# Modern Approaches in Fuzzy Sets, Intuitionistic Fuzzy Sets, Generalized Nets and Related Topics Volume II: Applications


Editors

Krassimir T. Atanassov
Władysław Homenda
Olgierd Hryniewicz
Janusz Kacprzyk
Maciej Krawczak
Zbigniew Nahorski
Eulalia Szmidt
Sławomir Zadrożny



SRI PAS IBS PAN

Systems Research Institute
Polish Academy of Sciences

# Modern Approaches in Fuzzy Sets, Intuitionistic Fuzzy Sets, Generalized Nets and Related Topics. Volume II: Applications

Editors


Krassimir Atanassov
Władysław Homenda
Olgierd Hryniewicz
Janusz Kacprzyk
Maciej Krawczak
Zbigniew Nahorski
Eulalia Szmidt
Sławomir Zadrożny



IBS PAN SRI PAS

Systems Research Institute
Polish Academy of Sciences
Newelska 6, 01-447 Warsaw, Poland
www.ibspan.waw.pl

**ISBN 83-894-7554-5**

# Computer-based support in multicriteria bargaining with use of the generalized Nash solution concept

**Lech Kruś**
Systems Research Institute, Polish Academy of Sciences,
Newelska 6, 01-447 Warsaw, Poland
e-mail: krus@ibspan.waw.pl

**Abstract**

A decision situation is considered in which two decision makers negotiate cooperation conditions to realize a joint project. Each decision maker has his own set of criteria measuring results of the cooperation. The situation is modeled as the multicriteria bargaining problem. Construction of the computer-based system supporting the negotiation process is proposed. A special multiround mediation procedure is presented. According to the procedure the system supports multicriteria analysis made by the decision makers and generates mediation proposals. The mediation proposals are derived on the basis of the original solution to the multicriteria problem, presented in the paper. The solution expresses preferences of the decision makers. It generalizes the classic Nash solution concept on the multicriteria case.

**Keywords:** computer-based intelligent systems, decision support, multicriteria analysis, cooperative games, mediation.

## 1 Introduction

The paper deals with computer intelligence problems related to construction of a computer-based system playing the role of a mediator in a bargaining process. The bargaining process is considered in the case of two decision makers discussing cooperation conditions to realize a joint project. The cooperation is possible if

***Modern Approaches in Fuzzy Sets, Intuitionistic Fuzzy Sets, Generalized Nets and Related Topics. Volume II: Applications*** *(K.T. Atanassow, W. Homenda, O. Hryniewicz, J. Kacprzyk, M. Krawczak, Z. Nahorski, E. Szmidt, S. Zadrożny, Eds.), IBS PAN - SRI PAS, Warsaw, 2014*

it is beneficial for both of them. It is assumed that each of the decision makers has his individual set of objectives which he would like to achieve. Achievements of the objectives are measured by given vectors of criteria, which are in general different for each decision maker. The criteria are conflicting in the case of the individual decision maker as well as between them. Each decision maker has also his individual preferences defined in the space of his criteria.

The bargaining process will succeed if the final cooperation conditions satisfy desirable benefits of each decision maker, measured by his criteria and valuated according to his individual preferences. Information about possibilities and preferences of each decision maker is confidential. In many situation, at beginning of the bargaining process, decision makers can not be conscious of their preferences if they have not enough information about the attainable payoffs.

Let us consider the simplest buying - selling bargaining problem.
A buyer and a seller propose prices of a good trying to find a consensus. The consensus is possible if there exists an interval of prices beneficial for both sides, called as an agreement set. Typically, the positional negotiation are applied in this case. The positional negotiations frequently lead to an impasse, and the negotiations can not succeed even if the agreement set is not empty. To resolve the problem in this case, but also in the case of more complicated negotiations, special procedures are applied with a mediation support. A mediator in the negotiations is an impartial outsider who tries to aid the negotiators in their quest to find a compromise agreement. The mediator can help with the negotiation process, but he does not have the authority to dictate a solution.

In the considered multicriteria bargaining process each of the two decision makers valuates variants of cooperations with use of his own vector of criteria. A compromise variant should be found which will be accepted by both sides despite the fact that the criteria are conflicting in the case of each decision maker as well between them. In practice of complicated international negotiations so called Single Negotiation Text Procedure is frequently applied. The procedure has been proposed by Roger Fisher during the Camp David negotiations to resolve an impasse which has occurred after several initial rounds of the positional negotiations (see Raiffa [14]). According to the procedure a negotiation process consists of a number of rounds. In each round a mediator prepares a package for the consideration of protagonists. Each package is meant as a single negotiation text to be criticized by protagonists then modified and remodified. Typically the negotiation process starts from the first single negotiation text which is far from the expectations of protagonists. The process is progressive for each of the protagonists.

A question arises: can a specially constructed computer-base system play a role of mediator and support the negotiation process? This question is discussed in the paper in the case of the mentioned decision situation formulated as the multicriteria bargaining problem. The problem is presented in Section 2. The multicriteria bargaining problem is a generalization of the bargaining problem formulated and discussed in the classic game theory by many researchers, including (Nash [10, 11], Raiffa [13], Kalai and Smorodinsky [3], Roth [15], Thomson [16], Peters [12], Moulin [9]) and others. In these papers many different solution concepts have been proposed and analyzed. In the classic theory the decision makers are treated as players playing the bargaining game and it is assumed that each of them has explicitly given utility function measuring his payoffs. The solution is looked for in the space of the utilities. In the multicriteria problem considered in this paper, the payoff of each decision maker (player) is measured by a vector of criteria and we do not assume that his utility function is given explicitly. The solution is looked for in the space being the cartesian product of the multicriteria spaces of the players. The solution concepts proposed in the classic theory do not transfer in a simple way to the multicriteria case. Let us see that looking for a solution in the multicriteria bargaining problem we have to consider jointly two decision problems: the first - the solution should be related to the preferences of each of the players, and the second the solution should fulfill fairness rules accepted by the players.

