POLSKA AKADEMIA NAUK
INSTYTUT BADAŃ SYSTEMOWYCH

# PROCEEDINGS OF THE 3rd ITALIAN-POLISH CONFERENCE ON APPLICATIONS OF SYSTEMS THEORY TO ECONOMY, MANAGEMENT AND TECHNOLOGY

WARSZAWA 1977

Redaktor techniczny
Iwona Dobrzyńska

Korekta
Halina Wołyniec

POLSKA AKADEMIA NAUK
INSTYTUT BADAŃ SYSTEMOWYCH

# PROCEEDINGS OF THE 3rd ITALIAN-POLISH CONFERENCE ON APPLICATIONS OF SYSTEMS THEORY TO ECONOMY, MANAGEMENT AND TECHNOLOGY

**BIAŁOWIEŻA, POLAND**

**MAY 26—31, 1976**

**EDITED BY J. GUTENBAUM**

WARSZAWA 1977

Redaktor techniczny
Iwona Dobrzyńska

Korekta
Halina Wołyniec

# PREFACE

The present volume comprises the papers read at the Italian-Polish Conference which was held in Warsaw in May [illegible]. The conference organized by the Centre of Control Systems and Computation of the Italian National Research Council in Rome and the Institute for Organization, Management and Control Sciences of Polish Academy of Sciences and the Ministry of Science, Higher Education and Technology in Warsaw was the third conference of this type.

The first one took place in Kraków (Poland) in 197[illegible], the second in Pugnochiuso (Italy) in 197[illegible].

This conference, like both the previous ones, gathered the leading representatives of various scientific groups from Italy as well as from Poland, representing also the different aspects of the system analysis applied to the economic, management, data processing and technological systems.

From the past, such meetings permit the exchange of views and experiences between the specialists from both countries what is the [illegible] to both sides. Above the general goal, the systematization of the notions and conceptions [illegible] contributing essentially to the development of the quantitative analysis and the [illegible] of large complex systems.

The meaning of such meetings however goes far beyond that what is happening during the conference because the established contacts are also fruitful in form of common scientific research works and — "last but not least" — build the friendly ties between the scientists of both countries which last for years.

The proceedings of the conference were divided into three parts:

1. Optimization and Control Theory,
2. System Theory in Economics,
3. Technological, Managerial and Information Systems.

While the first two parts are of rather homogeneous character, the third part contains the papers concerning the different types of models — for the economic, technological, management and data processing systems.

# III. TECHNOLOGICAL, MANAGEMENT AND INFORMATION SYSTEMS

*J. Babarowski*
*M. Inkielman*
*H. Pietkiewicz-Saldan*
*J. Wiśniewski*
Systems Research Institute,
Polish Academy of Sciences, Warsaw

## OPERATIVE CONTROL OF NITROGEN FERTILIZERS PRODUCTION

### 1. INTRODUCTION

The paper gives a brief description of the concept of operative control of continuous production processes. The principal aim of such a control is to minimize costs of implementing monthly schedules, i.e. to determine such a time distribution of loads of individual production units that incurred costs are minimal. Loads are determined on the basis of disturbance forecasts. Moreover, consideration is being given to stochastic characteristics of technical conditions of the process equipment and to contracts, commitments and transportation restrictions for sold products.

By way of example the above mentioned problem is solved for the Nitrogen Works in Włocławek. There are two production lines in the Works consipered. Common to both lines are inter-unit stores shown in Fig. 1.1.

### 2. DESCRIPTION OF THE STRUCTURE OF PRODUCTION OPERATIVE CONTROL

The problem under discussion consists in solving multivariable dynamic optimization problem, assuming the presence of disturbances and constraints. Two types of disturbances are distinguished. For those of the first group $z_r(t)$, short-term forecasts $z(t)$ and long-term forecasts $\tilde{z}(t)$ are available. The second group consists of disturbances with given stochastic characteristics (they are associated with breakdowns and varying technical conditions of the process equipment).

The constraints are divided into three groups:

(i) those related to physical and technological process properties (storage capacity, limiting rates of load changes)

(ii) those due to the cooperation with the supervisory unit as well as transportation units (data determining the way of delivery and demands for sold products)

(iii) those associated with the organization of miantenance units (spare parts and tools provisioning).

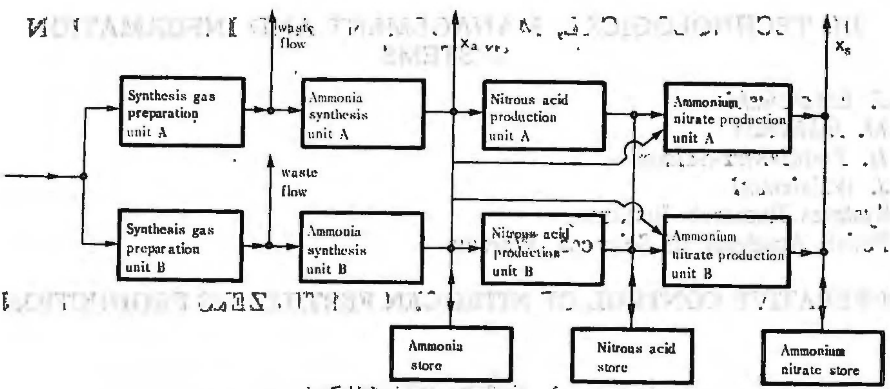


**Fig. 1.1.** Structure of the Nitrogen Works in Włocławek

The proposed structure of a production operative control makes as simple as possible solving of the discussed complex multivariable problem. Deterministic problems related to load optimization are separated from those having stochastic character. The later consists in burdensome computations of plant characteristics and determining essential stochastic characteristics of the process.

The deterministic problem is broken down into two parts

(i) the first one consists in optimizing short-term schedules (block C, Fig. 2.1.). This problem concerns static optimization of production schedules for individual units. It is done by taking into account long-term disturbance forecasts. Using it, the long planning horizon is divided into short stages e.g. several-days long intervals, during which the operating conditions are almost invariant. Solving this problem, finite storage capacities of interunit stores are taken into consideration.

