

**POLSKA AKADEMIA NAUK
INSTYTUT BADAŃ SYSTEMOWYCH**

**PROCEEDINGS OF THE 3rd
ITALIAN-POLISH CONFERENCE ON
APPLICATIONS OF SYSTEMS THEORY
TO ECONOMY,
MANAGEMENT AND TECHNOLOGY**

WARSZAWA 1977

Redaktor techniczny
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The present volume comprises papers from the 1980-1981 Conference which was held in Helsinki, Finland in 1981. The conference was organized by the Center for Economic Research and Education of the Finnish National Research Council and the Institute for Management and Control Sciences of the University of Helsinki. The main theme of the conference was "Optimization and Control Theory". The first two papers in this volume are by the authors of the present volume. The conference was held in Helsinki, Finland in 1981. The main theme of the conference was "Optimization and Control Theory". The first two papers in this volume are by the authors of the present volume.

The structure of the conference was divided into three parts:
1. Optimization and Control Theory
2. Systems Theory in Economics
3. Technological Management and Information Systems

While the first two parts are in other volumes, the third part contains the papers covering the different types of models — for the economic, technological, management and data processing systems.

PROBLEMS OF POLIOPTIMIZATION WITH SEPARATION OF DIFFERENT PERFORMANCE CRITERIA

1. INTRODUCTION

Among practical problems of optimal control of technological and economical systems there are many in which formulation of a single performance criterion is not possible. There may be various criteria characterizing various aspects of systems behaviour that are not comparable between one another. In such cases determination of optimal control is conceptually difficult since the control that is optimal with respect to one performance criterion is usually not optimal with respect to all the others. In other words, while in classical optimal control problems (single objective) a performance criterion introduces complete ordering into the set of controls, there is no such an ordering in case of uncomparable performance criteria, i.e. it is not implied in the formal statement of a problem. Only partial ordering may be introduced into the set of controls and that was done in XIX-th century by V. PARETO [5] for polioptimal (multiobjective) economical problems.

This ordering consists in separation from the set of admissible controls of a subset containing controls having the property that each control being an element of the subset is not worse than all the other admissible controls with respect to all the performance criteria simultaneously. This subset is called a PARETO set of optimal control.

Separation of the Pareto set does not however solve the problem if the set contains more than one element as we still face the problem of selecting one particular control from the Pareto set.

There may be many ways of introducing "deus ex machina" either partial or complete ordering into the Pareto set [6, 7, 8]. Each of them requires defining more or less explicitly on the set of performance criteria some norm ordering the Pareto set. By finding an extremum of this criterion a particular control from the Pareto set, which is regarded the best, is determined.

In the present paper some new type of dynamic polioptimization problems is presented, which to my knowledge, has not been considered by specialists before. It should be emphasised that the purpose of this paper is merely to draw attention to some new problems in the area of polioptimization.

In the polioptimization problems considered in the literature we have a mathematical model of the controlled plant, a set of admissible controls and a set of performance criteria containing more than one subset J_i . The elements of the subsets are values of performance criteria each of which is determined by a control $u(\cdot)$ and an initial event $(t_0, x_0) \in \{T \times X\}$, where T is a set of time instants and X is a set of states.

For the problems considered on this paper it is assumed that the control process is composed of n ($n > 1$) stages and that its performance on each stage is given by means of individual performance criterion J_i ($i = 1, 2, \dots, n$) whose values are determined by the initial event (t_{i-1}, x_{i-1}) for stage i and a control transforming this initial event into an element of the target set of stage i .

We can generalize the problem assuming that on each stage we are given an separate mathematical model of the plant and a separate set of admissible controls.

The polioptimization problem is nontrivial, i.e., it can not be reduced to n independent problems of scalar optimization if the initial event on stages $i = 2, 3, \dots, n$ are not uniquely determined by the statement of the problem but follows from its solution.

The examples given below should clarify the type of problems I have in mind.