A general structure of the proposed computer-based system is presented in Section 3. The system supports solving both the decision problems with use of a special mediation procedure. The procedure has been inspired by the mentioned Single Negotiation Text procedure. The system includes modules supporting multicriteria analysis made by the decision makers independently, in the phase called as unilateral analysis, and a module generating mediation proposals analyzed by the decision makers in consecutive rounds.

The following Sections 4, 5, 6 present respectively proposals including: the mediation procedure, the unilateral analysis support and the formulation of the solution to the multicriteria bargaining problem, which is used to generate mediation proposals. The solution generalizes the Nash solution concept on the case of multicriteria payoffs of players.

This paper continues the line of research dealing with multicriteria payoffs of players in bargaining presented in papers (Kruś and Bronisz [8], Kruś [7], [6], [5]).

## 2 Multicriteria bargaining problem

Let us consider two decision makers negotiating conditions of possible cooperation.

Each decision maker $i$, $i = 1, 2$ has defined decision variables, denoted by a vector $x_i = (x_{i1}, x_{i2}, \ldots x_{ik^i})$, $x_i \in I\!R^{k^i}$, where
$k^i$ is a number of decision variables of decision maker $i = 1, 2$, and
$I\!R^{k^i}$ is a space of his decisions.
Decision variables of all the decision makers are denoted by a vector $x = (x_1, x_2) \in I\!R^K$, $K = k^1 + k^2$, where $I\!R^K$ is the cartesian product of the decision spaces of the decision makers 1 and 2.

It is assumed that results of the cooperation are measured by a vector of criteria which is in general different for each decision maker. The criteria of the decision maker $i$, $i = 1, 2$, valuating his payoff are denoted by a vector $y_i = (y_{i1}, y_{i2}, \ldots y_{im^i}) \in I\!R^{m^i}$, where
$m^i$ is a number of criteria of the decision maker $i$, and
$I\!R^{m^i}$ is a space of his criteria.
The criteria of all the decision makers are denoted by $y = (y_1, y_2) \in I\!R^M$, $M = m^1 + m^2$, where $I\!R^M$ is the cartesian product of the citeria spaces of all the decision makers.

We assume that a mathematical model is given describing payoffs of the decision makers, being a result of the decision variables undertaken by them. The model implemented in a computer based system will be used to derive the payoffs of the decision makers for given variants of the decision variables.

Formally we assume that the model is defined by a set of admissible decisions $X_0 \subset I\!R^K$, and by a mapping $F : I\!R^K \to I\!R^M$ from the decision space to the space of the criteria. A set of attainable payoffs, denoted by $Y_0 = F(X_0)$ is defined in the space of criteria of all decision makers. However each decision maker has access to information in his criteria space only. In the space of criteria of $i$-th decision maker a set of his attainable payoffs $Y_{0i}$, can be defined, being an intersection of the set $Y_0$. The set of attainable payoffs of every decision maker depends on his set of admissible decisions and on the set of admissible decisions of other decision maker. An example illustrating the sets of admissible payoffs $X_{0i}$ of the decision makers $i = 1, 2$, as well as the sets of their attainable payoffs $Y_{0i}$ is presented in Fig. 1. In this example each decision maker has two different criteria. The set $Y_0$ is defined in 4 dimensional space. The sets $Y_{01}$, $Y_{02}$ represent intersections of the set in the criteria spaces of the decision makers 1 and 2 respectively. The sets $Y_{01}$, $Y_{02}$ are mutually dependent. A form of the set $Y_{01}$ depends on the payoff of the decision maker 2, and a form of the set $Y_{02}$ depends on the payoff of the decision maker 1.

A partial ordering is introduced in the the criteria spaces. Let $I\!R^m$ denote a space of criteria for an arbitrary number $m$ of criteria. Each of $m$ criterions can be maximized or minimized. However, to simplify the notation and without loss

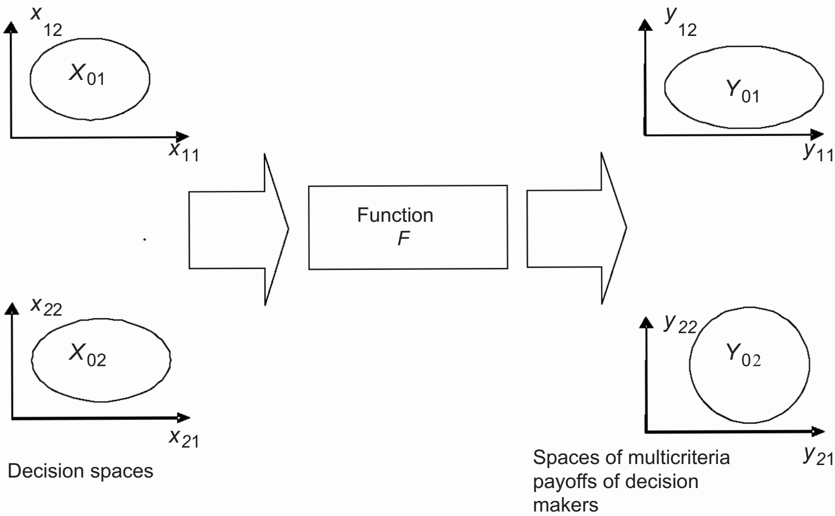


Figure 1: Sets of admissible decisions of the decision makers and the subsets of the attainable mulicriteria payoffs.

of generality we assume that decision makers maximize all their criteria.

