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Editors:

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Olgierd Hryniewicz

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The purpose of this publication is to present the information technology (IT) tools and techniques that have been developed at the Systems Research Institute of Polish Academy of Sciences in Warsaw (IBS PAN) and at the German Institute for Landscape System Analysis in Müncheberg (ZALF) in the area of applications of informatics in environmental engineering and environment protection. The papers published in this book were presented in the form of extended summaries during a special workshop organized by IBS PAN in Szczecin in September 2006 together with the conference BOS'2006 organized jointly by IBS PAN, University of Szczecin, and the Polish Society of Operational and Systems Research. In the papers the problems of mathematical modeling, approximation and visualization of environmental variables are described. Moreover, some questions concerning the environmental economy are also presented.

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CHAPTER 2

Mathematical Modeling

SIMPLIFIED MODELLING OF WATER TABLES IN THE LIBYAN DESERT AQUIFERS UNDER PRODUCTION PUMPING

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Abstract: *The paper is dealing with the problem of predicting the groundwater table levels in the Saharan aquifers in Libya under production pumping for transfering water to the heavily inhabited coastal belt. For this, the model for drawdown surface of a pumping well is used. Two wellfields, Sarir and Tazerbo, are considered. The model has been calibrated using the parameters from earlier studies and pumping tests, as well as from measurements from an already existing wellfield located in the same region. The forecasting results are positively compared with those obtained from much more complicated partial differential models.*

Keywords: Arid regions, groundwater aquifers, multiple well systems, water table levels.

1. Introduction

Surface water in Libya is limited. Water resources depend basically on the groundwater which forms more than 96% of total water resources in the country. In northern parts of the country there is an immense shortage of traditionally exploited water as a result of high population density and the concentration of industry. As a consequence of the greatly increased deficit, groundwater tables have substantially become lower in these regions and the hydraulic slope, which traditionally was from the southern Jabal Al Gharbi mountains into the northern Mediterranean Sea, has reversed along several kilometres wide coastal strip. Demand for freshwater in Libya will continue to increase because of economic development, population growth and an improving living standards, see Salem (1998).

Spotting shortages of fresh water and sea water intrusions at the coastal regions, the Libyan authorities decided to exploit huge amount of fresh water from the groundwater aquifers located at the Sahara south of Libya. Daddesh (1998c) in his PhD thesis analysed the cost of water gained this way and compared it to predicted costs of alternative solutions until year 2050. For this, change of water table levels at the desert wellfields were needed to estimate the costs of pumping the water to the ground level. This was done using simplified models of drawdowns caused by production wells. After superposition of drawdowns for all wells in the wellfield, the final model has been obtained. It was calibrated, as far as it was possible. It was also compared with other predictions computed elsewhere using partial differential models.

The original project assumed that further validation of the model would be done after several years of wellfields exploitation. However, Dr. Daddesh sudden health problems, which finally ended tragically, did not allow for terminating this phase. This paper summarizes the earlier stage of the project. Some other parts of Dr. Daddesh PhD thesis were published earlier (Daddesh, 1996; 1998a).

Serious, systematic works on groundwater resources in Libya started in the 1950s with FAO report by Pioger (1952) and compilations of existing data by United States Geological Survey (USGS) from 1956 onwards. The latter produced a series of monographs in the early 1960s, dealing with areas such as Tripoli, in 1960, Azzawia, in 1962, and Garabulli, in 1962. Then Jones (1964) compiled a series of maps showing groundwater level contours. The threat of salt water intrusion is emphasised in a later report by Jones (1969). A French consulting consortium Gefli (1972) published what is still the best and fullest hydrological data base for the Jeffara Plain. Between 1976 and 1982 a great number of technical reports were published, including those by Pallas (1976, 1978a, 1978b), Kruseman (1977), Floegel (1978, 1979), Welsh (1979), Gignoux (1981), Dula Navarrete (1981), Krummenacher (1982), and Polservice (1985), see also Ganous (1994).

The Institute of Geological Sciences of the Natural Environmental Research Council of Great Britain (IGS) conducted the first in-depth hydrogeologic studies in the Sarir region, initially for British Petroleum Exploration Co. (LIBYA) Ltd. and subsequently for the Libyan government. A finite-difference model was made of the Sarir region north of latitude 28° (Wright & Makridakis, 1974). It focused on the proposed Jalu-Awjilah wellfield. Tipton and Kalmbach, Inc., consulting engineers from Denver, Colorado, were the next to study the Sarir region. They retained Electrowatt, an European consulting firm, who modeled the Sarir region and made the first wellfield designs for the Sarir agricultural project in 1973 and 1974. They also made a design for the Jalu-Awjilah wellfield. From 1975 to 1977 Moid

Ahmad of Hygronics, Inc., Athens, Ohio, redesigned the Sarir agricultural wellfields and predicted their performance using the USGS model. These wellfields were finally constructed according to his designs. He later recommended roughly the same design for the CBWP wellfield at Sarir (Ahmad et al., 1979) and a modified, yet similar design for the CBWP wellfield at Tazerbo (Ahmad, 1979) based on the USGS model. Meanwhile, the Engineering Consultants Group (ECG), an Egyptian consulting firm from Cairo, directed the exploration of the CBWP Sarir wellfield area and designed a wellfield using a Theis digital model and also designed an aqueduct to the coastal belt. Finally, designs by Ahmad for both CBWP wellfields were used.

Background information on regional groundwater geology and hydrology can be found in the principal reports of IGS and a series of subsequent publications in technical journals by IGS personnel. The Soil Land Water Department of the Secretariat of Land Reclamation and Agricultural Development, Tripoli, is the source of several unpublished reports on the groundwater hydrology and results of exploratory work and pumping tests in the CBWP wellfields at Sarir and Tazerbo and Sarir agricultural project area. Data for the model, like the individual well transmissivities, monthly discharges, and piezometric water levels of the Sarir agricultural wellfields can be found in Hasnain et al. (1982). Summary reports of the Sarir region were made by El Ramly (1980) and Pallas (1980).