In process of heating in heating furnace or in certain chemical reactors two stages can be clearly distinguished. On the first stage a predetermined temperature zone should be reached in minimum time while on the second stage its should be maintained within some time interval at minimum energy cost. The present practice of determining optimal control for such type of processes is based on construction of mixed performance criterion to be minimized, that involves time and energy costs [2]. It is however difficult to set in a rational manner an appropriate value of the coefficient involved in the summation of the two criteria. The approach that leads to minimization of time on the first stage and minimization of the energy cost on the second seams to be more reasonable. It should be noted that the two stages can not be treated independently. Because of the plants dynamics, the time instant separating the first from the second stage must depend on controls implemented on both stages.

Second example concerns a problem of investment expenditure control in case of constructing an industrial plant or a road junction which may gradually come into use before construction process is completed. On the first stage the state that allows for exploitation of the plant is to be achieved in minimum time, while on the second stage losses due to a deviation between full and current efficiency are to be minimized.

Both examples represent problems in which different performance criteria are used in a given time sequence. One can also think about a different case. It deals with controller synthesis in a system which is required to be locally optimal for small deviations from the steady state and at the same time the stability domain around the steady state is required to be as large as possible

to lower the danger of large disturbances and possible break-down [4]. Thus we have two different performance criteria which in this case are valid in different areas of the state space.

2. MATHEMATICAL MODEL OF THE PROCESS AND THE DEFINITION OF POLIOPTIMAL CONTROL

Assume that the control process is carried on in $n(n > 1)$ stages and that each stage $i(i = 1, 2, \dots, n)$ is described by the following dynamic system

$$\sum_i = \sum_i (T_i, U_i, \Omega_i, X_i, Y_i, \Gamma_i, \varphi_i, \eta_i) \quad (1)$$

where

T_i — set of time instants

U_i — set of current values of control variables

$\Omega_i = \{\omega_i : T_i \rightarrow U_i\}$ — set of admissible controls

X_i — set of current values of states

Y_i — set of current values of outputs

$\Gamma_i = \{\gamma_i : T_i \rightarrow Y_i\}$ — set of admissible outputs

$\varphi_i : T_i \times T_i \times X_i \times \Omega_i \rightarrow X_i$ — state transition function whose values are expressed as

$$x_i(\tau_i) = \varphi_i[\tau_i, t_{i-1}, x_{i-1}, u_i(\cdot)] \quad (2)$$

$\eta_i : T_i \times X_i \rightarrow Y_i$ — output function whose values are

$$y_i(\tau_i) = \eta_i(\tau_i, \varphi_i) \quad (3)$$

Let us have n target sets $S_i, i = 1, 2, \dots, n$ which for all dynamic systems \sum_i are subsets of $\{T_i \times X_i \times Y_i\}$

$$S_i = \{T_i^s \times X_i^s \times Y_i^s\} \subset \{T_i \times X_i \times Y_i\} \quad (4)$$

Control $u_i(\cdot)$ transforms initial event (t_{i-1}, x_{i-1}) of stage i into S_i if the following set

$$\{ \{\tau_i, \varphi_i[\tau_i, t_{i-1}, x_{i-1}, u_i(\cdot)], \eta_i(\tau_i, \varphi_i)\} : (t_{i-1} < \tau_i \leq t_i) \} \quad (5)$$

has a nonempty intersection with S_i .

The initial event of stage $i+1, i = 1, 2, \dots, n-1$, is assumed to be an element of the following set

$$P_i = \{T_i^p \times X_i^p\} \subset \{T_i \times X_i\} \quad (6)$$

and it is assumed that P_i has a nonempty intersection with $\{T_i^s \times X_i^s\}$

$$P_i \cup \{T_i^s \times X_i^s\} = \emptyset \quad (7)$$

The above condition means that there exists a pair in the set P_i of initial conditions of stage $i+1$ that is an element of $\{T_i^s \times X_i^s\}$ — a set containing

pairs being parts of tripples from the target set S_i of the proceeding stage i (fig. 1).

The problem of polioptimization considered in this paper is nontrivial if for some i the target set S_i consists of more elements than one.

For each stage $i (i=1, 2, \dots, n)$ and for each control $u_i(\cdot)$ transforming the event (t_{i-1}, x_{i-1}) into S_i the criterion function is formulated in the form

$$J_i[t_{i-1}, x_{i-1}, t_i, x_i, u_i(\cdot)] = K_i(t_i, x_i) + \int_{t_{i-1}}^{t_i} L_i\{\varphi_i[\tau_i, t_{i-1}, x_{i-1}, u_i(\cdot)], u_i(\tau_i), \tau_i\} d\tau_i \quad (8)$$

Our task consists in determining the polioptimal control for a given (t_0, x_0) .

