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# Volume II: Applications

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# Possible application of new intuitionistic fuzzy set distance to game method for modelling of forest fire spread

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**Abstract**

In the present paper a new distance between intuitionistic fuzzy sets is defined – the result of which is an ordered intuitionistic fuzzy pair. A way of applying it to configurations resulting from modeling fire spread with game method for modeling with intuitionistic fuzzy estimates is proposed.

**Keywords:** Intuitionistic Fuzzy Set, Distance, game method for modelling, fire spread

## 1 Introduction

In a series of papers [3–8] several ideas for approximating (and predicting) forest fire spread through Game method for modelling [2] have been proposed.

The basic structure of these models lies in the following assumptions - the area of concern is represented by a grid (square or hexagonal) in which under certain rules the process (in this particular case - fire spreading) the process develops iteratively. A simple illustration of the way these types of models develop is taken from [5] is shown on Figure 1. Although in this particular case the values

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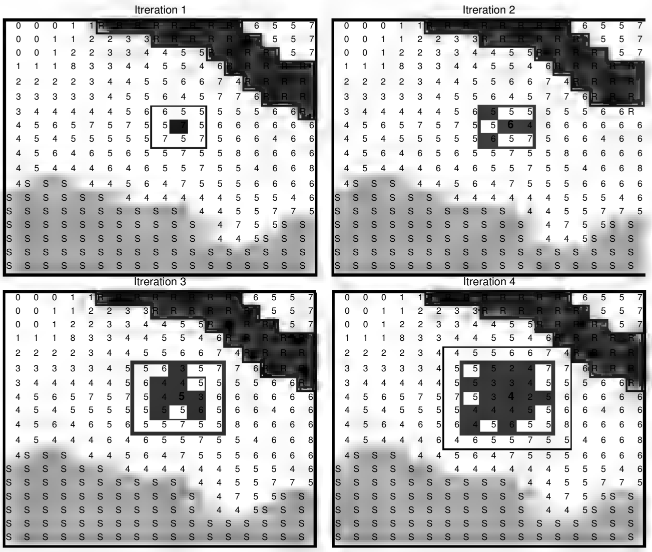

Figure 1: First iterations of a simplified fire spread model with GMM

describing the spread are not intuitionistic fuzzy pairs, it is entirely possible to implement such models. For this purpose we consider a new distance that could help us choose the most appropriate among them.

## 2 The proposed distance

Here we will introduce and investigate new distance the result of which is an ordered pair of values from the interval $[0, 1]$. Let the intuitionistic fuzzy sets $A$ and $B$ be given (defined over the same (discrete) universe $X$). For the basic definitions of intuitionistic fuzzy sets we refer the reader to [1].

**Definition 1.** *Given an intuitionistic fuzzy sets $A$ defined over a universe $X$ and $X^* \subset X$ let*

$$A/X^* \stackrel{\text{def}}{=} \{\langle x, \mu_A(x), \nu_A(x)\rangle | x \in X^*\}$$

Let $X^0(A, B)$ denote the maximal subset of $X$, such that for the two fixed *IFSs* $A$ and $B$

$$d(A/X^0(A, B), B/X^0(A, B)) = 0,$$

where $d$ : IFS$(X^0(A, B)) \times$ IFS$(X^0(A, B)) \to [0, 1]$ is a distance (normalized with a coefficient inversely proportional to the number of elements of $X^0$). Of course, if $X^0(A,B) = \emptyset$, then we agree that

$$A/\emptyset \overset{\text{def}}{=} \emptyset.$$

and thus

$$|X^0(A,B)| \overset{\text{def}}{=} 0.$$

We can now define another type of distance between two intuitionistic fuzzy sets

**Theorem 1.** *Let $d$ be a distance between $A$ and $B$ defined over $X$ (for simplicity, we will only consider discrete universe sets $X$). Then*

$$D(A,B) = \left\langle 1 - \frac{|X^0(A,B)|}{|X|}, |X \setminus X^0(A,B)| d(A/X \setminus X^0(A,B), B/X \setminus X^0(A,B)) \right\rangle$$