Let $z, y \in I\!R^m$, we say, that
a vector $z$ **weakly dominates** $y$, and denote $z \geq y$, when $z_i \geq y_i$ for $i = 1, 2, \ldots, m$,
a vector $z$ **dominates** $y$, and denote $z > y$, when $z_i \geq y_i, z \neq y$ for $i = 1, 2 \ldots, m$,
a vector $z$ **strongly dominates** $y$, and denote $z \gg y$, when $z_i > y_i$ for $i = 1, 2 \ldots, m$.

A vector $z \in I\!R^m$ is **weakly Pareto optimal** (weakly nondominated) in set $Y_0 \subset I\!R^m$ if $z \in Y_0$ and does not exist $y \in Y_0$ such, that $y \gg z$.
A vector $z \in I\!R^m$ jest **Pareto optimal** (nondominated) in set $Y_0 \subset I\!R^m$ if $z \in Y_0$ and does not exist $y \in Y_0$ such, that $y \geq z$.

A bargaining problem with multicriteria payoffs of decision makers (multicriteria bargaining problem) can be formulated by a pair $(S, d)$, where the element $d = (d_1, d_2) \in Y_0 \subset I\!R^M$ is a disagreement point, and the set $S$ is an agreement set. The agreement set $S \subset Y_0 \subset I\!R^M$ is the subset of the set of the attainable payoffs dominating the disagreement point $d$. The agreement set defines payoffs attainable by all decision makers but under their unanimous agreement. If such agreement is not achieved, the payoffs of all decision makers are defined by the disagreement point $d$.

The multicriteria bargaining problem is analyzed under the following general conditions:

**C1** agreement set $S$ is compact and convex,

**C2** agreement set $S$ jest nonempty and includes at least one point $y \in S$ such, that $y \gg d$,

**C3** disagreement point $d \in Y_0$, additionally for any $y \in S$, we have $y > d$.

We assume, that each decision maker $i$, $i = 1, 2$, defines the vector $d_i \in I\!R^{m^i}$ as his reservation point in his space of criteria. Every decision maker, negotiating possible cooperation, will not agree for payoffs decreasing any component of the vector. A decision maker can assume the reservation point as the status quo point. He can however analyze some alternative options to the negotiated agreement and he define it on the basis of the BATNA concept presented in (Fisher, Ury [1]). The BATNA (abbreviation of Best Alternative to Negotiated Agreement) concept, is frequently applied in processes of international negotiations in a prenegotiation step. According to the concept, each side of negotiations should analyze possible alternatives to the negotiated agreement and select the best one according to its preferences. The best one is called as BATNA. It is the alternative for the side (decision maker), that can be achieved if the negotiations will not succeed.

A question arises, how each decision maker can be supported in the processes of decision analysis and in looking for the agreeable solution. The support should enable valuation of payoffs for different variants of his own decisions and the decisions of the second decision maker. It should also aid derivation of the agreeable, nondominated solution defining the payoffs of the decision makers in the agrement set. The solution should fulfil fair play rules such that it could be accepted by both the decision makers.

In this paper an interactive procedure is proposed including multicriteria decision support of each decision maker and applies an idea of the Nash cooperative solution for derivation of mediation proposals. The multicriteria decision support is made with use of the reference point method developed by A.P. Wierzbicki (see Wierzbicki [17], (Wierzbicki, Makowski, Wessels [19]). The Nash solution (Nash [10],[11]) has been originally formulated under axioms describing the fair play distribution of cooperation benefits, that can be accepted by rational players. It has been proposed by Nash to the bargaining problem under assumptions of the scalar payoffs of players. It can not be applied directly in the multicriteria bargaining problem considered here. This paper presents a construction enabling application of this idea in the case of the multicriteria payoffs of decision makers.

# 3 General structure of the computer-based system

The proposed system includes a model representation, modules supporting unilateral analysis made by the decision makers (DMs), a module generating mediation proposals, as well as modules including an optimization solver, respective data bases, procedures enabling interactive sessions realizing the mediation procedure and a graphical interface. A general structure of the system is presented in Fig. 2.