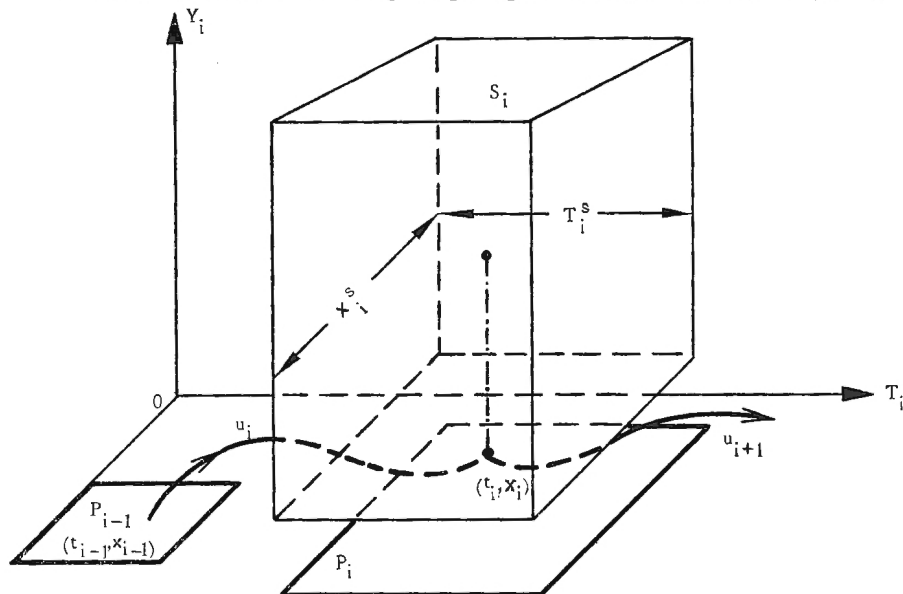


Fig. 1. Graphic representation of the target set $S_i = \{T^S \times X^S \times Y^S\}$ of the i -th stage and the initial conditions set $P_i = \{T^P \times X^P\}$ of the $i+1$ stage

Definition of polioptimal control (a proposal). Control \hat{u} will be called the polioptimal control if it satisfies the following conditions:

$$(i) \quad \hat{u}(\cdot) \in \hat{U} = \{U_1^0 \times U_2^0 \times \dots \times U_n^0\} \quad (9)$$

where U_i^0 is a set of optimal controls on stage i , i.e. a set whose elements $u_i^0(\cdot) \in \Omega_i$ transform events $(t_{i-1}, x_{i-1}) \in P_i$ into $\hat{S}_i \subset S_i$ and minimize the value of J_i along the trajectory φ_i , $(i=1, 2, \dots, n)$.

(ii) Subset $\hat{S}_i \subset S_i (i=1, 2, \dots, n)$ select from the sets S_i such targets $(\hat{t}_i, \hat{x}_i, \hat{y}_i)$, for which the corresponding control $u(\cdot)$ is an element of the Pareto set of optimal controls. Formal definition of the sets S_i may be the following:

$$\hat{S} = \{[\hat{t}_i, \hat{x}_i, \hat{y}_i]: J_i[\hat{t}_{i-1}, \hat{x}_{i-1}, \hat{t}_i, \hat{x}_i, u_i^0(\cdot)] \geq J_i[\hat{t}_{i-1}, x_{i-1}, t_i, x_i, u_i^0(\cdot)], \forall (t_i, x_i, y_i) \in S_i/\hat{S}_i, \forall i\} \quad (10)$$

(iii) Free coordinates of the points $(\hat{t}_i, \hat{x}_i, \hat{y}_i)$, i.e. points at which the optimal control is switched from u_i^0 to u_{i+1}^0 , are determined from the condition of minimization of a chosen norm of vector $\Delta \bar{u}$ which consists of $n-1$ vectors $\Delta \bar{u}_i$

$$\Delta \bar{u}_i = \bar{u}_i[(\hat{t}_i, \hat{x}_i, y_i) \in \hat{S}_i] - u_{i+1}[(\hat{t}_i, \hat{x}_i) \in P_i]; \quad i = 1, \dots, n-1 \quad (11)$$

The vectors \bar{u}_k ($k = i, i+1; i = 1, 2, \dots, n-1$) are constructed in the following way.