*is a distance between the IFSs $A$ and $B$ defined over the same universe set $X$.*

*Proof.* The first property is obvious, i.e.

$$D(A,B) = \langle 0, 0 \rangle \Leftrightarrow A = B$$

and also

$$D(A,B) = \langle a, b \rangle \geq \langle 0, 0 \rangle$$

where $\langle a, b \rangle \geq \langle c, d \rangle$ is to be understood as $a \geq c$ and $b \geq d$. The symmetry is ensured since $d$ is a distance. It remains only to prove the triangle inequality, i.e.

$$D(A,B) + D(B,C) \geq D(A,C) \tag{1}$$

If any two of these sets coincide the inequality is trivial. Let $A, B$ and $C$ be different. Then let us denote $D(A,B) = \langle a_1, b_1 \rangle$, $D(B,C) = \langle a_2, b_2 \rangle$ and $D(A,C) = \langle a_3, b_3 \rangle$. We have to show that $a_1 + a_2 \geq a_3$ and $b_1 + b_2 \geq b_3$.

Let us look at $a_1 + a_2 \geq a_3$. We have

$$a_1 + a_2 = 2 - \frac{|X^0(A,B)| + |X^0(B,C)|}{|X|}; a_3 = 1 - \frac{|X^0(A,C)|}{|X|}$$

Thus $a_1 + a_2 \geq a_3$ is equivalent to

$$1 \geq \frac{|X^0(A,B)| + |X^0(B,C)|}{|X|} - \frac{|X^0(A,C)|}{|X|}$$

i.e. to

$$|X| \geq |X_0| + |X_1| - |X_2| \tag{2}$$

where $X_0 = X^0(A, B)$, $X_1 = X^0(B, C)$, $X_2 = X^0(A, C)$. Let us consider

$$|X^*| \stackrel{\text{def}}{=} |X_0| + |X_1|. \tag{3}$$

We have two cases:

$(a)$ $|X^*| \leq |X|$

$(b)$ $|X^*| \geq |X|$

Case $(a)$ is trivial since (2) is obviously true.

Let us consider case $(b)$. Let us first suppose that $|X_0 \cap X_1| = 0$, i.e. $X_0$ and $X_1$ have no common elements, and since $X_0 \subset X$, $X_1 \subset X$ we have $X_0 \cup X_1 \subseteq X$, thus we have

$$|X^*| = |X_0| + |X_1| \leq |X|,$$

hence at best they are equal and (2) is fulfilled.

Finally let $|X_0 \cap X_1| \neq 0$. Then

$$|X| = |X_0| + |X_1| - |X_0 \cap X_1|$$

since $X_0 \setminus (X_0 \cap X_1)$ and $X_1$ have no common elements and are both subsets of $X$.

Now using the fact that $X_0 \cap X_1 \subseteq X_2$, i.e. $|X_0 \cap X_1| \leq |X_2|$, we obtain that

$$|X| \geq |X^*| - |(X_0 \cap X_1)| \geq |X^*| - |X_2|$$

hence (2) is fulfilled.

Let us now consider the second component $b_1 + b_2 \geq b_3$. We have to show that:

$$\begin{aligned}(|X| - |X_0|)(d(A/(X \setminus X_0), B/(X \setminus X_0)))+\\ (|X| - |X_1|)(d(B/(X \setminus X_1), C/(X \setminus X_1))) \geq\\ (|X| - |X_2|)(d(A/(X \setminus X_2), C/(X \setminus X_2)))\end{aligned}$$

The last is obviously true since $d$ is a normalized distance and its normalizing coefficient is inversely proportional to the number of elements in the sets, which happen to coincide exactly with the coefficients in the above inequality.