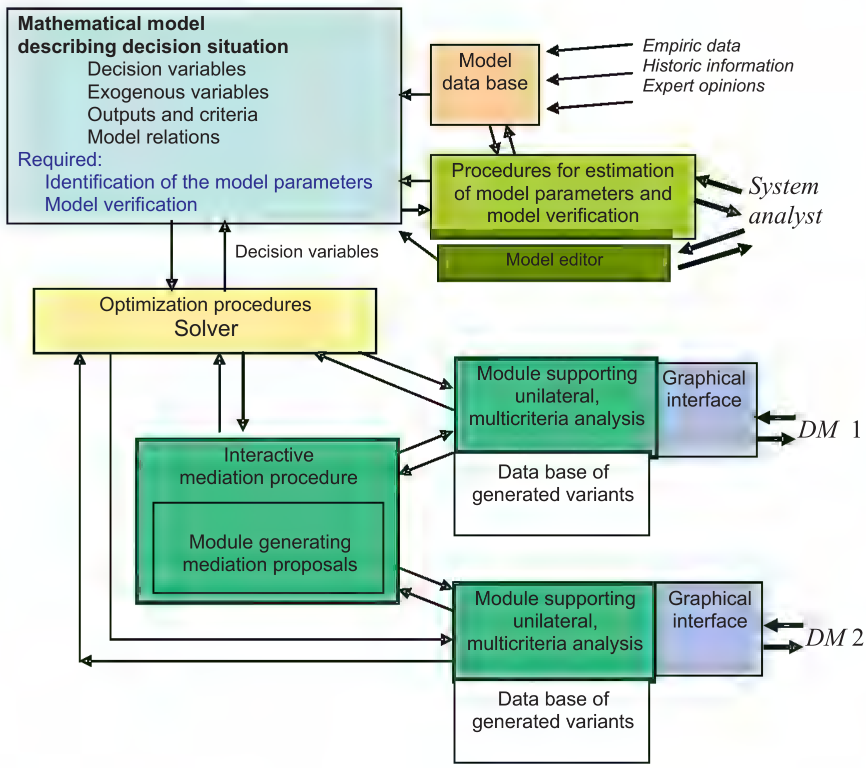


Figure 2: General scheme of the computer-based system.

The model describing the considered decision situation of the decision makers is the base for the decision analysis and support. The model is constructed by a system analyst with use of the gathered information according to the rules of the system sciences. It includes the specification of decision variables, exogenous variables, output quantities, criteria, model relations. With use of the model values of the criteria of the decision makers are derived for given assumed values of the decision variables. The criteria depend also on the exogenous variables representing quantities describing external conditions, not dependent on the decision makers. These variables are typically evaluated by experts in the forms of scenarios. The model parameters are identified on the base of the collected data. The model should be verified and validated. Therefore modules containing respective data base, a model editor, procedures for estimation of model parameters and for model verification are included in the system.

The module supporting unilateral analysis enables each DM to obtain independently information about possible multicriteria payoffs for assumed scenario, and look for the preferred option. The analysis is made in an interactive way.

The system generates also mediation proposals. The mediation proposals are derived with use of selected solution concepts of the theory of cooperative games and on the base of the preferences expressed by DMs. The mediation proposals are generated and presented to the DMs in a special mediation procedure.

Optimization techniques are utilized in the system: in the modules supporting unilateral multicriteria analysis made by the decision makers and in the module generating the mediation proposals. The respective optimization procedures are included in the solver module.

# 4 Interactive procedure supporting mediation process

The procedure has been proposed under inspiration of the mentioned Single Negotiation Text Procedure frequently applied in the international negotiations. In the considered case the role of mediator is played by the computer based system. A general scheme of the procedure is presented in Fig. 3 .

The procedure is realized in some number of rounds $t = 1, 2, ..., T$. In each round $t$:

- each decision maker makes independently interactive analysis of nondominated payoffs in his multicriteria space of payoffs (the analysis is called further as unilateral) and indicates a direction improving his payoff in comparison to the disagreement point. The direction is selected by him according to his preferences as an effect of the multicriteria analysis.

- computer-based system collects improvement direction indicated by both decision makers and generates on this basis a mediation proposal $d^t$,

- decision makers analyze the mediation proposal and correct the preferred improvement directions, afterwards system derives next mediation proposal.

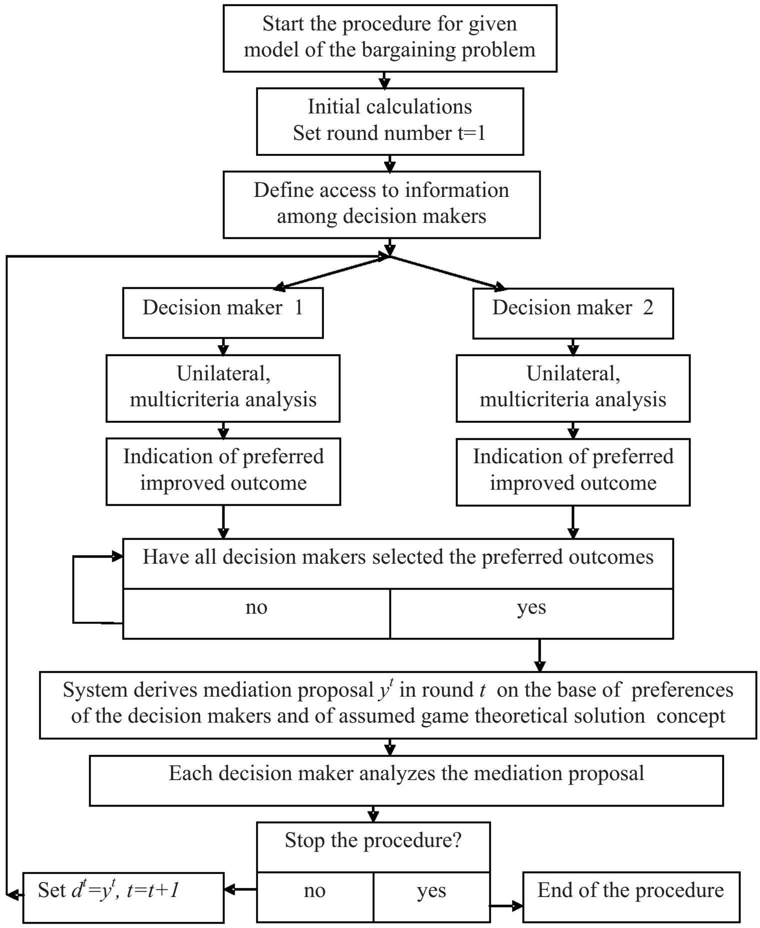


Figure 3: Scheme of the procedure.