2. Water supply situation in Libya

2.1. Climatic considerations and water resources

Libya is located in Northern Africa between Egypt and Tunisia, extending from the Mediterranean Sea in the north to the Tibesti mountains in the south. The total surface area is 1 775 500 km^2. The climate of Libya changes from north to south under the effects of the Mediterranean Sea and the Sahara. Air temperature varies from summer to winter ranging from zero to over 48°C. The highest rainfall occurs in the northern Tripoli region (Jabal Nafusa and Jeffara Plain) and in the northern Benghazi region (Al Jabal Al Akhdar), these two areas being the only ones where the average yearly rainfall exceeds the minimum values (250-300 mm) which are considered necessary to sustain the rain-fed agriculture. Otherwise, rainfall average is less than 100 mm/year over 93% of the land surface, and less than 10 mm/year over 60%. High rates of evaporation which are in the order of 1700 mm towards the coast and of 6000 mm in the oases reduce the groundwater recharge.

The available water resources consist mostly of non-renewable (fossil) groundwater, limited amounts of renewable groundwater recharged by annual rainfall, limited surface water stored in impounding reservoirs and desalinated sea water. There are no permanent rivers or lakes.

Surface water resources are limited and contribute only a small amount to the total water consumption. Runoff volumes are estimated at 77 Mm^3 per year from about 30 hydrographical basins in the northern Jabal Nafusa. In addition, 30 Mm^3 of runoff occurs on the northern and south-eastern slopes of Jabal Nafusa and 100 Mm^3 on the Al Jabal Al Ahkdar, making a total annual amount of runoff in the vicinity of 210 Mm^3. A few springs of small and medium discharge are located at different parts of the country. The largest of these are Ayn Zayana near Benghazi with an annual discharge of 90 Mm^3 and Ayn Tawargha to the east of Misurata with an annual discharge of 60 Mm^3.

Many desalination plants in the country along the coastal line in the areas with heavy residential density have been established. But they provide only limited amount of freshwater.

Treated water is used for irrigating orchards, fodder and other indirectly consumed agricultural crops. For this purpose many plants of purifying sewage water have been constructed close to the cities.

Groundwater is responsible for more than 98% of the total water consumption in the country. It occurs in aquifers of varying thickness, age and lithological composition. These aquifers can be classified according to their recharge into two groups. The first group represents the aquifers which receive current recharge, including the Quaternary, Miocene, upper Cretaceous and Triassic aquifers in the north, mostly in the Jeffara Plain and in the Al Jabal Al Akhdar plateau, as well as in the Benghazi plain area. The second group of aquifers is those with no current recharge, including the lower Cretaceous, Triassic, and Cambro-Ordovician aquifers in the south and Tertiary aquifers in the Sarir basin. Overall groundwater recharge has not been specified. But, it is estimated to be in the order of 500 Mm^3 per year. Of that, recharge in the Jeffara Plain is estimated to be 200 Mm^3/year and in the Al Jabal Al Akhdar and Benghazi areas another 200 Mm^3/year. The majority of the Saharan aquifer groundwater was collected between 30 000 and 14 000 years ago.

As mentioned before, the traditional water resources do not meet the current level of exploitation. Most of the water requirements are met from the coastal aquifers which represent the zones of heavy demand. Over-exploitation of these aquifers has resulted in continuous water-table decline accompanied by an overall deterioration in water quality and sea water intrusion along the coast. Particularly in Jeffara and Benghazi Plains their groundwater resources have been over-exploited and partly contaminated, so that their exploitation potential during the next 100 years is very limited indeed. An example of the grave situation of water-table decline at the Tripoli region (Bin Gashir south of Tripoli) is shown in Figure 1.

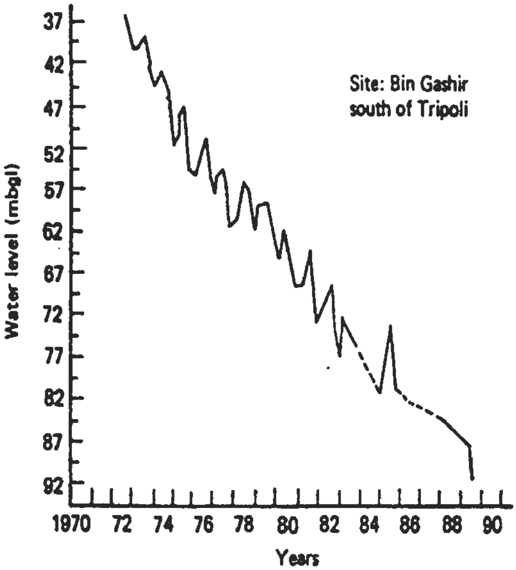

Figure 1. Water level decline in an observation well south of Tripoli. Source: Salem (1992)

2.2. Groundwater basins

The Libyan ground water supply situation can be conveniently described by six groundwater hydrological water basins, see Figure 2.

Jeffara Plain basin. The Jeffara plain is located north-west of Libya. This area, which is roughly triangular in shape, is bounded by the Mediterranean Sea on the north and Jabal Nafusa escarpment forming the southern boundary of the plain. The width of this plain is about 100 km at the Libyan-Tunisian border and pinches out at Khoms area, where the Jabal reaches the coast. In this area the plain is formed of Pleistocene and Holocene sand and gravels underlain by Miocene and Mesozoic rocks, which are exposed at the escarpment. The water in the northern aquifers in Jeffara Plain is either related to rainfall on Nafusa escarpment or it may come from the southern basin.