(ii) The second one is defined by dynamic optimization problems (block D) solved for short planning horizons. In this case constraints related to stores can be neglected. These problems can be solved locally for separate sets of units or individual units of a production line for given short-term optimal schedules taking into account short-term forecasts of disturbances.

Fig 2.2. provides an example of determining planning horizons for both deterministic problems. The division of the long-term planning horizon into short-term stages is due to:

(i) planned changes of the process structure (in particular during maintenance work caried out),
(ii) the fact that only short-term forecasts of all disturbances having impact upon the process are reliable,
(iii) the possibility to assume that for purposes of short-term schedule optimisation disturbances are constant over short intervals,
(iv) the presence of constraints on stock levels which are substantial for long-term planning horizons, but can be neglected for short-term ones.

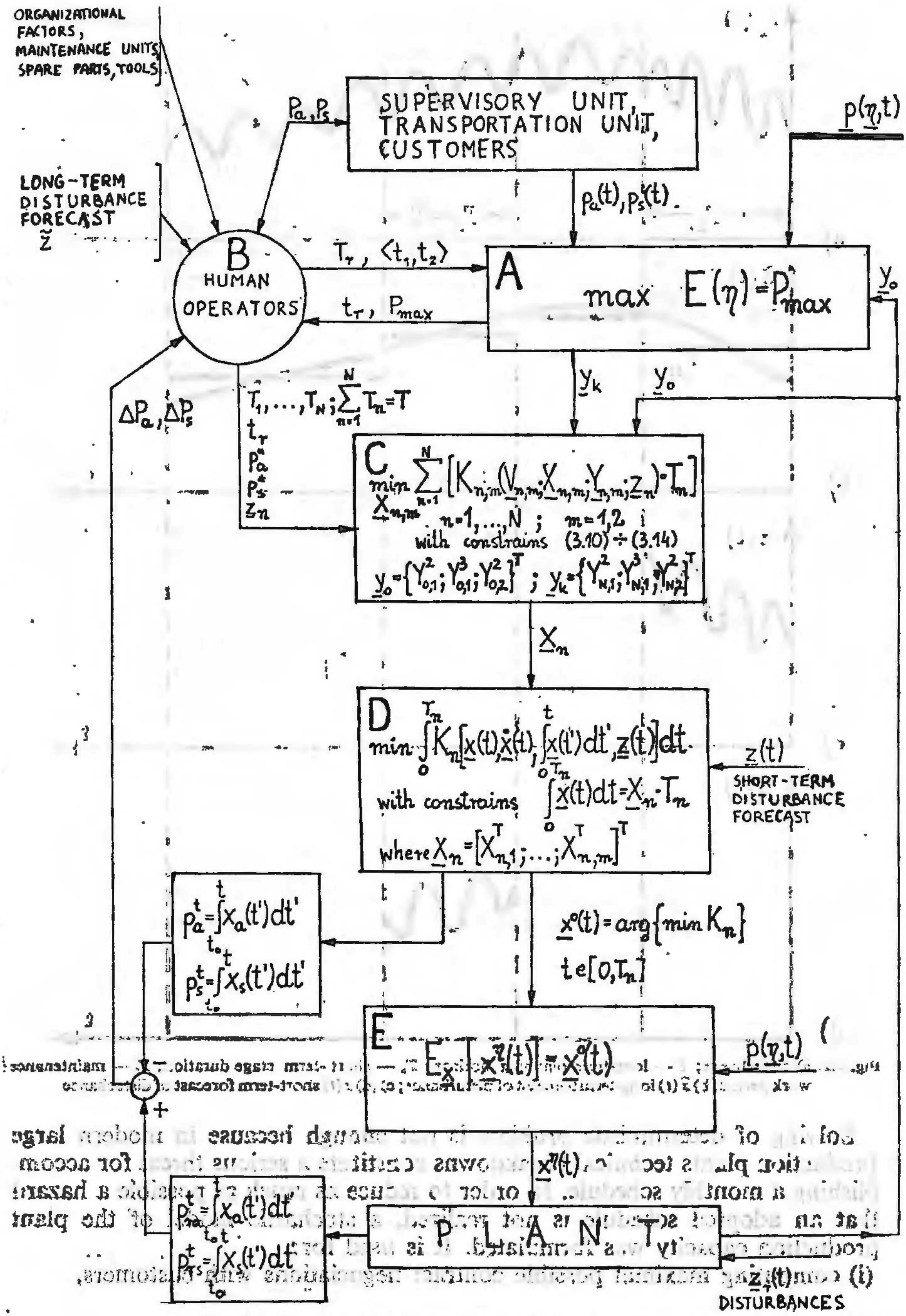


Fig. 2.1. Structure od a system for production operative control

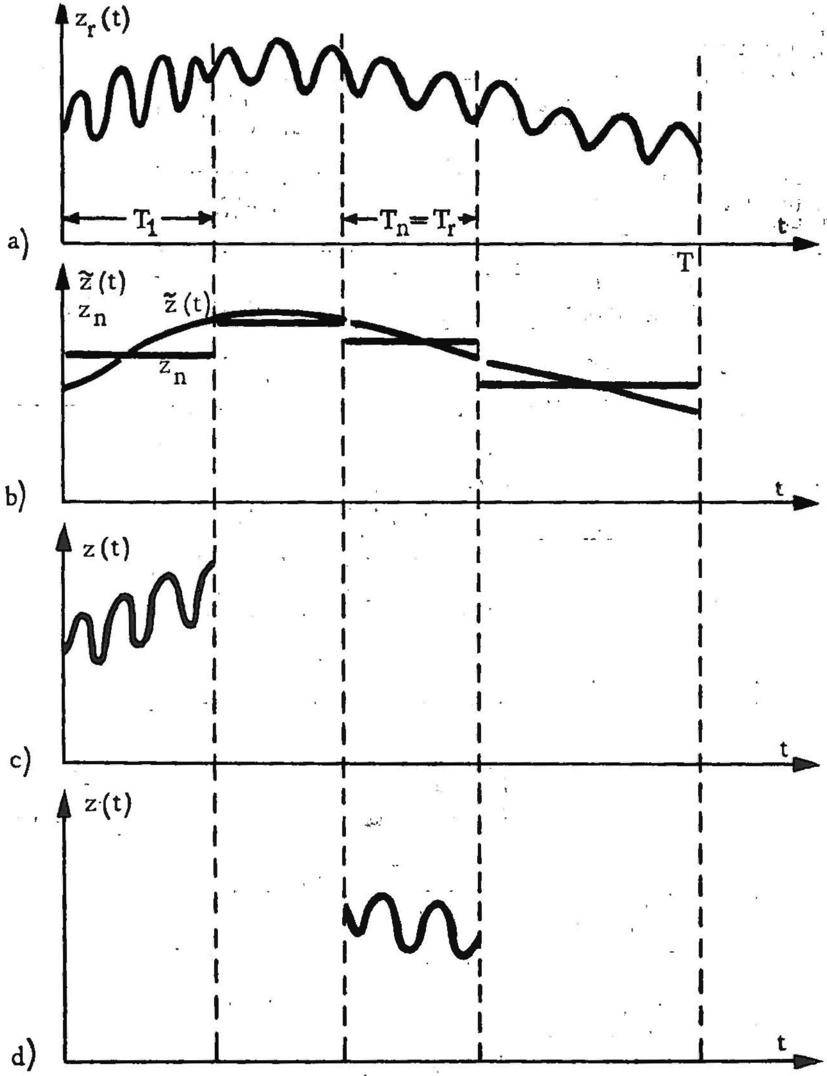


Fig. 2.2. a) disturbance; $T$ — long-term-planning horizon; $T_n$ — short-term stage duration; $T_r$ — maintenance work period; b) $\tilde{z}(t)$ long-term forecast of disturbance; c), d) $z(t)$ short-term forecast of disturbance

Solving of deterministic problem is not enough because in modern large production plants technical breakdowns constitute a serious threat for accomplishing a monthly schedule. In order to reduce as much as possible a hazard that an adopted schedule is not realized, a stochastic model of the plant production capacity was formulated. It is used for:

(i) computing maximal possible contract negociations with customers,

(ii) computing such policies for production load control that the optimal loads resulting from solving the deterministic problem (so called tactic goals) are to be achieved in the presence of breakdowns in terms of the expected values of separate production flows (block E).

In the block—diagram shown in Fig. 2.1. production management functions accomplished by human operatos are represented by block B. The main one constists in correcting the monthly schedule $P_a$ and $P_s$ for ammonia and ammonium nitrate and related contracts in case when such a schedule can not be implemented (it is greater than the maximal output computed by block A) or it has not been realized during the preceding interval. According to procedures for receiving orders adopted at the Nitrogem Works in Włocławek it is planned to repeat long-run computations every 15 days. It is assumed that the long-term schedule horizon is one month or one and half month by turns. Such a planning system makes it possible to correct not accomplished fortnight schedules and to assure the continuity of optimal maintenance scheduling.