All mediation proposals $d^t$ are generated on the basis of the improvement directions indicated by the decision makers and with application of an assumed solution concept of the multicriteria bargaining problem:

$d^t = d^{t-1} + \alpha^t[G^t - d^{t-1}]$, for $t = 1, 2, ...T$,

where

$d^0 = d$,

$\alpha^t$ is so called confidence coefficient assumed by the decision makers in the round $t$,

$G^t$ is a solution of the multicriteria bargaining problem. The solution is derived in the round $t$ and defines payoffs of the decision makers. In this case a multicriteria solution concept is proposed which is a generalization of the Nash solution concept to the case of multicriteria payoffs of the decision makers.

Each decision makers can in each round reduce improvement of the payoffs (his own payoffs and at the same time payoffs of other decision maker) assuming respectively small value of the confidence coefficient $\alpha^t$.

# 5 Unilateral analysis

The unilateral analysis is made independently by each decision maker. It should lead the given decision maker $i$, $i = 1, 2$ to selection of the Pareto optimal element in the set $S$ according to his in mind preferences. The element defines the required direction improving his current payoff.

The unilateral analysis is supported in the computer-based system with use of the reference point method and with application of the respective achievement function (Wierzbicki [17], Wierzbicki, Makowski, Wessels [19]). The decision maker can obtain some number of the Pareto optimal points in the set $S$ using this method and can select the preferred point.

Any Pareto optimal point $\overline{y}_i$ of the set $S$ in the criteria space of the decision maker $i$ $i = 1, 2$, can be derived as the solution of the following optimization problem:

$$\max_{x \in X_0} s(y_i, r_i), \tag{1}$$

where

$r_i \in I\!R^{m_i}$ is a reference point of the decision maker $i$ in his space of criteria,
$x$ is the vector of the decision variables,
$y_i = v_i(x)$ defines the vector of criteria of the decision maker $i$, as dependent on the decision variables $x$ due to the mapping $F$, under additional constraints that the criteria of the second decision maker are on the level of his reservation point i.e. $y_{3-i} = d_{3-i}$,
$s(y_i, r_i)$ is the achievement function approximating order in the space $I\!R^{m_i}$.

A representation of the Pareto frontier of the set $S$ can be obtained by solving the optimization problem for different reference points $r_i$ assumed by the decision maker $i$.

A general achievement function (Wierzbicki, Makowski, Wessels [19]) has the form:

$$\overline{s}(y_i, y_i^a, y_i^r) = \min_{1 \le k \le m_i} \sigma_{i,k}(y_{i,k}, y_{i,k}^a, y_{i,k}^r) + \rho \sum_{k=1}^{m_i} \sigma_i(y_{i,k}, y_{i,k}^a, y_{i,k}^r) \tag{2}$$

where $y_i = v_i(x)$, and $y^a_{i,k}, y^r_{i,k}$ denote respectively aspiration and reservation levels defined by decision maker $i$. Functions $\sigma_{i,k}(.)$ are of the form

$$\sigma_{i,k}(y_{i,k}, y^a_{i,k}, y^r_{i,k}) = \\ = \begin{cases} \beta(y_{i,k} - y^r_{i,k})/(y^r_{i,k} - y^{lo}_{i,k}), & \text{if} \quad y^{lo}_{i,k} \le y_{i,k} \le y^r_{i,k} \\ (y_{i,k} - y^r_{i,k})/(y^a_{i,k} - y^r_{i,k}), & \text{if} \quad y^r_{i,k} \le y_{i,k} \le y^a_{i,k} \\ 1 + \gamma(y_{i,k} - y^a_{i,k})/(y^{up}_{i,k} - y^a_{i,k}), & \text{if} \quad y^a_{i,k} \le y_{i,k} \le y^{up}_{i,k} \end{cases} \tag{3}$$

In the considered case $s(y_i, r_i) = \overline{s}(y_i, y^a_i, y^r_i)$, when the reference points $y^a_i = r_i$ but the reservation point is assumed on the level of the disagreement point $y^r_i = d_i$. The parameters $\rho, \beta, \gamma$ are the assumed coefficients of the reference point method , $\rho$ - is a relatively small number, $0 < \beta < 1 < \gamma$, $y^{up}_i$ and $y^{lo}_i$ are relatively a point dominating the ideal point, and a point dominated by the reservation point in the space $\mathbb{R}^{m_i}$. The points $y^{up}_i$ and $y^{lo}_i$ are assumed to normalize the optimization problem.