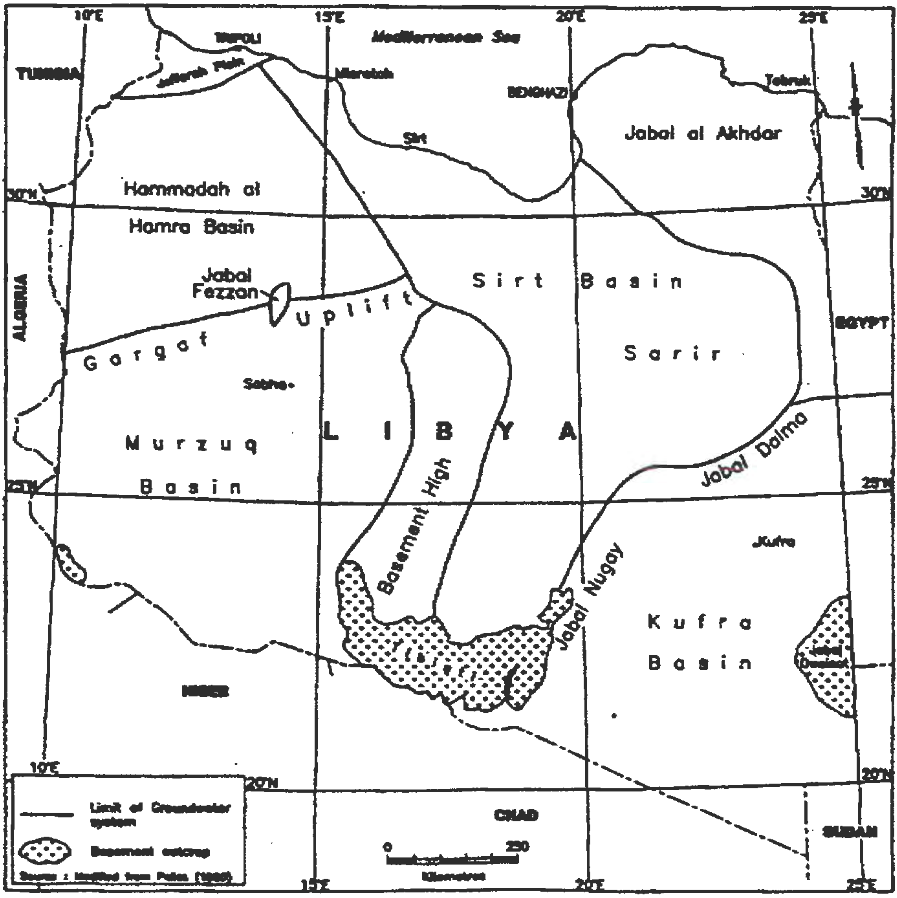

Figure 2. Main groundwater basins in Libya.

Al Hammada Al Hamra basin. This basin is located at the western part of Libya and forms a large basin in the North African shield. The basin is limited to the north by Al Garyat Uplift and on the east by Sirt basin, whereas to the west, the basin extends into Tunisia and Algeria. It contains a thick sequence of Palaeozoic strata that range in thickness from about 1000 to 3000 meters, and is overlain by a thinner section of Mesozoic and Cainozoic sediments. In Al Hamada Al Hamra basin the main water bearing formation in hydraulic continuity from south to north is found. The Palaeozoic sandstones in the south around Jabal Fazzan are essentially of upper Silurian and lower Devonian and may also play a role in the transfer of water. The other water bearing formation is represented by lower Mesozoic sandstone which makes transfer of water towards the north. The origin of the water in this basin is attributed either to local recharge of Palaeozoic aquifer during rainfall events or inflow from the south (Murzuk basin).

Murzuk basin. This basin is located south-west of Libya and it covers an area of about 450 000 km^2. It is bounded on the north by Jabal Fazzan and to the west by a basin in Algeria. The basin contains a thick sequence of Paleozoic sediments overlain by Mesozoic sediments mostly of continental origin. The maximum thickness of these sediments is about 4000 km at the center of the basin. Two main groundwater reservoirs are reported in the Murzuk basin. The lower groundwater reservoir is represented by Silurian-Devonian and Cambro-Ordovician sandstone. The Silurian Acacus sandstone - Devonian (tadrart) sandstone - are thicker and well developed only in the southern part of the basin. It is isolated form Cambro-Ordovician sandstone by a thick series of shale(s). In the north and north east of the basin (Wadi Asshati) the Silurian sandstones are often missing and the Devonian sandstone reduced in thickness and shows a good hydraulic connection with the Cambro-Ordovician aquifer which is the main hydraulic unit of the lower groundwater reservoir. The upper groundwater reservoir is represented by the continental formation of undifferentiated (Triassic, Jurassic) and lower Cretaceous strata, formally known as the post-Tassilian and Nubian series. It consists of alternating clay-lose sand and sandstones. The total saturated thickness of this reservoir exceeds 1000 meters in the central part of the basin. Available piezometric data suggest a groundwater flow towards the north from the south (Pallas, 1978), as indicated by discharge through the sobkhas (salt lakes), along depression and in the Ubari sand sea north east of Ubari located along the northern part of the basin. The Murzuk basin is estimated to hold 4 800 km^3 of water.

Kufra basin. Kufra basin is located south-east of Libya covering an area of about 350 000 km^2. It is bounded by Jabal Eghi from the west, Jabal Azalma Uplift from the north and Jabal Alawinat from the east. The central part of the basin is filled by sediments of Palaeozoic and Mesozoic age, mostly of continental origin. Huge sand dunes formally known as Rabyana sand sea cover most north-western part of the basin. The Kufra basin is well delineated on the periphery by Palaeozoic outcrops, mostly of continental sandstones. The central part is mainly occupied by thick Mesozoic continental sandstones (over 2000 meters) formally known as Nubian sandstones.

This basin forms a large freshwater reservoir, estimated for 20 000 km^3 capacity in the Libyan sector.. The hydraulic behaviour of the reservoir based on few piezometric data available on the Mesozoic sandstone aquifers suggest a groundwater flow directed from south (Chad-Sudan) to the north (Sarir).

Sarir and Sirt basin. At the Sarir basin located north of Kufra basin a Cretaceous and lower Tertiary marine transgression overlies the Mesozoic and Palaeozoic sandstones. These post-Eocene deposits consist mainly of sand, sandstone and clay with some limestone. The total thickness is about 800-900 meters documented in the centre of Sarir which form another important freshwater reservoir. Further north the post-Eocene continental deposits pass gradually to the shallow marine carbonaceous formation with evaporatic intercalations reflecting lagoonal and estuarine conditions. The piezometric data indicate a groundwater flow directed towards the north. The sobkhas located at the north of the Sarir basin between Ajdabia to Al Jaghbub are the main natural outlet of the groundwater system. The Sarir basin water-bearing formations form two main aquifers: (i) the post middle Miocene, (ii) the lower and middle Miocene and Oligocene. The first aquifer ranges in thickness from a few meters to 210 meters and consists of medium to coarse-grained sands grading to calcareous sandstones with thin interbeds of clay. The second aquifer varies in thickness from 150 to 800 meters and consists in the north of marine carbonates and clays with evaporates and in the central zone of interbeded clays, carbonates of shoreline complex. In the south and south-west fluvial sands and sandstones with thin clay beds predominate. The Oligocene ranges in thickness from 240 to 730 meters and consists in the south and south-west of coarse-grained sands and sandstones with clay interbeds. In this zone, the Oligocene is in contact with the second aquifer. Further north, it consists of calcareous sandstones, limestones, clays and evaporates. The Tertiary deposits are well developed in Hun Graben and in the Sirt basin. These Tertiary deposits are mainly of evaporate and carbonaceous sediments. The Oligo-Miocene aquifer occurs along the coast of Sirt and consists of limestone and dolomite alternating with marls. Oligocene aquifer in Al Jufra area where the deposits consist of sand, clay-gypsum and calcarenite contain brackish water found in Sukna, Hun and Waddan. The origin of these water is either infiltration of runoff water from Jabal Sawdda and Jabal Waddan present in the central part of the Sirt basin or upward leakage from the upper Cretaceous aquifers through the western fault of the Hun Graben. The Sirt basin is estimated to hold in total over 10 000 km^3 of water.