First, vector \bar{U}_k is constructed of coordinates that are various fiscal and material factors belonging to sets of controls Ω_i and Ω_{i+1} , e.g. streams of energy, finances and materials, labour force etc.

Then vector $\bar{u}_k(\cdot)$ is created from elements of \bar{U}_k in such a way that the elements of $\bar{u}_k(\cdot)$ are equal to $u_k(\cdot)$ if the corresponding elements of \bar{U}_k are in Ω_k and zeros otherwise.

Summarizing, polioptimal control is composed of segments of controls that are optimal with respect to performance criteria characterizing subsequent stages of the system and that satisfy two conditions: firstly the polioptimal control must be an element of the Pareto set and secondly it must switch from one segment to another in such a way that a certain global measure of discontinuity of the control is minimized.

The second requirement represents the arbitrary norm introducing on ordering to the Pareto set.

It should be noticed that in our case the norm is defined on the set of controls rather than on the set of performance criteria used in formulation of polioptimal control problems.

The choice of the norm may be justified by the following reasoning. If the formulation of a problem does not involve any premises on which ordering of the Pareto set could be based (when it does then the problem is no longer actually polioptimal) then either any control from the Pareto set may be picked up by chance or such an ordering may be based on more general indications following from more general characteristics than those involved in the problem statement. Such a general indication, following from systems point of view, may be the fact that significant rapid changes in control streams, although indifferent from the point of view of the particular problem, may be damaging for the environment. For example rapid changes in energy flows are disturbances affecting all receivers of the network; jumps in materials flows require buffer stores, rapid changes in demand for labour may result in perturbations on the labour market which are dangerous for the society.

Remarks on determining of optimal control. The way of obtaining the polioptimal control follows from its definition. First, stage optimal controls must be determined in a parametric form for all stages. Free coordinates of target points from sets S_i ($i = 1, 2, \dots, n-1$) are the parameters in the stage optimal controls.

Then the free coordinates (parameters) of the switching points are determined from the condition expressing the fact that the global control is an element of the Pareto set and from the condition of minimization of a chosen measure of vector $\Delta \bar{u}(\cdot)$.

This procedure is typical for obtaining solutions of optimization problems in two-level systems [1] (Fig. 2).

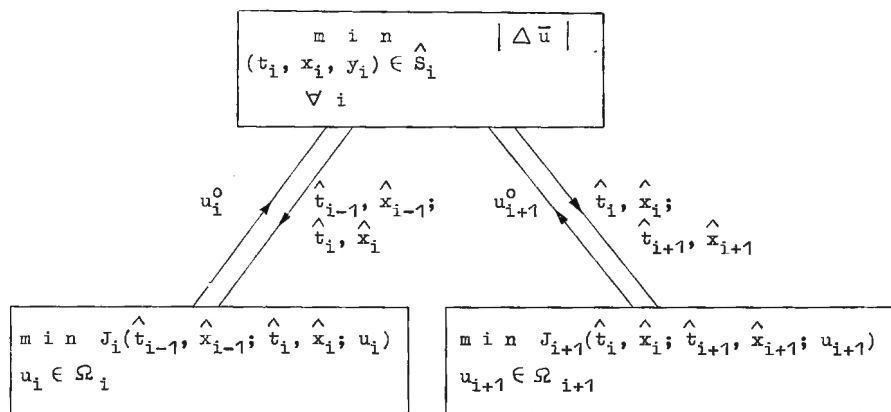


Fig. 2. Determination of polioptimal control in two-level system

On the lower level local optimization is carried on with respect to criterion functions J_i which yields $u_i^0(\cdot) \in \Omega_i$ for target points $(t_i, x_i, y_i) \in S_i, i = 1, 2, \dots, n-1$ specified by the upper level (the target point of the last stage is assumed to be given univocally).