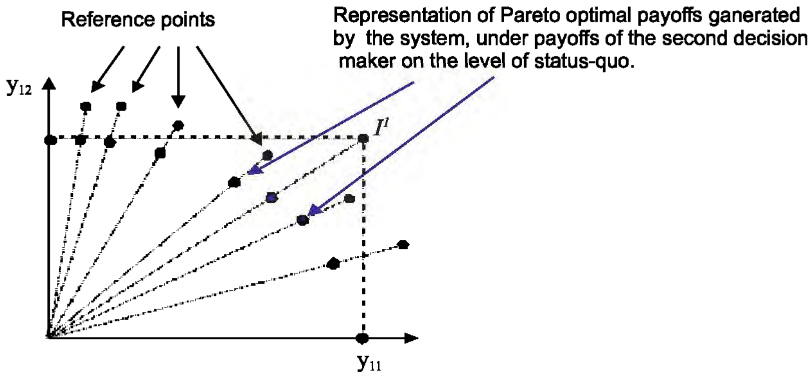


Figure 4: Generation of the nondominated payoffs of the decision maker 1 for assumed reference points

Fig. 4 illustrates how a given decision maker can generate and review his attainable nondominated payoffs. He assumes different reference points and then the system derives the respective Pareto optimal solutions. The reference points assumed by the decision maker and the Pareto optimal payoffs $\overline{y}_i$ derived by the system are stored in a data base, so that the decision maker can obtain a representation of the Pareto optimal frontier of the set $S$ and can analyze it.

It is assumed that each decision maker $i$, $i = 1, 2$, finishing the multicriteria analysis, indicates his preferred nondominated payoff $\widehat{y}_i$ in his space of criteria. The payoff $\widehat{y}_1$ corresponds to the element $y^1 = (\widehat{y}_1, d_2) \in S$ in the case of the decision maker $i = 1$ and the payoff $\widehat{y}_2$ corresponds respectively to the element

$y^2 = (d_1, \widehat{y}_2) \in S$ in the case of decision maker $i = 2$. The last elements are defined in the space of criteria of both the decision makers. The stage of unilateral analysis is finished when both the decision makers have indicated their preferred payoffs.

The unilateral analysis can be realized in different ways with respect to access to information available for decision makers. In the presented way it is assumed that each decision maker makes unilateral analysis not knowing the criteria nor the reservation point of the second decision maker. The mediator has only access to the full information. This information is obviously used in calculations of the computer based system. In general any decision maker has not permission to see any data introduced and generated by the other one.

# 6 Derivation of the mediation proposal

A mediation proposal is derived by the system when both decision makers have indicated their preferred payoffs $\widehat{y}_1, \widehat{y}_2$ in their spaces of criteria and when respective points $y^1, y^2 \in S$ have been calculated by the system.

Let us construct a two dimensional hyperplane $H^2$ defined by points $d, y^1, y^2$ in the criteria space $I\!R^M$. Each point $y \in H^2$ may be defined as

$$y = d + a_1(y^1 - d) + a_2(y^2 - d).$$

Let $A$ denote mapping from $H^2$ to $I\!R^2$ defined by $A(y) = A[d + a_1(y^1 - d) + a_2(y^2 - d)] = (a_1, a_2)$. A two person bargaining problem $(A(S^H), A(d))$ can be considered on the hyperplane $H^2$. Set $S^H = S \cap H^2$ in the problem.

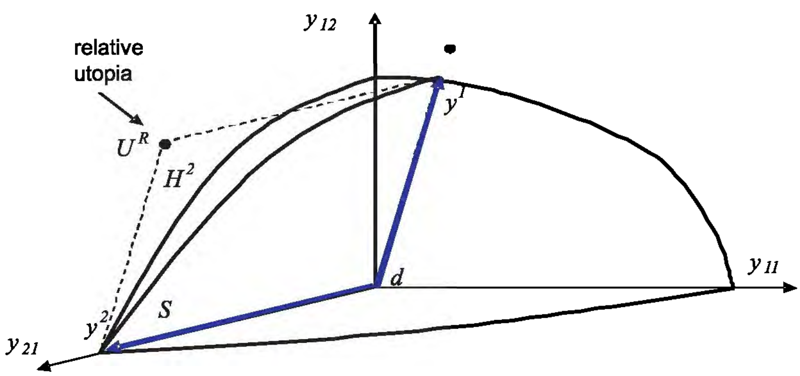


Figure 5: Construction of the hyperplane $H^2$.

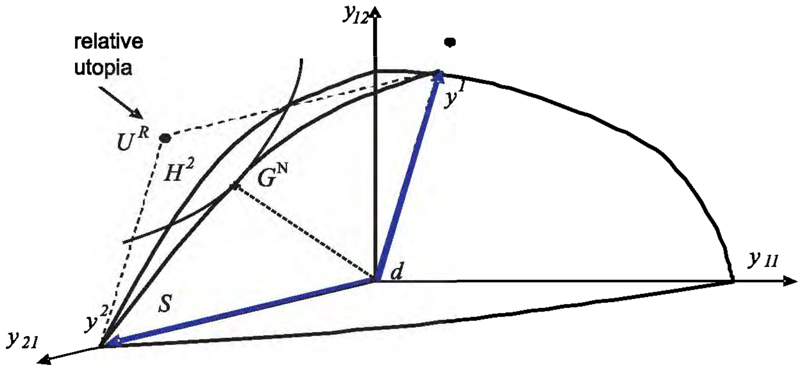


Figure 6: Construction of the generalized Nash solution to the multicriteria bargaining problem

A generalization of the Nash cooperative solution concept can be constructed using the hyperplane $H^2$. Fig. 5 presents a construction of the plane $H^2$ for a multicriteria bargaining problem of two decision makers. In this example the decision maker 1 has two criteria $y_{1,1}$ and $y_{1,2}$ respectively, the decision maker 2 has only one criterion $y_{2,1}$. Let the point $y^1$ be defined according to the preferences of the first decision maker. The preferred point $y^2$ of the second one is defined by the maximal attainable value of his payoff. The hyperplane $H^2$ is defined by the points $d$, $y^1$ and $y^2$.