Al Jabal Al Akhdar basin. This area includes that part of east Libya located north of latitude 30° and west of Egyptian border. This area includes

Al Jabal Al Akhdar sensu stricto and the whole stretch of land between the latitudes 30° and 32° north. Al Jabal Al Akhdar is a crescent-shaped ridge attaining a height of more than 850 meters above sea level in its central part. The northern flank consists of step-like plateau bordered by escarpments. The southern flank dips gently towards a depression extending from Ajdabiya to Aljaghbub which is marked by several large sobkhas. To the east and mostly to the west, a coastal plain is well developed between the foot of the first escarpment and the sea. The groundwater reservoir is composed of the thick calcareous Tertiary series. In correspondence with a bigger development of marl layers in the Oligocene or lower Miocene, perched aquifers may occur locally. These perched aquifers are usually connected by vertical leakage (dripping) with the main water body. Groundwater flow is mainly related to a system of microfissures and micropores. The recharge of the aquifers is due to direct infiltration of rainwater and to the seepage of runoff water along the wadi beds. The natural outlets of the aquifers are either springs for the northern flank, and for the southern flank large sobkhas where water evaporates. Most of the aquifers described above are in one way or another in hydraulic connection, either directly or by leakage through semipervious layers.

2.3. The Man-Made River project

A project known as the Man-Made River for conveying groundwater through the network of buried underground pipes, with a total length of 4000 km has been planned to convey freshwater from underground reservoirs in Sarir, Tazerbo, Hassawana Mountain and Kufra, from the arid desert to the fertile coastal strip areas for agricultural irrigation, general utilities and industrial activities, see GMR (2006).

The project consists of five major phases. From the first phase a total of 2 Mm^3/day is conveyed from As-Sarir and Tazerbo to to the coastal areas extending from Benghazi to Sirt. The wellfields provide around 1 Mm^3/day water each. The wellfields are properly located in areas where dewatering of overlying unconfined aquifer will minimize pumping lifts (Fisk et al., 1983). From the second phase, 1 Mm^3/day of freshwater is conveyed to the Jeffara Plain in the north-west of Libya from 127 wells tapping the Cambro-Ordovician aquifer at the northern-eastern part of Murzuk basin in the wellfield at Sarir Qattusah in the Fezzan region. The third phase of this project will increase water flow in the first phase system by 1.68 Mm^3/day from the wellfields near Kufra, connecting them to the main conveyance system of the first phase. Another project of this phase is oriented toward further extensions of the conveyance lines of the first phase eastward to reach Tobruk and westward to link with the second phase along the western coast. The last two phases involve further extensions and connections of the distribution network. Upon the completion of the Man-Made River project it will be capable of providing about 6.18 Mm^3/day of water. Two existing well-fields, Sarir North and Sarir South, are supplying water for Secretariat of Land Reclamation and Agricultural Development farming projects in the Sarir region.

The new water source from the Man-Made River project is therefore expected to provide a good opportunity for: (i) the coastal aquifers to recover a considerable part of the underground water lost during the past years, (ii) support the existing industries, effectively increasing their production, (iii) agricultural expansion, (iv) creation of new fields of employment.

3. Modelling and forecasting the groundwater level changes

3.1. Partial differential models of aquifer systems

Aquifer models are used for studying proposed management policies. Each such policy, comprising a list of timings and locations of new installation and specification of the temporal and spatial distribution of pumping and artificial recharge, can be tested for physical feasibility by using a model. This is the simulation approach. To seek an optimal policy the aquifer model becomes a constraint to the optimization criterion.

To study behaviour of aquifer systems in Libya mainly the partial differential equation models were applied. A partial differential equation model must be well defined. The definition should be based on the detailed geometry of the aquifer, information about its physical parameters boundaries, inputs, outputs, etc. The geometric factors include aquifer bottom elevations, aquifer thickness, horizontal extent of the aquifer including the location of no-flow boundaries, constant head, and constant flux boundaries. All this information is derived from geological studies and from the observations in the real aquifer system. Whenever information is not available, it must be assumed on the basis of experience and then verified during the calibration process. The calibration, or identification, of a model is the process in which the various model parameters (that may include also its geometry, inputs, etc.) are determined, if no previous knowledge of them is available, or verified (if such information is available). The calibration or identification procedure is often referred to as the inverse problem. The calibration is based on data obtained from observations of the behavior of the aquifer in the past.

Such data usually include water levels, pumping and recharging rates, etc. Finally we end up with a calibrated model with identified model parameters.

By comparing the multiple sources of data and eliminating the inconsistencies, it was possible to establish for the region to be modeled a reliable data base of over 300 static water elevations, ground surface elevations, and well depths. Data, not quite so extensive but well distributed throughout the region, were geologic cross sections, chemical analyses, and aquifer pumping tests. The latter data were extremely valuable in establishing the aquifers transmissivities, storativities, and water quality. Pumping tests were performed at six wells in the area tested for the CBWP wellfield at Sarir. In Tazerbo CBWP area eight pumping tests were made at six locations. Moreover, tests were performed at Jalu-Awjilah agricultural project and at Marada agricultural project. The foregoing figures of pumping test results, depicted in Table 1, were taken from ElRamly (1980, Table 16).

Table 1. Aquifers hydraulic parameters obtained from pumping tests.

Wellfield	Transmissivity m^2/day	Storativity	Hydraulic conductivity m/day
Sarir	1090-6500	$5\cdot10^{-4}$-$3\cdot10^{-2}$	15
Tazerbo	180-4500	$3\cdot10^{-4}$-$5\cdot10^{-2}$	
Jalu-Awjilah	1000-2000	$1\cdot10^{-5}$-$2\cdot10^{-2}$	
Marada		$6\cdot10^{-3}$-0.4	

Similar model building process can be used with the model presented in the sequel. This model is considerably simpler than the partial differential model mentioned above and needs only limited aquifer parameters, mainly connected with the ground hydrological properties. It consists of equations for the pumping wells and can be useful when the water level is to be predicted.