On the upper level minimization of the chosen norm of the vector $\Delta \bar{u}$ is carried on yielding the free coordinates of the target points under the condition $(t_i, x_i, y_i) \in S_i$.

3. EXAMPLE

Consider an example illustrating the above discussion.

Let us assume that the temperature control in a heating furnace is implemented in two stages. On the first stage the furnace is to be heated in the shortest possible time. The second stage has a goal of maintaining prespecified temperature with the minimum heating cost. Maximum admissible overshoot is prespecified to avoid damaging the brick setting.

Let us assume for simplicity, that in the interesting range of temperatures the furnace may be regarded as an integrating element. All quantities express transient states and are related to some quantities characterizing the steady state.

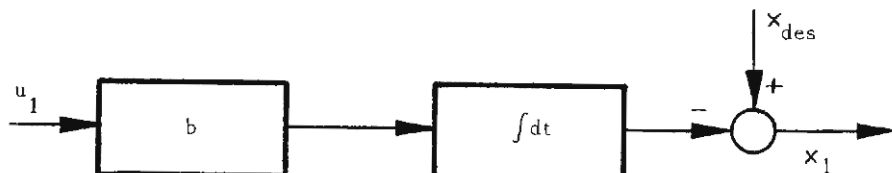


Fig. 3. Block diagram of the heating store at the first stage of heating

At the first stage the model and the performance criterion are the following (Fig. 3)

$$\frac{dx_1}{dt} = bu_1, \quad x_1(0) = X_{des} \quad (12)$$

$$|u_1| \leq L \quad (13)$$

$$J_1(t_1) = \int_0^{t_1} dt \quad (14)$$

where $x_1(t)$ denotes temperature deviation from the desired value X_{des} .

The optimal control in this case has the form (Fig. 4a).

$$u_1^0 = L \quad (15)$$

and the optimal trajectory (Fig. 4b) is given by the equation

$$x_1^0(t) = -bLt + X_{des}; \quad 0 \leq t < t_1 \quad (16)$$

Assume that the target set for the first stage has the following form

$$S_1 = \{(t_1, x_1) : -\varepsilon \geq x_1(t_1) < \gamma\} \quad (17)$$

where $\varepsilon \geq 0$ — admissible overshoot

$\gamma > 0$ — admissible deviation of the temperature below the desired value at time t_1 of switching from the first to the second stage.

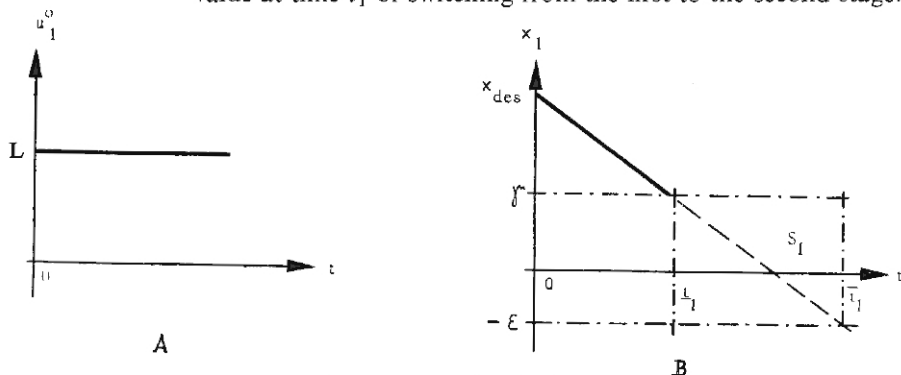


Fig. 4. Time profiles of $u_1(t)$ and $x_1(t)$

t_1^- — lower limit of t_1

t_1^+ — upper limit of t_1

On the second stage temperature control system with a (proportional + integral action) *PI* controller is switched on (Fig. 5).