A human operator has to determine a time period $[t_1, t_2]$ during which maintenance work are to be started and the period of maintenance work $T_r$. Block $A$ accomplishes maximization of the plant output with respect to a time instant $t_r \in [t_1, t_2]$ defining the start of maintenance work, and with respect to the stock level policy.

The problems mentioned will be discussed one after the other in order presented above {i.e. deterministic problems first (blocks $C$ and $D$) and then stochastic ones (blocks $A$ and $E$)} in the following sections of the paper.

## 3. DECOMPOSITION AND COORDINATION OF MONTHLY PRODUCTION JOB

In the general form the problem of optimizing the plant output during the period $T$ (e.g. one month) can be presented as the following problem of profit maximization

$$\max_{x_a(t),\, x_s(t),\, x(t)} \Big[\int_0^T \Big(x_a(t)\, C_a + x_s(t)\, C_s - \sum_1^L k_i\big(x_i(t),\, z(t),\, t\big)\Big)\, dt - K_s\Big] \tag{3.1}$$

subject to

$$\boldsymbol{d}(t) \leqslant \boldsymbol{x}(t) \leqslant \boldsymbol{g}(t) \tag{3.2}$$

— constraints on control variables (loads)

$$\boldsymbol{m} \leqslant \int_0^t \varphi\big(\boldsymbol{x}(t)\big)\, dt + \varphi^0\big(\boldsymbol{x}(0)\big) \leqslant M \tag{3.3}$$

— constraints on stock levels

$$T(x(t), x_a(t), x_s(t)) = 0 \tag{3.4}$$

— constraints due to the model of a production line

$$d_a(t) \leqslant x_a(t) \leqslant g_a(t) \tag{3.5}$$

$$d_s(t) \leqslant x_s(t) \leqslant g_s(t) \tag{3.6}$$

where $x_a(t)$ — ammonia sale flow (price $C_a$)
$x_s(t)$ — ammonium nitrate sale flow (price $C_s$)
$x_i(t)$ — load of the $i$-th production unit
$z(t)$ — disturbance forecast
$k_i(.)$ — function of variable production costs in the $i$-th production unit
$K_s(.)$ — fixed costs
$d, g$ — constraints on loads
$m, M$ — inventory constraints
$L$ — the number of process units with independently controlled loads.

If monthly plans of ammonia sale ($P_a^*$) or (and) ammonium nitrate sale ($P_s^*$) are given, then new constraints have to be introduced

$$\int_0^T x_a(t)\, dt = P_a^* \tag{3.7}$$

or (and)

$$\int_0^T x_s(t)\, dt = P_s^* \tag{3.8}$$

In such a case the primary problem is reduced to cost minimization

$$\min_{x_a(t),\, x_s(t),\, x_i(t)} \int_0^T \sum_{i=1}^{L} k_i(\dot{x}_i, x_i(t), z(t), t)\, dt \tag{3.9}$$

subject to constraints mentioned above.

The problem formulated can be transformed into a static optimization problem under assumption that the period $T$ can be devided into $N$ stages of the duration $T_n$ $(n=1, \ldots, N)$ due to the mentioned considerations (Fig. 2.2).

The obtained static model for each stage can be decomposed into two subprocesses couples by the ammonia flow. The storage capacity of the ammonia store is relatively large. In consequence solutions to both subprocesses have small sensitivity with respect to this variable. Such a situation makes the coordination of solutions easier.