3.2. Hydraulics of pumping wells

Pumping from phreatic aquifer removes water from the void space leaving there only a limited quantity of water which is held against gravity. As a result the water table at each point is lowered with respect to its initial position by a vertical distance called *drawdown* $s(x,y,t)$, see Figure 3. The drawdown surface in the vicinity of a pumping well forms a cone of depression. The cone of depression, with pumping at a rate Q_w from an isotropic phreatic aquifer, is radially symmetric between circular equipotential boundaries at $r = R$ and $r = r_w$ where r_w is the well radius and R is the well influence radius defined in the sequel. The drawndown $s(r)$ for a single well, for $r_w \leq r \leq R$, can be described by the equation, see e.g. Bear (1979),

$$s(r) = \phi(R) - \phi(r) = \frac{Q_W}{2\pi T} \ln\left(\frac{R}{r}\right) \tag{1}$$

where Q_w is the pumping rate. At the well wall, for $r = r_w$, the drawdown is

$$s(r_w) = \phi(R) - \phi(r_w) = \frac{Q_W}{2\pi T} \ln\left(\frac{R}{r_w}\right) \tag{2}$$

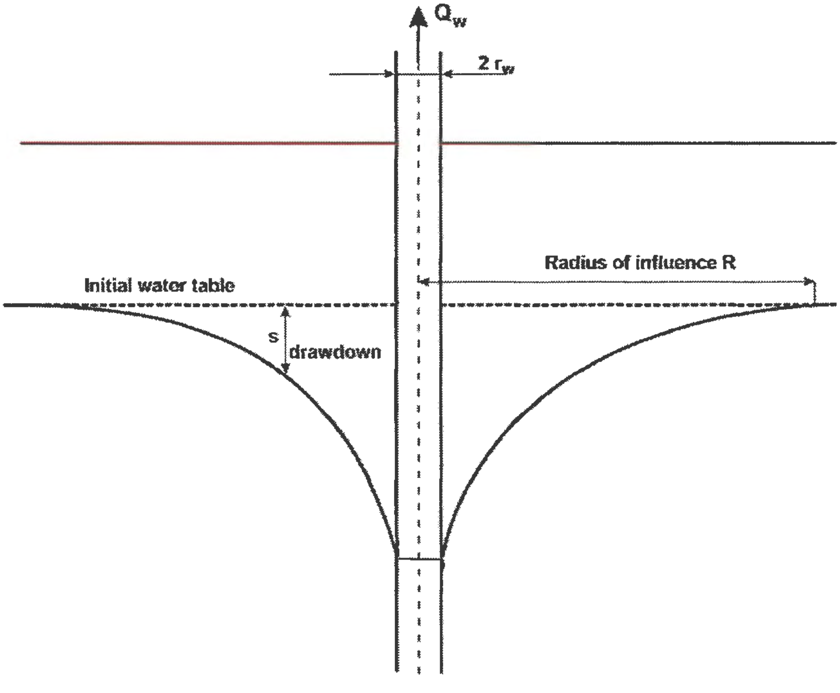

Figure 3. Geometry of a pumping well.

By definition, the distance R where the drawdown is zero or unobservable is the radius of influence of the well. In general, this parameter has to be estimated from past experience. Various attempts have been made to relate the radius of influence, R, to the well aquifer and flow parameters in both steady and unsteady flow in confined and unconfined, or phreatic, aquifers. Some relationships are purely empirical, others are semi-empirical. For example Bear (1979) quotes the following semi-empirical formula

$$R = \alpha \sqrt{\frac{H_0 K t}{n_e}} \quad \text{with } \alpha = 1.9 \text{ or } \alpha = 2.45 \tag{3}$$

where R and H_0 (initial water table depth) are in meters and K (hydraulic conductivity) is in meters per second, and n_e is the specific yield, which is equivalent to storativity for the phreatic aquifers.

3.3. Multiple well system model

When wells are spaced at distances smaller than their radius of influence R, they affect each other's drawdown and discharge rate. The common drawdown can be obtained by summing the drawdowns from different wells. Consider steady flow in an infinite aquifer (or in a bounded one, with homogeneous boundary and initial conditions) in which N wells are operating at a constant pumping rate Q_j at a point (x_j, y_j). Let S_i be the total drawdown at an observation well located at (x_i, y_i) and s_{ij} be the drawdown caused at observation point (x_i, y_i) by the pumping well at (x_j, y_j), as if that well is operating alone in the considered field. Then

$$S_i = \sum\nolimits_{j=1}^{N} s_{ij} \tag{4}$$

For steady flow in an infinite confined aquifer the drawdown s_{ij} for a single well is given by an expression which has the general form

$$s_{ij} = \frac{Q_j}{2\pi T} \ln\left(\frac{R_j}{r_{ij}}\right)$$

where $T = KH_0$ is the aquifer transmissivity and r_{ij} the distance between the two wells. Hence equation (4) can be written as

$$S_i = \sum_{j=1}^{N} \frac{Q_j}{2\pi T} \ln\left(\frac{R_j}{r_{ij}}\right) \tag{5}$$

assuming a different R_j for every pumping well.

For special case $Q_1 = Q_2 = \ldots = \mathrm{Q} = \text{constant}$ and $R_1 = R_2 = \ldots = R$ equation (5) becomes

$$S_i = \sum_{j=1}^{N} \frac{Q_j}{2\pi T} \ln\left(\frac{R}{r_{ij}}\right) = \frac{NQ}{2\pi T} \ln\left(\frac{R}{r^*}\right) \tag{6}$$

where $\left(r^*\right)^N = r_{i1} r_{i2} \ldots r_{iN}$. This means that the drawdown S_i is equal to that produced by a single well pumping at a rate NQ localized at an equivalent - geometrical mean - distance r_i^* from (x_j, y_j).

3.4. Identifiability analysis of the model

Not all of the parameters in model (6) are known with sufficiently good accuracy. The parameters N, Q, r^* depend on the field topology and assumed water abstraction. They are known accurately. However, the parameters H_0, K, n_e depend on the soil properties and are known only approximately. Also the coefficient α in the formula for the radius of influence (2.45 or 1.9) is not known exactly and different values of it are taken in the literature.

Conceptually, these parameters, known only approximately, could be estimated from the measurements of the drawdowns available in different years in the future. We show below that it is not possible to determine these parameters in this way completely and that only some of their combination can be estimated. The reason is that the parameters are not identifiable.