Systems equations on this stage are the following

$$\frac{dx_1}{dt} = -a_1 x_2 - bu_2 \quad x_1(t_1) = x_{10} \quad (18)$$

$$\frac{dx_2}{dt} = x_1 \quad x_2(t_1) = x_{20} \quad (19)$$

$$u_2 = fx_1 \quad (20)$$

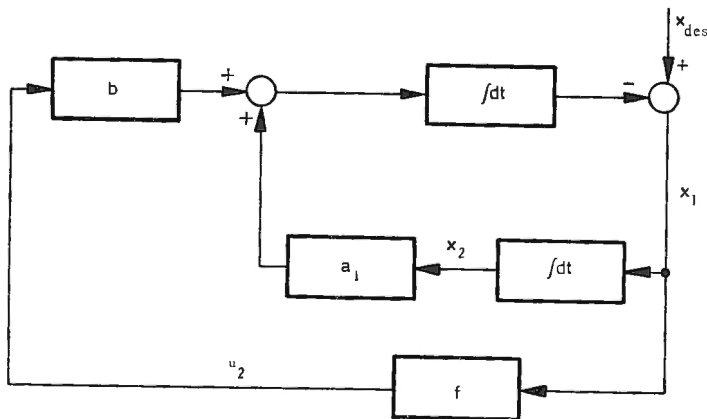


Fig. 5. Block diagram of the control system at the second stage of heating

The performance criterion is assumed to be of the form

$$J_2(t_1) = \int_{t_1}^{\infty} (x_2^2 + \alpha u_2^2) dt \quad (21)$$

The set of initial conditions is given by the equation

$$P_1 = \{(x_{10} = -bL t_1 + X_{des}, \alpha_1 \leq x_{20} < \alpha_2) : x_1(t) \geq -\varepsilon, t_1 \leq t < \infty\} \quad (22)$$

where α_1, α_2 — given parameters

Let the choice of the optimal control consists in selection of the optimum value of parameter f .

After appropriate calculations we get the following result [3]

$$J_2(t_1) = \frac{\alpha f^2 a_1 + 1}{2bf a_1} x_1^2(t_1) - \frac{1}{a_1} x_1(t_1) x_2(t_1) + \frac{1}{2} \left(\frac{\alpha f^2 a_1 + 1}{bf} + \frac{bf}{a_1} \right) x_2^2(t_1) \quad (23)$$

The performance criterion $J_2(t_1)$ assumes its minimum value for $f = f^0$

$$f^0 = \sqrt{\frac{\frac{1}{a_1} x_1^2(t_1) + x_2^2(t_1)}{\alpha x_1^2(t_1) + \left(\alpha a_1 + \frac{b^2}{a_1} \right) x_2^2(t_1)}} \quad (24)$$

Solution of equations (18)–(20) yields

$$x_1(\tau - t_1) = \exp[\varrho(\tau - t_1)] \{ x_{10} \cos[\omega(\tau - t_1)] + \frac{1}{\omega} (\varrho x_{10} + a_1 x_{20}) \sin[\omega(\tau - t_1)] \}; \quad t_1 \leq \tau < \infty \quad (25)$$

$$\text{where } \varrho = -\frac{bf}{2}; \quad \omega = \sqrt{a_1 - \frac{b^2 f^2}{4}} \quad (26)$$

The condition expressing the overshoot limit has the form (Fig. 6)

$$x_i(T) = -\varepsilon$$

$$\left. \frac{dx_1}{dt} \right|_{t=T} = 0 \quad \text{for } t_1 \leq T < \frac{\pi}{\omega} + t_1 \quad (27)$$