This completes the proof. □

# 3 Application of the new distance

Let there be given a finite number of rulesets with intuitionistic fuzzy estimates for the forest fire development $\{R_1, R_2, \ldots, R_k\}$, where $k > 1$ is a natural number. Let the result of applying the $i$-th rule at $j$-the iteration result in a configuration denoted by $C_{i,j}$.

We can represent the actual (discretized in iterations) development of the forest fire (or one predicted by another method/model/simulation tool) with intuitionistic fuzzy sets whose membership functions are strictly 1s or 0s for the elements, and the non-membership are the complementary. We will denote these sets by $F_j$.

What we will be further interested in are two things: how close are the results produced by our rules to the actual forest fire development and how similar they are to each other.

In other words we consider the distances $D(F_j, C_{i,j})$ and $D(C_{i,j}, C_{m,j})$ with $m \neq i$.

For each iteration $j$ we have a preference towards such rule $C_i$ that has a score $\langle a_{i,j}, b_{i,j} \rangle$ which is close to $\langle 0, 0 \rangle$. We are only interested in such scores that have $b \leq \varepsilon$, where $\varepsilon$ is preliminarily chosen threshold value. Among all these preference is to be given to those that have $a < \theta$, where $\theta$ is another threshold value. Let us define:

$$B(i,j) = \begin{cases} 1 & \text{if } b_{i,j} \leq \varepsilon \\ 0, & \text{otherwise} \end{cases} \quad (4)$$

$$A(i,j) = \begin{cases} \frac{1}{2} & \text{if } a_{i,j} \leq \theta \\ 0, & \text{otherwise} \end{cases} \quad (5)$$

Thus we can assign a rank of a rule as follows:

$$r(R_i) = \sum_{j=1}^{N} B(i,j) + \sum_{j=1}^{N} A(i,j), \quad (6)$$

where $N$ is the number of total iterations.

This allows us to sieve through rules which are inadequate (in terms of good enough score) and discard them.

# 4 Conclusion

We have proposed a method for ranking rulesets aimed at representing forest fire spread. This also allows for discarding rulesets that fall beneath a certain threshold of accuracy (compared to real or simulated data).

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The papers presented in this Volume 2 constitute a collection of contributions, both of a foundational and applied type, by both well-known experts and young researchers in various fields of broadly perceived intelligent systems.

It may be viewed as a result of fruitful discussions held during the Twelfth International Workshop on Intuitionistic Fuzzy Sets and Generalized Nets (IWIFSGN-2013) organized in Warsaw on October 11, 2013 by the Systems Research Institute, Polish Academy of Sciences, in Warsaw, Poland, Institute of Biophysics and Biomedical Engineering, Bulgarian Academy of Sciences in Sofia, Bulgaria, and WIT - Warsaw School of Information Technology in Warsaw, Poland, and co-organized by: the Matej Bel University, Banska Bystrica, Slovakia, Universidad Publica de Navarra, Pamplona, Spain, Universidade de Tras-Os-Montes e Alto Douro, Vila Real, Portugal, Prof. Asen Zlatarov University, Burgas, Bulgaria, and the University of Westminster, Harrow, UK:

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The consecutive International Workshops on Intuitionistic Fuzzy Sets and Generalized Nets (IWIFSGNs) have been meant to provide a forum for the presentation of new results and for scientific discussion on new developments in foundations and applications of intuitionistic fuzzy sets and generalized nets pioneered by Professor Krassimir T. Atanassov. Other topics related to broadly perceived representation and processing of uncertain and imprecise information and intelligent systems have also been included. The Twelfth International Workshop on Intuitionistic Fuzzy Sets and Generalized Nets (IWIFSGN-2013) is a continuation of this undertaking, and provides many new ideas and results in the areas concerned.

We hope that a collection of main contributions presented at the Workshop, completed with many papers by leading experts who have not been able to participate, will provide a source of much needed information on recent trends in the topics considered.

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