Fig. 6 presents how the solution to the multicriteria bargaining problem is constructed. The arrows drown on the figure present the improvement directions leading to the nondominated payoff $y^1$ selected by the decision maker 1 and to the nondominated payoff $y^2$ selected by the decision maker 2 respectively. The Nash cooperative solution $y^N = f^N(S^H, d)$ to the bargaining problem $(S^H, d)$ is defined as the point of the set $S^H$ maximizing the product of the payoffs increases for the decision makers 1 and 2 on the hyperplane $H^2$.

Let the preferences of decision makers are expressed by points $y^1$ i $y^2$. Then the final payoffs defined by this point fulfill the following axioms:

**(A1)** Pareto-optimality
$y^N = f^N(S^H, d)$ is Pareto-optimal in set $S^H$,

**(A2)** Individual rationality
For every bargaining game $(S^H, d)$, $y^N = f^N(S^H, d) \geq d$.

**(A3)** Symmetry
We say, that bargaining problem $(S^H, d)$ is symmetric, if $d_1 = d_2$ and

$(x_1, x_2) \in S$, then $(x_2, x_1) \in S$. We say, that a solution fulfills symmetry property, if for symmetric $(S^H, d)$ problem, $f_1^N(S^H, d) = f_2^N(S^H, d)$.

**(A4)** Independence of equivalent utility representation
Let $L$ be a affine mapping, i.e. such that $Lx = (a_1x_1 + b_1, a_2x_2 + b_2)$ for any $x \in R^2$, where $a_i, b_i \in R, a_i > 0, i = 1, 2$. We say, that a solution is independent of equivalent utility representation, if $Lf^N(S^H, d) = f^N(LS^H, Ld)$.

**(A5)** Independence of irrelevant alternatives
Let $(S^H, d)$ and $(T^H, d)$ be bargaining problems such that $S^H \subset T^H$ and $f^N(T^H, d) \in S^H$.
Then $f^N(S^H, d) = f^N(T^H, d)$.

The last axiom means that if the decision makers have agreed the solution $f^N(T^H, d)$ in the bargaining problem $(T^H, d)$, then decreasing of the agreement set $T^H$ to a set $S^H$ which includes the solution, i.e. $f^N(T^H, d) \in S^H$, should not change the final payoffs of the decision makers.

According to the Nash theorem (Nash 1950), for any bargaining problem $(S^H, d)$ satisfying assumptions C1 - C3
there exists one and only one solution $f^N(S^H, d)$ of the form:

$$f_N(S^H, d) = \arg\max_{y \in S^H} ||y_1 - d_1|| \cdot ||y_2 - d_2||,$$

satisfying the axioms A1 - A5.

$||.||$ is a distance measured on hyperplane $H^2$.

The axioms A1 - A5 can be treated as fair play rules satisfied by the mediation proposal constructed according to the Nash solution concept. The axiom A1 assures efficiency of the solution in the set $S$. The solution is individually rational according to the axiom A2. The axiom A3 means that the decision makers are treated in the same way. The axiom A4 prevent possible manipulation of the decision makers. Any decision maker will not benefit by changing scales measuring his payoffs.

The system derives the point $f_N(S^H, d)$ in the set $S$ according to the above formula. The tentative mediation proposal is calculated improving the current status quo in the direction of the point, but the improvement is limited by the confidence coefficient. In the particular rounds of the mediation procedure the decision makers repeat unilateral multicriteria analysis. Each one can correct the preferred direction improving his payoffs, having more information about the form of the agreement set and knowing the tentative mediation proposal. Finally a sequence of mediation proposals is generated. It can be shown that the sequence converges to the Pareto optimal point in the agreement set (Kruś [4]) and the payoffs defined by the point have the properties analogical to the properties of the classic Nash solution.

Different solution concepts to the multicriteria bargaining problem can be applied in the procedure to derive the mediation proposals. In particular the solutions based on the ideas of the Raiffa-Kalai-Smorodinsky and Lexicographic solution concepts have been proposed for the multicriteria bargaining problems and analyzed in the papers (Kruś and Bronisz [8], Kruś [7], [6], [5]). The proposals relating to the Nash solution concept presented in this paper and in (Kruś [4]) complete the previous results.

# 7 Conclusions

The computer base system has been proposed supporting negotiation in the case of the multicriteria bargaining problem. The problem describes the decision situation of two decision makers negotiating conditions of possible cooperation in realization of a join enterprize and each of them valuates effects of the cooperation by his own different set of criteria. The problem is defined by the disagreement point and the agreement set formulated in the space being the cartesian product of the criteria spaces of the decision makers. The system acts according to the specially constructed mediation procedure. With use of the procedure the decision makers look independently for the preferred variants of cooperation using reference point method. The preferred variants indicated by them are used to generate mediation proposals. The mediation proposals are derived on the basis of the proposed solution concept satisfying respective fair play rules to the multicriteria bargaining problem. The solution has been constructed on the basis of the Nash solution concept originally formulated for the classic bargaining problem with scalar payoffs of players. It can be considered as a generalization of the Nash cooperative solution on the multicriteria case.

The computer based system plays a role of mediator in the negotiation process. It supports in particular rounds of the process, the multicriteria analysis made by each of the decision makers. It enables exploration of the agreement set and selection of the preferred variants of payoffs. The preferred variants selected by the decision makers express information about preferences of the decision makers. The system derives the mediation proposal expressing the preferences but also satisfying rules defined by the applied cooperative solution concept. The decision makers can control the speed of the negotiation precess with use of the confidence coefficients. The smaller confidence coefficient results in the grater number of rounds of the negotiation process and the decision makers can more precisely explore the agreement set.