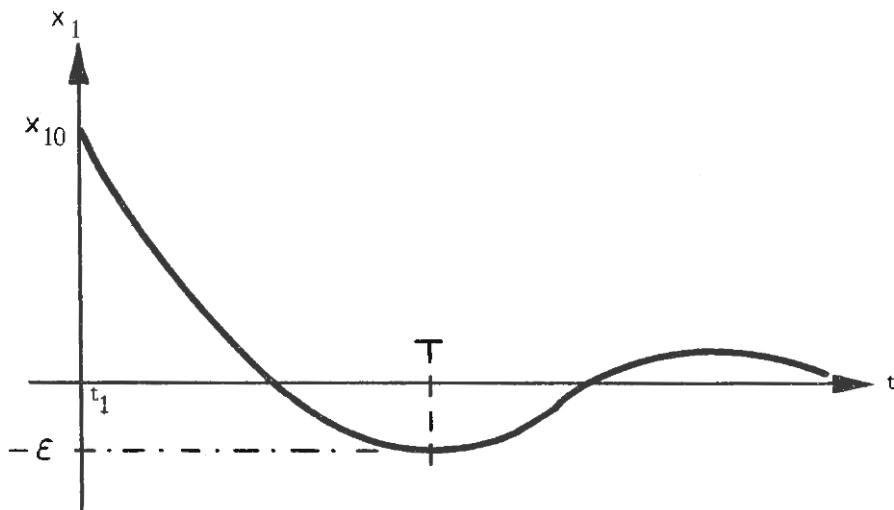


Fig. 6. Example of $x_1(t)$ profile illustrating equations (27)

More computations have been done on a computer for concrete values of all parameters involved in the equations:

— for given range of values of x_{20} , values of f^0 and x_{10} satisfying equations (24) and (27) have been computed. The results make possible choice of t_1 and x_{10} minimizing the control jump at time t_1 from $u_1^0 = L$ to $u_2^0 = f^0 x_{10}$.

— for values x_{10} and x_{20} satisfying equations (24) and (27) J_2 have been calculated. On the basis of these results it can be found out whether the range of x_{10} ($-\varepsilon \leq x_{10} < \gamma$) results in polioptimal controls, i.e. controls being elements of the Pareto set. As

$$\left. \frac{\delta J_1}{\delta x_{10}} \right|_{u_1 = u_1^0} < 0 \quad (28)$$

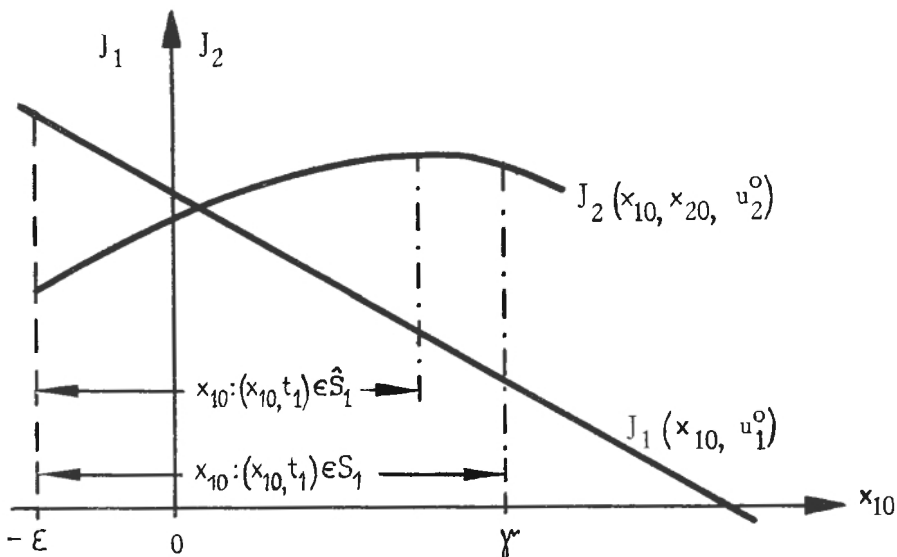


Fig. 7. Example of possible plots of $J_1(x_{10}, u_1)$ and $J_2(x_{10}, x_{20}, u_2)$ versus x_{10} , illustrating the separation of region \hat{S}

the Pareto set contains only those controls that switch from u_1^0 to u_2^0 at the values of x_{10} satisfying the inequality (Fig. 7)

$$\left. \frac{\delta J_2}{\delta x_{10}} \right|_{u_2 = u_2^0} > 0 \quad (29)$$

Note that in the problem of choice x_{10} and x_{20} there is only one excess variable, as we have three equations given by (24) and (27) that are to be satisfied and four variables involved: f^0, T, x_{10}, x_{20} .

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SUMMARY

In the paper a new kind of polioptimization problem is presented. It concerns the situations in which the control process consists of a number of stages and the particular stages have different performance criteria.

The problems of this appear during design and realization of new projects and in some discontinuous technological processes.

