega}^{-1} - \sum_{j=1}^{N} (\boldsymbol{v}(j)^T \otimes \boldsymbol{v}(j)^T)(\boldsymbol{\Omega}^{-1} \otimes \boldsymbol{\Omega}^{-1})\right) = 0 \tag{29}$$

which for the properties of the Kronecker product can be written:

$$\boldsymbol{\Omega}^{-1} = \frac{1}{N}\boldsymbol{\Omega}^{-1}\sum_{j=1}^{N} (\boldsymbol{v}(j)\boldsymbol{v}(j)^T)\boldsymbol{\Omega}^{-1} \tag{30}$$

$$\boldsymbol{\Omega} = \frac{1}{N}\sum_{j=1}^{N} (\boldsymbol{v}(j)\boldsymbol{v}(j)^T) \tag{31}$$

Substituting (19) in (16) we obtain:

$$J = -\frac{N}{2}\ln(\det \boldsymbol{\Omega}) - \frac{1}{2}\sum_{j=1}^{N}\left(\boldsymbol{v}(j)^T\left(\frac{1}{N}\sum_{i=1}^{N}\boldsymbol{v}(i)\boldsymbol{v}(i)^T\right)^{-1}\boldsymbol{v}(j)\right) =$$

$$= -\frac{N}{2}\ln(\det \boldsymbol{\Omega}) - \frac{1}{2}\operatorname{tr}\left\{\sum_{j=1}^{N}\left(\boldsymbol{v}(j)^T\left(\frac{1}{N}\sum_{i=1}^{N}\boldsymbol{v}(i)\boldsymbol{v}(i)^T\right)^{-1}\boldsymbol{v}(j)\right)\right\} =$$

$$= -\frac{N}{2}\ln(\det \boldsymbol{\Omega}) - \frac{1}{2}\operatorname{tr}\left\{\left(\sum_{j=1}^{N}\boldsymbol{v}(j)\boldsymbol{v}(j)^T\right)\left(\frac{1}{N}\sum_{i=1}^{N}\boldsymbol{v}(i)\boldsymbol{v}(i)^T\right)^{-1}\right\} =$$

$$= -\frac{N}{2}\ln(\det \boldsymbol{\Omega}) + \text{cost} \tag{32}$$

We can now formulate the problem of the estimation of unknown entries of $\boldsymbol{A}, \boldsymbol{B}, \boldsymbol{K}$ with a maximum likelihood criterion, as follows:

$$\min_{\Theta} J = \frac{1}{2}\ln\left(\det\left(\frac{1}{N}\sum_{j=1}^{N}(\boldsymbol{v}(j)\boldsymbol{v}(j)^T)\right)\right) \tag{33}$$

subject to (5).

In this case gradient and Hessian became:

$$\nabla J = \frac{1}{2}(\det \boldsymbol{\Omega})^{-1}\frac{\partial}{\partial \boldsymbol{\theta}}(\det \boldsymbol{\Omega}) = \frac{1}{2}\boldsymbol{\Omega}^{-1}\frac{\partial \boldsymbol{\Omega}}{\partial \boldsymbol{\theta}} =$$

$$= \frac{1}{2N}\boldsymbol{\Omega}^{-1^T}\sum_{j=1}^{N}\left(\frac{\partial(\boldsymbol{v}(j)\otimes\boldsymbol{v}(j))}{\partial \boldsymbol{v}(j)}\frac{\partial \boldsymbol{v}(j)}{\partial \boldsymbol{\theta}}\right) =$$

$$= \frac{1}{2N}\sum_{j=1}^{N}\left(\boldsymbol{\Omega}^{-1^T}[(\boldsymbol{I}\times\boldsymbol{v}(j)) + (\boldsymbol{v}(j)\times\boldsymbol{I})]\right)\frac{\partial \boldsymbol{v}(j)}{\partial \boldsymbol{\theta}} =$$

$$= \frac{1}{N}\sum_{j=1}^{N}\left(\boldsymbol{v}(j)^T\boldsymbol{\Omega}^{-1}\frac{\partial \boldsymbol{v}(j)}{\partial \boldsymbol{\theta}}\right) \tag{34}$$

$$\boldsymbol{H} \approx E\left\{\left(\frac{\partial}{\partial \boldsymbol{\theta}}\right)\left(\frac{\partial J}{\partial \boldsymbol{\theta}}\right)^T\right\} \approx \sum_{j=1}^{N}\left(\left(\frac{\partial \boldsymbol{v}(j)}{\partial \boldsymbol{\theta}}\right)^T\boldsymbol{\Omega}^{-1}\left(\frac{\partial \boldsymbol{v}(j)}{\partial \boldsymbol{\theta}}\right)\right) = \boldsymbol{M} \tag{35}$$

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SUMMARY

In this paper the problem of building economic short term dynamical model of Italian fiberglass industry has been tackled in its multiple facets: formulation of the mathematical model on the basis of economic hypothesis about the

interaction among the variables taken into account, parameters identification and economic and statistical analysis of numerical results.

A discrete state space linear representation of the system has been utilized to express the economic hypothesis. In this representation some of the parameters have been assumed to be zero or known physical constants and the others known functions of some unknown parameters. The steady state Kalman filter representation of the model has been used to estimate unknown parameters with a maximum likelihood criterion. This simultaneous identification method, very heavy from computational point of view, must be utilized to take into account the strong interaction among variables and to obtain an efficient estimate. The simulated endogenous variables have been plotted against the data time plot.

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