The notion of parameter identifiability can be roughly explained as a possibility to calculate an unknown parameter from the perfect (unobstructed) measurements. Such parameter is called to be identifiable globally if only one solution exists in the region considered. If the solutions are many but countable, then the parameter is called to be identifiable locally. If the solutions are uncountable, the parameter is unidentifiable.

To check the parameter identifiability of model (6) we insert the formula for the radius of influence (3) to (6) and drop the subscripts to get

$$S = \frac{NQ}{2\pi KH_0} \ln\left(\frac{\alpha}{r^*}\sqrt{\frac{KH_0 t}{n_e}}\right)$$

This formula can be also written in the form

$$S = \frac{NQ}{2\pi KH_0} \ln\left(\frac{\alpha}{r^*} \sqrt{\frac{KH_0}{n_e}} \right) + \frac{NQ}{4\pi KH_0} \ln t$$

or as

$$S = A + B \ln t \tag{7}$$

where

$$A = \frac{NQ}{4\pi KH_0} \ln \frac{KH_0 \alpha^2}{(r^*)^2 n_e} \quad \text{and} \quad B = \frac{NQ}{4\pi KH_0} \tag{8}$$

Coefficients A and B can be estimated from observations of the drawdown S in different years using the formula (7). But they are not enough to determine four unknown parameters K, H_0, n_e, α. Thus the model parameters are unidentifiable. It can be easily checked that observations of the drawdown in different space points do not change the identifiability considerations and conclusions.

Let us notice, however, that from equations (8) we can calculate

$$KH_0 = \frac{NQ}{4\pi B} \qquad \frac{\alpha^2}{n_e} = \frac{4\pi (r^*)^2 B}{NQ} e^{\frac{A}{B}}$$

so that combination of some parameters can be determined from observations, provided N, Q and r^* are known. The first formula gives the value of transmissivity $T = KH_0$.

Because of unidentifiability of the model parameters, here another procedure, called calibration, is used. The initial values of parameters are chosen in the range of values quoted in the literature and then manipulated to get as good fit as possible. In general, calibration will not give the optimal set of parameters but only a satisfactory one.

4. Water production from CBWP wellfields

4.1. Sarir wellfield

Sarir wellfield, called *the coastal belt water project (CBWP) wellfield at Sarir*, is located about 370 km south-southeast of the Gulf of Sirte, in the southern end of the Sirte basin, which is a large structural basin flanking the stable African shield. It consists of 126 production wells, 450 meters deep, tapping the post-Eocene aquifers, in three parallel rows set in east-west direction. Each row consists of 42 wells, 1.3 km apart from each other, see

Figure 4. The distance between rows is 10 km. The static water level varies from 60 to 90 meters below ground level (mbgl). Each well is designed to produce 92 l/s of water, 114 wells being operated at the same time, with 12 wells in standby. The wellfield was designed to produce 1 Mm^3/day. The wells were drilled in 1987-1990.

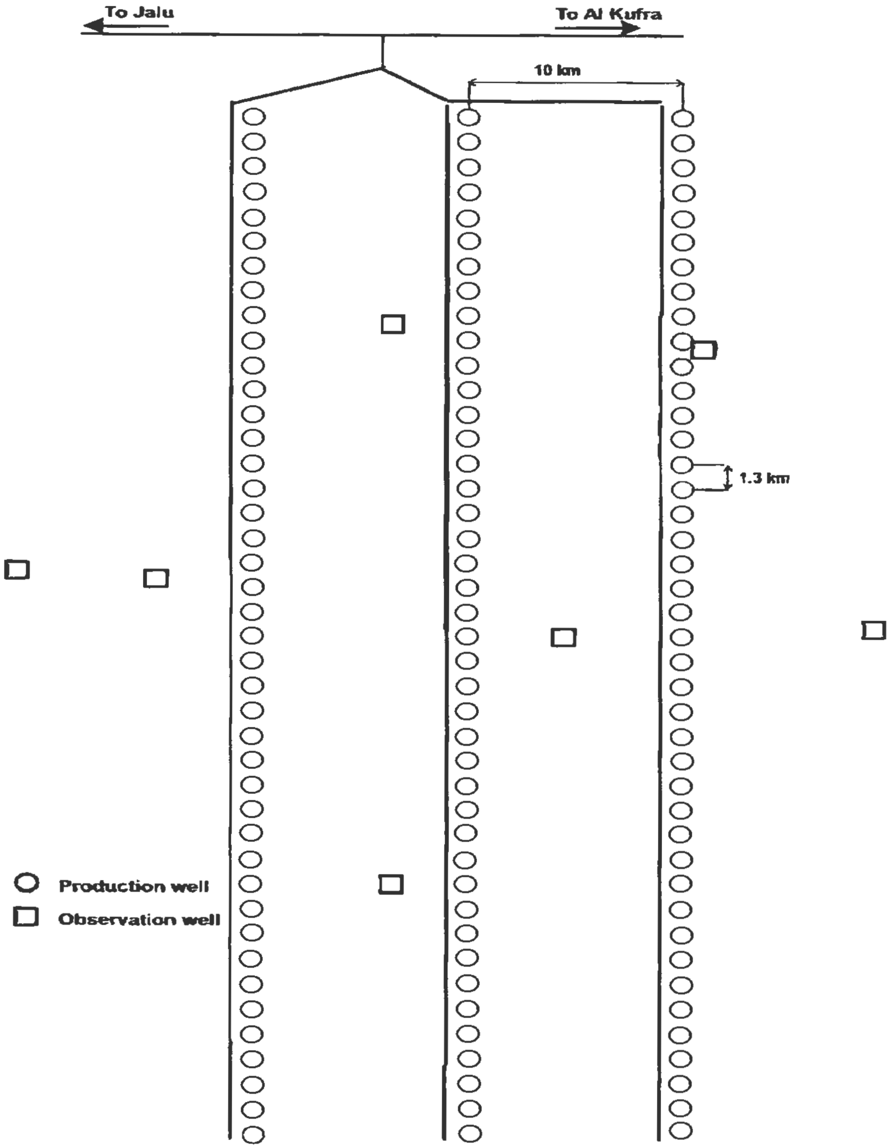

Figure 4. Sarir wellfield outline.
Source: Madi (1993)

Apart of CBWP, in Sarir region there are Sarir production projects (SPP) wellfields, situated approximately 30 to 50 km east of CBWP wellfield. The two wellfields were set up in the 1970s and aimed at agricultural production. Because of the similarity of the geological and hydrological situations of CBWP and SPP some conclusions can be drawn from the results of 15 to 20 year operation of SPP to be applied to CBWP wellfield.