The polioptimization problem was formulated and as its solution control satisfying the following conditions is presented:

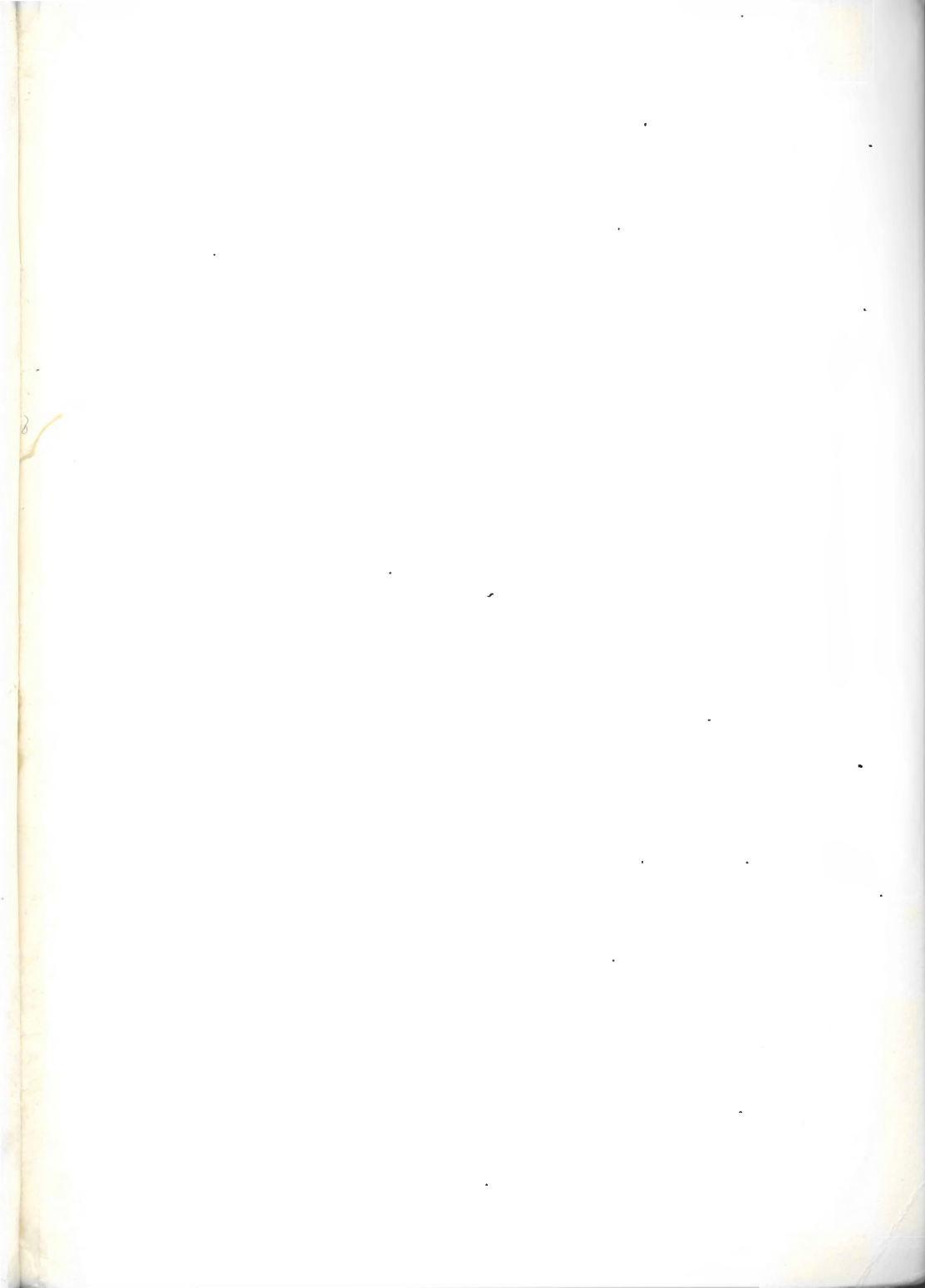
- it consists of pieces of locally optimal controls on consecutive stages,
- it belongs to the Pareto set,
- it minimizes a global measure of discontinuity in passing from one locally optimal control to the next one. The polioptimal control can be formulated in two-level optimization system.

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