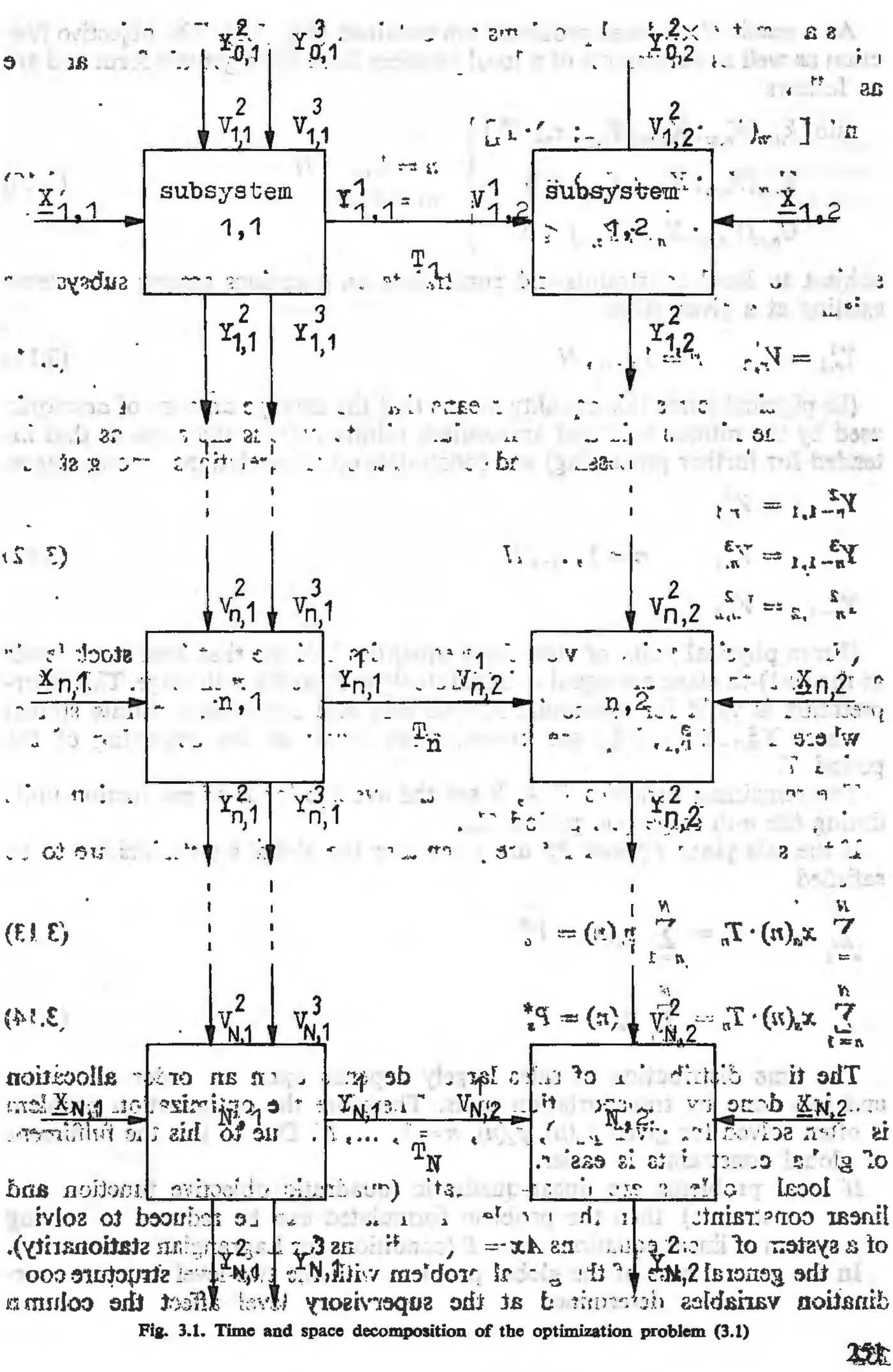


**Fig. 3.1. Time and space decomposition of the optimization problem (3.1)**

As a result $N \times 2$ local problems are obtained (Fig. 3.1). The objective function as well as constraints of a local problem have the algebraic form and are as follows

$$\left.\begin{aligned} &\min\left[k_{n,m}(V_{n,m};\, X_{n,m};\, Y_{n,m};\, z_n)\cdot T_n\right] \\ &\quad T_{n,m}(V_{n,m};\, X_{n,m};\, Y_{n,m}) = 0 \\ &\quad G_{n,m}(V_{n,m};\, X_{n,m};\, Y_{n,m}) \leqslant 0 \end{aligned}\right\} \begin{aligned} &n = 1, \ldots, N \\ &m = 1, 2 \end{aligned} \tag{3.10}$$

subject to local constraints and constraints on couplings among subsystems existing at a given stage

$$Y^1_{n,1} = V^1_{n,2} \qquad n = 1, \ldots, N \tag{3.11}$$

(In physical terms this equality means that the average amount of ammonia used by the nitrous acid and ammonium nitrate units is the same as that intended for further processing) and constraints on interrelations among stages.

$$\begin{aligned} &Y^2_{n-1,1} = V^2_{n,1} \\ &Y^3_{n-1,1} = V^3_{n,1} \qquad n = 1, \ldots, N \\ &Y^2_{n-1,2} = V^2_{n,2} \end{aligned} \tag{3.12}$$

(From physical point of view these equalities indicate that final stock levels at the ($n$-1)-th stage are equal to initial stock levels at the $n$-th stage. This interpretation is valid for ammonia, nitrous acid and ammonium nitrate stores) where $Y^2_{0,1}$, $Y^3_{0,1}$, $Y^2_{0,2}$ are known stock levels at the beginning of the period $T$.

The remaining variables $V$, $X$, $Y$ are the average loads of production units during the $n$-th stage (i.e. period $T_n$).

If the sale plans $P^*_a$ and $P^*_s$ are given then the global constraints are to be satisfied

$$\sum_{n=1}^{N} x_a(n)\cdot T_n = \sum_{n=1}^{N} p_a(n) = P^*_a \tag{3.13}$$

$$\sum_{n=1}^{N} x_s(n)\cdot T_n = \sum_{n=1}^{N} p_s(n) = P^*_s \tag{3.14}$$

The time distribution of sales largely depends upon an order allocation and job done by transportation units. Therefore the optimization problem is often solved for given $p_a(n)$, $p_s(n)$, $n = 1, \ldots, N$. Due to this the fulfilment of global constraints is easier.