The total drawdown in the CBWP wells predicted by Fisk (1983) model is ranging from 45 to 60 meters after 50 year operation, assuming an abstraction of 1 Mm^3/day.

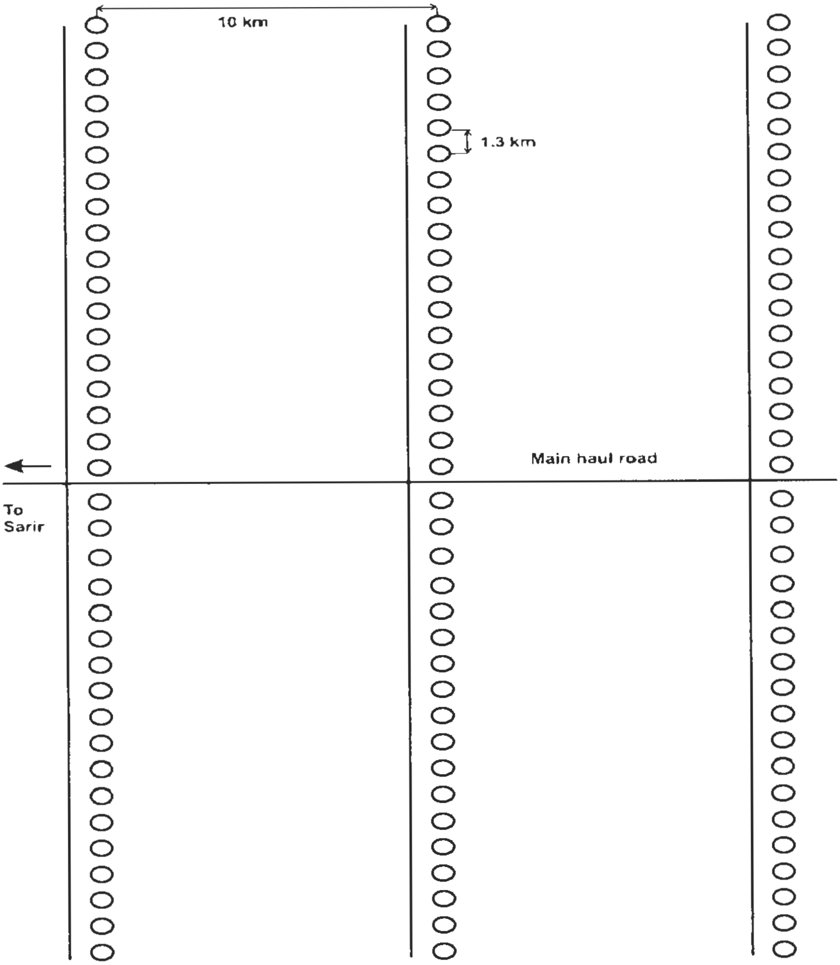

Figure 5. Tazerbo wellfield outline.
Sorce: Madi (1993)

4.2. Tazerbo wellfield

Tazerbo wellfield, called *the coastal belt water project (CBWP) wellfield at Tazerbo*, is located about 200 km south of the Sarir wellfield and roughly 40 km southeast of the oasis of Tazerbo. The CBWP wellfield at Tazerbo is in a transitional area that separates the Sirte basin from the Kufra basin. There are widely divergent interpretations of the subsurface geology in the Tazerbo area because of meager exploratory drilling done there. It consists of 108 wells tapping the Palaeozoic aquifer in three parallel rows. Each row consists of 40 wells, 1.3 km apart from each other, see Figure 5. The distance between rows is 10 km. The wells range in depth from 500 to 800 meters with a static water level ranging 7 and 24 (mbgl). Each well was originally designed to produce 120 l/s, 98 wells being operated at the same time, with 10 wells in standby. The wellfield was designed to produce 1 Mm^3/day.

Several computer models, Ahmad (1979), Fisk (1983), Abufila (1984), Brown & Root (1992), were prepared to tentatively predict the response of Tazerbo aquifer to a daily abstraction of 1 Mm^3/day. However, neither of them could definitely remove the uncertainties regarding a fundamental parameter - the vertical permeability of the mudstone and siltstone layers (aquitard). The latter model has proved some indication that maximum 50 year drawdown in Tazerbo wellfield is likely to be about 109 meters - only slightly greater than the 96.2 meters predicted by General Water Authority (GWA).

5. Predictions of groundwater levels using simplified models

CBWP Sarir and Tazerbo wellfields have been modelled using Thiem equation (6) to predict their water level drawdown in 50 years of the project life using an abstraction quantities of $Q_1 = 0.9062$ Mm^3/day and $Q_2 = 0.9953$ Mm^3/day for both wellfields, respectively. These predicted figures of wellfields water level drawdowns were simulated by choosing the following four parameters: transmissivity (T), hydraulic conductivity (K), initial height of water table (H_0), and specific yield (n_e).

It was assumed that the aquifers at the coastal belt water project wellfield at Sarir had similar hydrologic properties as those at Sarir South agricultural project wellfield. This justifies modeling the CBWP wellfield at Sarir based upon aquifer characteristics similar to those determined for the Sarir South wellfield from its 7-year production history. Then, the parameters were chosen from the ranges published for the both Sarir and Tazerbo wellfields, respectively. The final parameters chosen are given in Table 2.

The prediction results are shown on Figures 6 and 7. The values presented comprise smallest drawdowns, for the wells in the corners, and the biggest ones, for the wells in the center of the wellfiels.

Table 2. Literature parameters and the chosen parameters.

	Sarir wellfield		Tazerbo wellfield	
Parameter	literature	chosen	literature	chosen
K [m/d]	10 - 20	15	5 - 30	20
T [m^2/d]	400 - 6000	5000	180 - 4500	2000
H_0 [m]	400 - 500	500	260 - 500	300
n_e	0.005 - 0.4	0.04	0.005 - 0.3	0.2

We concluded from the simulation processes that predictions of water table drawdown for those wellfields is very sensitive with respect to parameter changes especially for parameters T, K and H_0, see Daddesh (1998b).