# References


[1] Fisher R., Ury W. (1981), Getting to Yes. Hougton Mifflin, Boston.

[2] Imai H., (1983), "Individual Monotonicity and Lexicographical Maxmin Solution", *Econometrica*, Vol.51, 389-401.

[3] Kalai E., M. Smorodinsky, (1975), Other Solutions to Nash's Bargaining Problem, *Econometrica*, Vol. 43, 513-518.

[4] Kruś L.(2013), Iterative Solution to the Multicriteria Bargaining Problem Based on the Nash Solution Concept. Research Report, Systems Research Institute, Polish Academy of Sciences, RB/44/2013, Warsaw.

[5] Kruś L.(2011), Multicriteria cooperative decisions, methods of computer-based support (in Polish: Wielokryterialne decyzje kooperacyjne, metody wspomagania komputerowego). Instytut Badań Systemowych PAN, Seria: Badania systemowe. Tom 70. Warszawa 2011, 248 str.

[6] Kruś L.(2001), Multicriteria decision support in bargaining, a problem of players's manipulations. In: T . Trzaskalik, J. Michnik (Eds.): Multiple objective and goal programming recent developments. Physica Verlag, Heidelberg, 143-160.

[7] Kruś L., (1996), Multicriteria Decision Support in Negotiations, *Control and Cybernetics*, Vol. 25, No. 6, 1245-1260.

[8] Kruś L., Bronisz P.(1993), Some New Results in Interactive Approach to Multicriteria Bargaining. In: User Oriented Methodology and Techniques of Decision Analysis, A. P. Wierzbicki et al. (Eds.), Lecture Notes in Economics and Mathematical Systems, Vol. 397 , Springer Verlag, Berlin, 21-34.

[9] Moulin H. (1988) Axioms of Cooperative Decision Making. Cambridge University Press, Cambridge.

[10] Nash J.F., (1950), The Bargaining Problem, *Econometrica*, Vol. 18, 155-162.

[11] Nash J.F., (1953), Two-Person Cooperative Games, *Econometrica*, Vol. 21, 129-140.

[12] Peters H. (1986), Bargaining Game Theory. Ph.D. Thesis, Catholic University of Nijmegen, The Nederlands.

[13] Raiffa H., (1953), Arbitration Schemes for Generalized Two-Person Games, *Annals of Mathematics Studies*, No. 28 361-387, Princeton.

[14] Raiffa H. (1982), The Art and Science of Negotiations. Harvard Univ. Press, Cambridge.

[15] Roth A.E., (1979), Axiomatic Model of Bargaining, Lecture Notes in Economics and Mathematical Systems, Vol. 170, Springer-Verlag, Berlin.

[16] Thomson W., (1980), Two Characterization of the Raiffa Solution, Economic Letters, Vol. 6, 225-231.

[17] Wierzbicki A.P.(1986), On the Completeness and Constructiveness of Parametric Characterizations to Vector Optimization Problems. *OR Spectrum*, Springer Verlag, Vol. 8, 73-87.

[18] Wierzbicki A.P., Kruś L., Makowski M. (1993), The Role of Multi-Objective Optimization in Negotiation and Mediation Support. *Theory and Decision*, special issue on "International Negotiation Support Systems: Theory, Methods and Practice" Vol. 34, (2).

[19] Wierzbicki A.P., Makowski M., Wessels J. (2000), Model-based Decision Support Methodology with Environmental Applications. Kluwer Academic Press, Dordrecht, Boston.

The papers presented in this Volume 2 constitute a collection of contributions, both of a foundational and applied type, by both well-known experts and young researchers in various fields of broadly perceived intelligent systems.

It may be viewed as a result of fruitful discussions held during the Twelfth International Workshop on Intuitionistic Fuzzy Sets and Generalized Nets (IWIFSGN-2013) organized in Warsaw on October 11, 2013 by the Systems Research Institute, Polish Academy of Sciences, in Warsaw, Poland, Institute of Biophysics and Biomedical Engineering, Bulgarian Academy of Sciences in Sofia, Bulgaria, and WIT - Warsaw School of Information Technology in Warsaw, Poland, and co-organized by: the Matej Bel University, Banska Bystrica, Slovakia, Universidad Publica de Navarra, Pamplona, Spain, Universidade de Tras-Os-Montes e Alto Douro, Vila Real, Portugal, Prof. Asen Zlatarov University, Burgas, Bulgaria, and the University of Westminster, Harrow, UK:

Http://www.ibspan.waw.pl/ifs2013

The consecutive International Workshops on Intuitionistic Fuzzy Sets and Generalized Nets (IWIFSGNs) have been meant to provide a forum for the presentation of new results and for scientific discussion on new developments in foundations and applications of intuitionistic fuzzy sets and generalized nets pioneered by Professor Krassimir T. Atanassov. Other topics related to broadly perceived representation and processing of uncertain and imprecise information and intelligent systems have also been included. The Twelfth International Workshop on Intuitionistic Fuzzy Sets and Generalized Nets (IWIFSGN-2013) is a continuation of this undertaking, and provides many new ideas and results in the areas concerned.

We hope that a collection of main contributions presented at the Workshop, completed with many papers by leading experts who have not been able to participate, will provide a source of much needed information on recent trends in the topics considered.

ISBN 838947554-5

9 788389 475541