If local problems are linear-quadratic (quadratic objective function and linear constraints), then the problem formulated can be reduced to solving of a system of lineat equations $\boldsymbol{A}x = \boldsymbol{B}$ (conditions for Lagrangian stationarity).

In the general case of the global problem with the two-level structure coordination variables determined at the supervisory level affect the column vector $\boldsymbol{B}$ only, while the matrix $\boldsymbol{A}$ is unchanged. It facilitates numerical computations needed to solve the system $\boldsymbol{Ax}=\boldsymbol{B}$, which are repeated in successive iterations generated by the coordinator.

The coordination of interreactions by means of stock levels can be successfully accomplished using the method of price coefficients. Computational difficulties arising in this approach are not significant (solutions are admissible even in the case when determined couplings among subsystems are not fully consistent).

In the case when the ammonia flow is considered as the unit coupling variable the coordination has to be accomplished by parametric or mixed methods. The latter, despite the larger number of parameters, can provide faster convergence of a coordination algorithm.

An obtained solution to the static optimization problem stated for $N$ stages determines production schedules for individual subprocesses at each stage. These schedules can be used as equality (integral) constraints in dynamic optimization problems considered separately for each stage. It makes possible to satisfy the inequality constraints imposed upon stock levels as well as the global constraints $P_a^*$ and $P_s^*$ to be fulfilled during the whole period $T$.

In practice the planning horizon $T$ is subjected to changes. Firstly, having orders specified for one month, a problem with $T$ equal one month is solved. As accomplishing of the adopted schedules proceeds, a deviation of the production output from the accepted values is often observed. After some time (approximately 2 weeks) orders for the next month are also known. It makes possible to correct schedules in the course of 6 weeks period taking into account shortages or surplus gained during the first two weeks of the first month. Hence the planning of a production to be obtained at each stage is realized with a shifting horizon (the shift is of a step type).

## 4. DYNAMIC OPTIMIZATION OF PRODUCTION LOADES FOR SHORT PLANNING HORIZONS

The solution to the static optimization problem discussed in the previous sections determines only stage schedules, i.e. the average policy for control of a production intensity level at successive several-days-long intervals.

For every such an interval there is latitude in the choice of instantaneous values of the production intensity.

It should be mentioned that the process is under the influence of periodic external disturbances changing the operating conditions of the process equipment as well as production costs.

Considering this, it is reasonable to adjust instantaneous production load to disturbances in such a way that each individual stage schedule is accomplished with minimal costs.

The main disturbances taken into account are surrounding temperature fluctuations and different charges for the use of electric energy during day and night times. Temperature fluctuations have impact on most of chemical processes. In the example of the Nitrogen Works considered they significantly influence the operating conditions of compressors and coolers. This has a great impact on manufacturing costs as well as an instantaneous production capacity of an individual production unit.

It is assumed that for several-days-long periods (dynamic optimization horizon) temperature forecasts can be obtained with a sufficient accuracy. Such forecasts are much more accurate that those obtained for a period equal to the horizon of the above mentioned static optimization problem (approximately one month).

It can be assumed that a function describing temperature changes has a given form. e.g. it can be the sum of sinusoidal functions and simple aperiodic function representing variations of the average daily temperature. Unknown coefficients of this function can be determined on the basis of information supplied by weather forecasts.

Let us suppose that the production cost function $K_i(x_i, z)$ is given for each unit. It is such that the expression

$$\int_{t_0}^{t_1} K_i(x_i(t), z(t))\, dt \tag{4.1}$$

represents the total production costs incurred during the interval $[t_0, t_1]$, $x_i(t)$ denotes a production intensity of the $i$-th unit and $z(t)$ is the forecasted vector of external disturbances. Solution to the static problem provides a schedule $P_i$ determining the total production of a given unit at the time interval $T_n$, equal to the $n$-th planning stage, i.e.

$$\int_{0}^{T_n} x_i(t)\, dt = P_i \tag{4.2}$$

A simple dynamic optimization problem consists in determining for a given forecast of disturbances $z(t)$ such production loads $x_i(t)$ of the unit $t \in [0, T]$, that the objective function $Q_i$ (production costs) is minimized

$$Q_i = \int_{0}^{T_n} K_i(x_i(t), z(t))\, dt \tag{4.3}$$

The problem stated can be written in the form of a conventional optimization problem

$$\min_{x_i(t)} \int_{0}^{T_n} K_i(x_i(t), z(t))\, dt \tag{4.4}$$

subject to

$\dot{y}_i = x_i(t)$ — state equation

$y_i(0) = 0$ — initial and final conditions

$y_i(T_n) = P_i$

where $x_i$ — control; $y_i = \int_0^t x_i(t')\, dt'$ — state;

$T_n$ — fixed terminal time; $z(t)$ is a given time function.

Such a problem can be solved with a relative ease for a fairly arbitrary form of the cost function $K_i$ and arbitrary disturbance function $z(t)$. Using the Pontriagin maximum principle, the necessary conditions are derived. The function $x_i(t)$ satisfying these conditions is to be found by numerical methods. The problem formulated was solved on a digital computer for an arbitrarily chosen cost function, but providing a fairly good approximate of the process properties.

For a real production process, variations of instantaneous production intensities are rather troublesome because they should be often accompanied by changes of settings for the whole set of controllers. In general, it can be said that rapid load changes are inexpedient and that step-type changes are often physically unrealizable. For the problem formulated, solutions having the form of step functions can be obtained. Therefore the problem has to be modified in order to eliminate solutions of this type. It can be done by introducting to the problem (4.4) the following constraints imposed on the derivative of the function $x_i(t)$

$$|\dot{x}_i(t)| \leqslant \delta_i, \quad \delta_i = \text{constant}, \quad \delta_i > 0 \tag{4.5}$$

Such an approach involves difficulties and makes solving of the problem much more difficult. Other method consists in the modification of the production cost function by introducing an additional term depending upon the square of the derivative of the function $x_i(t)$ (penalty for rapid load changes)

$$K_i^*(x_i, z) = K_i(x_i, z) + \frac{1}{2\varepsilon_i}(\dot{x}_i)^2, \quad \varepsilon_i > 0 \tag{4.6}$$

Comparing this problem with that of (4.4,) it is evident that the former dimensionality of the state vector is increased from 1 to 2 (state functions $x_i(t)$, $y_i(t)$; control $u_i = \dot{x}_i(t)$). In practice such a modification complicates to some extent a method of solving and results in the increase of a computation effort. However, these differences are not significant.