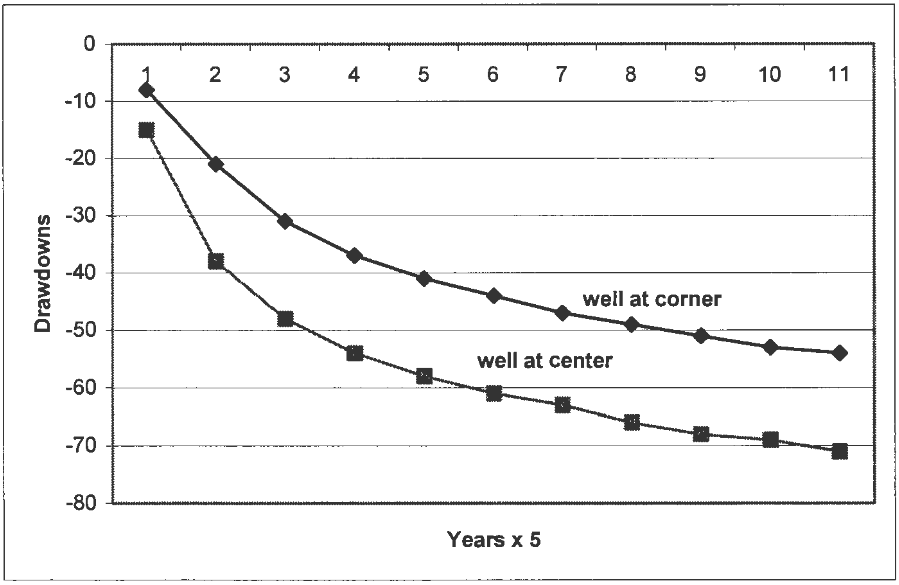

Figure 6. Predicted drawdowns in Sarir wellfield.

Having set the suitable parameters, finally we have compared these prediction results with different previous predictions published by others. The figures are shown in Table 3. The difference between these predictions of the wellfields can be attributed to the different modelling techniques as well as the use of different aquifer characteristics or parameters. The models obtained here were also used to predict drawdown values for other wellfields (not presented in this paper).

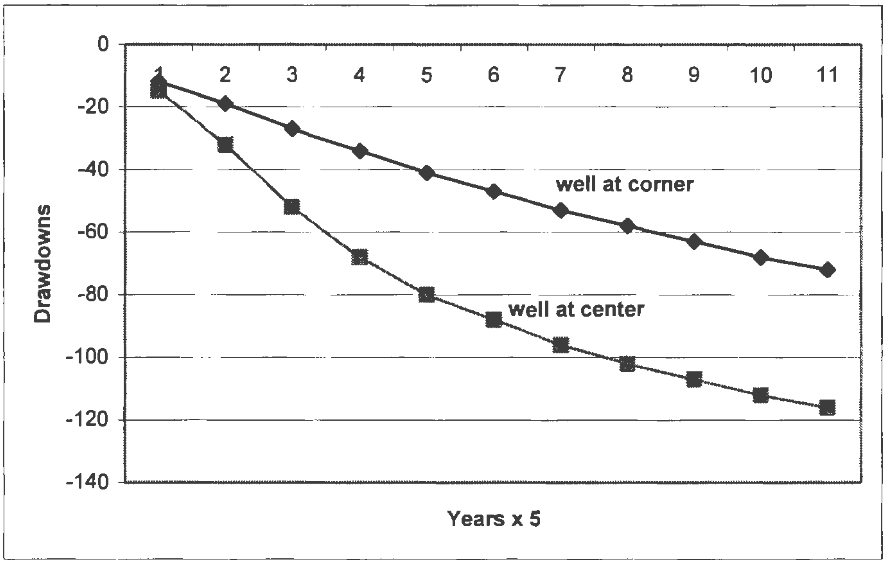

Figure 7. Predicted drawdowns in Tazerbo wellfield.

Table 3. The CBWP wellfields drawdown predictions using different computer models in 50 year project life.

wellfield	predicted drawdown in m	modeled by
Sarir wellfield	45 - 60	P. Fisk
	50 - 86	M. Ahmad
	54 - 71	**this study**
Tazerbo wellfield	75 - 95	B&R and GWA
	90 - 110	B&R
	72 - 101	M. Ahmad
	71 - 116	**this study**

6. Conclusions

The modelling performed led to the following results: (i) regional drawdown at Sarir wellfield was estimated to range between 54 and 71 meters at the end of 50 years, (ii) regional drawdown at Tazerbo wellfield was estimated to range between 72 and 116 meters at the end of 50 year, (iii) there are ample quantities of groundwater with good and satisfactory quality of water to sustain the proposed CBWP wellfields at Sarir and Tazerbo, each producing 1 Mm^3/d for the 50 year design life of the project. The results obtained agree quite well with those predicted previously by using the partial differential models.

This paper is also meant as a tribute to the memory of late Dr. Abdalla Maalul Daddesh who spent 5 years in Poland working on the above problem when preparing his PhD thesis.

Obituary

Late Abdalla Maalul Daddesh was born in 21 May 1945 in Sul Guima in Libya. In 1967 he terminated the secondary school in Tripoli. In 1971 he graduated from the Libyan University obtaining BSc in mathematics. Then he spent 3 years working as a teacher of mathematics in a secondary school. In 1974 he started to read in the Pensylwania State University in USA from where he graduated in 1977 with MSc in education of mathematics and statistics. Since then he was with Al Fatah University in Tripoli as a didactic and scientific worker. From 1993 he started his scholarship in Poland where he prepared and defended his PhD thesis *Modelling and Forecasting the Groundwater Vulnerability in Libya* under supervision of Zbigniew Nahorski. After returning to Libya he suddenly fell ill and despite of a longer treatment passed away.

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ECO – INFO AND SYSTEMS RESEARCH

This book presents the papers that describe the most interesting results of the research that have been obtained during the last few years in the area of applications of informatics in environmental engineering and environment protection at the Systems Research Institute of Polish Academy of Sciences in Warsaw (IBS PAN) and at the German Institute for Landscape System Analysis in Müncheberg (ZALF). The papers were presented in the form of extended summaries during a special workshop organized by IBS PAN in Szczecin in September 2006 together with the conference BOS'2006 dedicated to the applications of systems research in science, technology and economy and organized jointly by IBS PAN, University of Szczecin, and the Polish Society of Operational and Systems Research. They deal with mathematical modeling, approximation and visualization of environmental variables and with development of computer aided decision making systems in the area of environmental informatics.

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