Problems as stated above are formulated for separate units of the production process and do not take into account all the interconnections among units (series-connected, parallel-connected) as well as inter-unit stores. A complete short-term dynamic optimization problem corresponding to the production process under consideration is as follows

$$\min_{u_i(t)} \int_0^{T_n} \left\{ \sum_i \left[ K_i(x_i(t), z(t)) + \frac{1}{2\varepsilon_i} u_i^2(t) \right] + \right.$$
$$\left. + \sum_l K_{Ml}[y_i^d(t) - y_i(t) + V_{0,l}, z(t)] \right\} dt \tag{4.7}$$

where $l = 1, 2, 3$ — the number of a store
$K_{Ml}(\ldots)$ — invertory carrying cost

$V_{0l}+y_l^d(t)-y_l^0(t)$ — stock level in the $l$-th store at time $t$

$y_l^d(t), y_l^0(t)$ — integrals of flows entering ($d$) and leaving (o) stores. The upper limit is the current instant $t$. They are sums of appropriate functions $y_i(t) = \int_0^t x_i(t')\,dt'$.

$V_{0l}$ — stock level in the $l$-th store at time $t=0$, subject to constraints:

(i) $\forall i \int_0^{T_n} x_i(t)\,dt = P_i$ – stage schedules

(ii) $\forall t\; m_l \leqslant y_l^d(t)-y_l^0(t)+V_{0l} \leqslant M_l$ – constraints on storage capacity

(iii) $H(x)=0$, where $x=[x_i]$ — equation describing interrelations among units.

In the general form this problem is very complicated and multidimensional. What is more, solving of it is very difficult because of the inequality constraints (4.2) imposed on the state variables.

To simplify the problem some assumptions have to be made.

(i) Let us assume that the inventory carrying cost functions $K_{Ml}(\ldots)$ are linear with respect to the first argument, i.e. the stock level

$$K_{Ml} = \alpha_l(z(t))\cdot[y_l^d(t)-y_l^0(t)+V_{0l}]+\beta_l(z(t)) \tag{4.8}$$

Such a simplification seems to be reasonable. For example, as far as the ammonia store is concerned, the amount of energy consumed for cooling is approximately proportional to the ammonia stock level in a store.

(ii) Let us assume that the inequality constraints on storage capacity are not active. This is the case when storage capacities ($M_l$—$m_l$) are sufficiently large and the initial stock levels $V_{ol}$ are well below the maximal ones and well above the zero level. Stock levels $V_{ol}$ are determined by the statistic analysis of process breakdown occurrance as well as properties of finished products transportation and also as a result of the previously discussed static optimization.

The dynamic optimization horizon is at most several-days-long. Due to this the possibility of large variations of the stock level is excluded.

It is easy to show that under such assumptions the problem (4.7) is subdivided into several problems characterized by fewer dimensions. This subdivision is conditioned by the possibility of decomposing the constraint $H(x)=0$ into sets of equalities, such that each specific set contains only a part of the components $x_i$ of the vector $x$ and that there are no components belonging to two groups simultaneously. This follows from the fact that the objective function (4.7) is additive with respect to variables describing different production units and the state equation of a given unit ($u_i, x_i, y_i$ variables have indices $i$ fixed) does not depend upon variables $u, v, y$ corresponding to any other unit. Hence the partial problems correspond to subprocesses containing units coupled by energy or mass flows. This is represented by the constraint $H(x)=0$. If the form of a function $H(x)$ is simple (the small number of chain-or parallel-connected production units), then the problems discussed differ from that of (4.4) with the modified cost function (4.6) only in dimensionality. These problems can be solved on a digital computer.

# 5. STOCHASTIC MODEL OF PRODUCTION CAPACITY

This section is aimed at the description of a particular production capacity model. The model presented makes it possible to take into account, when determining interval schedules, that production capacities are lowered due to technological breakdowns. On the other hand, it allows to use more effectively in the process of dynamic scheduling those periods during which the number and importance of breakdowns are small and the plant is running at full capacity.

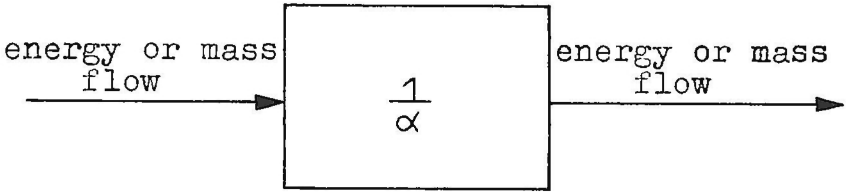


Fig. 5.1. A production unit

As it has been mentioned a production unit sequence consists of units which in the general case transform energy and mass flows (Fig. 5.1). Such processing will be called throughout a production of a given unit.

In the general case a relation between input and output flows involves a coefficient $\alpha$ (5.1)

$$x_{we} = \alpha x_{wy} \tag{5.1}$$

In the breakdown model accepted for the purpose of this paper this coefficient is assumed constant. It is not related to breakdowns (consider for example a stechiometric coefficient of a chemical reaction). However, to simplify further considerations it is assumed that $\alpha = 1$.

A maximal production level achieved at a given time instant (i.e. levels of input or output flows of a given unit) is called a production capacity $\eta$ of a given unit.

Let us assume that $\eta(t)$ is a stationary ergodic stochastic process with independent instantaneous values. Moreover, it is supposed that the probability density function $p(\eta)$ is given for each production unit (Fig. 5.2). $\eta_{\max}$ denotes the maximal production capacity. The dashed area in Fig. 5.2. represents the probability that technical conditions determining the production capacity of a given unit allows to generate an output-flow $x$. Similar density functions can be used to describe raw material sources with variable efficiency as well as transportation units in the case when transport contracts are disturbed by random factors.

If a policy $x^*$ is determined by other constraints an operator supervisioning the process can carry on it in such a way that the production capacity of a unit is not used in full. Hence, in the general case the output flow of a unit is a random variable

$$x = \min\{x^*, \eta\} \tag{5.2}$$

with the probability density function

$$p^*(x) = \delta(x-x^*) \int_x^\infty p(\eta)\, d\eta + p(\eta)\, 1[x-x^*] \tag{5.3}$$

where $\delta(.)$ is the Dirac function and $1[.]$ is a step function.

If a production unit sequence consists of chain-connected units, then the overall production capacity is equal to

$$\eta = \min\{\eta_1, \eta_2\} \tag{5.4}$$

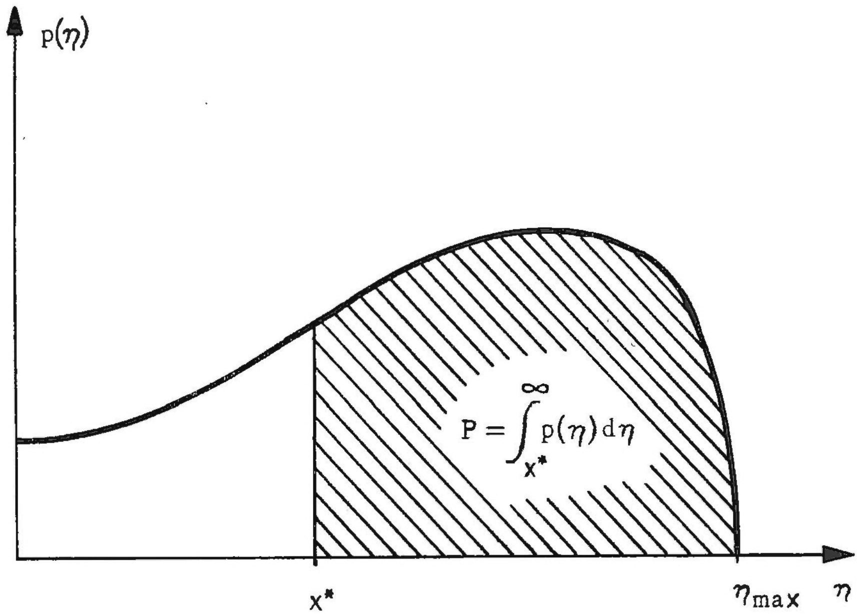


Fig. 5.2. Probability density function of separate unit production capacity

and the probability density function $p(\eta)$ is as follows

$$p(\eta) = p_1(\eta)\, \mathscr{L} p_2(\eta) \overset{df}{=} p_1(\eta)\,[1-F_2(\eta)] + p_2(\eta)\,[1-F_1(\eta)] \tag{5.5}$$

where $p_1, F_1$ — the probability density function and cumulative distribution for the first unit,

$p_2, F_2$ — the probability density function and cumulative distribution for the second unit,

$\mathscr{L}$ — denotes the operator defined in (5.5.)

If a production unit sequence consists of parallel-connected units, then production capacities are summed

$$\eta = \eta_1 + \eta_2 \tag{5.6}$$

and the probability density function is

$$p(\eta) = p_1(\eta) * p_2(\eta) \stackrel{df}{=} \int_{-\infty}^{\infty} p_1(\eta)\, p_2(\eta-\eta_1)\, d\eta_1 \tag{5.7}$$

where * denotes the convolution operator

Aggregation of the density functions of production capacities or pdoduction flows can be accomplished in an arbitrary order (operators $\mathscr{L}$ and * are symmetric and associative)

The role of inter-unit stores consists in:

(i) forming a buffer for asynchronous optimal loads of units preceding and following a store,

(ii) forming a buffer for local breakdowns occurring in units preceding or following a store.

It should be noted that a store can be considered as a buffer only in this case when minimal or maximal stock levels are not achieved.

Let $P_M$ and $P_m$ denote the probabilities of achieving maximal or minimal stock level respectively. Having these probabilities computed (or estimated in the case of computing difficulties) it is possible to determine density functions for all the flows occurring in a production unit sequence containing stores. In Fig. 5.3. a simple example of such a sequence is shown.

Let functions $p_1^*(x)$ and $p_2^*(x)$ stand respectively for the separately computed flow density function of a unit preceding a store and that for one following it. Taking into account that a stock level depends not upon the current

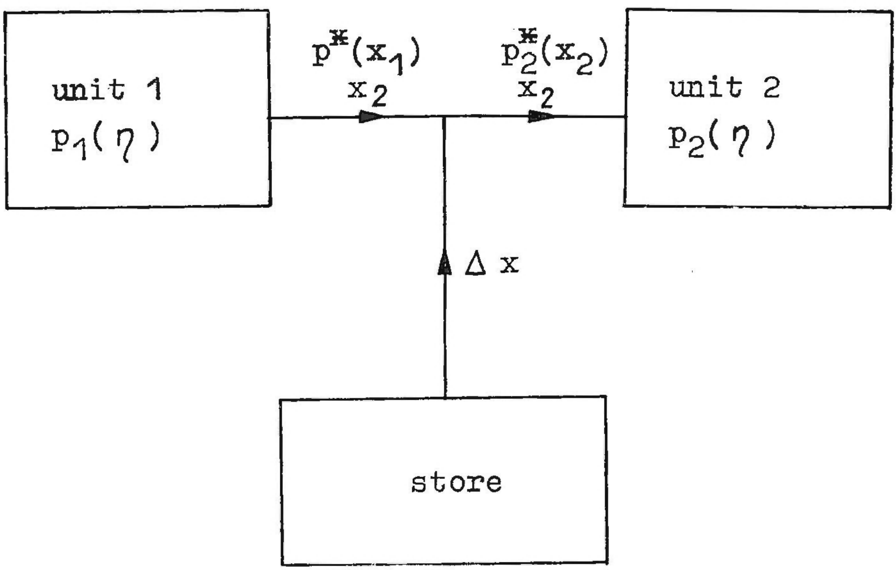


Fig. 5.3. Example of a production unit with a store

values of the flows $x_1$ and $x_2$ but upon history, the following relations are obtained

$$p_1(x_1) = (1-P_M)\,p_1^*(x_1) + P_M[p_1^*(x_1)\,\mathscr{L}\,p_2^*(x_1)] \tag{5.8}$$

$$p_2(x_2) = (1-P_m)\,p_2^*(x_2) + P_m[p_1^*(x_2)\,\mathscr{L}\,p_2^*(x_2)] \tag{5.9}$$

The estimation of the probabilities of achieving maximal or minimal stock levels is based on computing the density function $p(\Delta x)$ of the flow $\Delta x = x_2 - x_1$ (Fig. 5.3). It is done under assumption that the flows $x_1$ and $x_2$ are completely independent. In other words there are no constraints imposed on inventory. The function $p(\Delta x)$ — forms a basis for computing the density function $p(\Delta y)$ of a deviation, determined at the end of the interval $T$, of a stock level in a store with no constraints, from the initial level $y_0$.

Performing the integration for $\Delta y < m - y_0$ and $\Delta y > M - y_0$, the upper estimates of the probabilities $P_M$ and $P_m$ are obtained

$$P_m \leqslant \int_{-\infty}^{m-y_0} p(\Delta y)\,d(\Delta y) \tag{5.10}$$

$$P_M \leqslant \int_{M-y_0}^{\infty} p(\Delta y)\,d(\Delta y) \tag{5.11}$$

The density functions of production flows (or production capacities) determined in such a way form a basis for

(i) computing the expected value of production capacity of the whole production sequence $E(\eta)$ or in other words, the maximal production, maximized with respect to the desired initial stock levels

$$y_k = \arg\{\max_{y_0} E(\eta)\} \tag{5.12}$$

(ii) determining time for maintenance work optimal with respect to the above mentioned expected value of production capacity, computed for the actual initial stock levels $y_0$

$$P_{\max} = \max_{t_r \in [t_1, t_2]} E(\eta) \quad \text{for} \quad y_0 \tag{5.13}$$

(iii) searching for a policy to be applied by an operator in order to achieve — in terms of the expected values — the tactic goal determined by the optimal load computed for the deterministic case

$$E_x[x^\eta(t)] = x^0(t) \tag{5.14}$$

The discussion presented constitutes only outline of the production capacity model and its application. It should be noted that assumptions taken and forms of functions used are often inconsistent with real situations. For example, the value of maximal production capacity $\eta_{\max}$ is subjected to disturbances which are not associated with breakdowns. Hence, $\eta_{\max}$ is a time function and $\eta(t)$ is not a stationary process.

Other important question arises from the necessity of taking into account spectral characteristics of breakdowns. The assumption that instantaneous values of the production capacity are independent is sometimes inconsistent with characteristics of breakdowns. For example the duration of a total breakdown (zero load) is at least as long as time needed to stop and then to start the process.

Of course not all of the problems mentioned are completely solved. Hence the model presented needs further work.

## 6. CONCLUDING REMARKS

Questions discussed in the paper give a cross-section of problems connected with production control at the Nitrogen Works in Włocławek done with respect to the optimization algorithm. The ideas presented are new. Hence it is difficult to estimate advantages resulting from such a method of control. At the Works considered the dependence of operation of separate units upon temperature is not the same for all of them. Significant effects are supposed to be obtained by using stores as buffers in the case of asynchronous operation of production units.

Taking into account that the plant considered consists of two production lines, each having large output of 750 tons daily, and that breakdowns occur frequently, it seems necessary to have methods of evaluating the production capacity.

It seems that under adopted monthly and 6-week schedules (for the described "shifting system") results of such an evaluation can form a good approximate of the real production capacity.

However it should be noted that in order to perform all the computations needed for model implementation, a permanent acquisition and up-dating of data on process equipment conditions are necessary. Hence, the application of the presented production management system depends upon the efficiency of a computer information system used at the Works in Włocławek.

## REFERENCES

[1] FINDEISEN W.: Wielopoziomowe układy sterowania. PWN, Warszawa 1974.

[2] TITLI A.: Commande hierarchisée et optimisation des processus complexes. Dunod, Paris 1975.

[3] BIELUSZKO A., WIŚNIEWSKI J.: Alokacja produktów gotowych oraz komputeryzacja gospodarki remontowej w ZA Włocławek. An internal report of Institute for Organization, Management and Control Sciences, 1974.

[4] BABAROWSKI J., INKIELMAN M., PIETKIEWICZ-SAŁDAN H. WIŚNIEWSKI J.: Metody dekompozycji i optymalizacji komputerowego zarządzania i sterowania złożonymi obiektami na przykładzie zakładu produkcji amoniaku.
An internal report of Institute for Organization, Management and Control Sciences, 1975.
[5] ATHANS M., FALB P. L.: Optimal Control. Mc Graw Hill 1966.
[6] KOPOCIŃSKI B.: Zarys teorii odnowy i niezawodności. PWN, Warszawa 1973.
[7] FABRYCKY W. J., TORGERSEN P. E.: Operations Economy. Prentice--Hall, Inc. Englewood Cliffs, New Jersey 1966.
[8] HANSSMAN F.: Operations research in production and inventory control. John Wiley and Sons, Inc. 1962.

## SUMMARY

The paper concerns the problem of production flows (in different divisions of a fertilizer plant) minimizing production costs.

It is assumed that there are given:

i) monthly production and maintenance plans,

ii) long-term and short—term forecasts of disturbances

iii) stochastic characteristics of production capacities,

iv) plans and constraints on product transport.

The formulated problem is solved by decomposition of the monthly production plan into short-term plans.
Interdivision stock levels are used as the coordination variables. Next, short--term dynamic independent optimal problems are solved.

The obtained results are the basis for direct control and possible on-line optimization of technological parameters.

Instytut Badań Systemowych PAN
Nakład 300 egz. Ark. wyd. 25,0. Ark. druk. 23,75. Papier druk. sat. kl. III 80 g 61×86. Oddano do składania 8 X 1976 Podpisano do druku w sierpniu 1978 r. Druk ukończono w sierpniu 1978 roku

CDW — Zakład nr 5 w Bielsku-Białej zam. 62/K/77 J-124