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SUPPORT SYSTEMS FOR DECISION AND NEGOTIATION PROCESSES

Preprints of the IFAC/IFORS/IIASA/TIMS Workshop
Warsaw, Poland
June 24-26, 1992

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VOLUME 2:

Names of first authors: **L-Z**

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Preprints, IFAC/IFORS/IIASA/TIMS Workshop, June 24-26, 1992, Warsaw, Poland

CONFLICT RESOLUTION WITH ROBUSTNESS IN INTERNATIONAL NEGOTIATIONS: A GAME THEORETIC APPROACH

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Abstract: A robust game theoretic approach for constructing effective international concords for conflict solving is discussed. An n-person cooperative game in characteristic function form is used for international conflict solving via formation of coalitions. Nucleolus and augmented nucleolus as the solution concepts of the game are derived on its alternative forms and robustness of the solutions when the evaluation of the coalition values is varied is examined. For solving this problem, parametric linear programming is used. This approach provides in a resemblant form an alternative device to solving the fuzzy linear programming based on interval analysis when diversified evaluation is introduced.

Keyword: Cooperative game, Robust game, International conflict solving, Nucleolus, Angmented nucleolus, Parametric program, Fuzzy linear programming

I. Introduction

A robust game theoretic approach is discussed for constructing effective international concords for conflict-solving. An n-person cooperative game in characteristic function form is used for the international conflict-solving via formation of coalitions and the robustness of its solutions is examined.

The two-layer hierarchical systems configuration is constructed for international conflict solving. The nucleolus and augmented nucleolus concepts as solutions of a cooperative game are derived and the robustness of their values is examined. Generally the evaluation of the characteristic function as the coalition value is not unique, but varies in an interval due to variant assessment. Then the effects on the solution of a change in the given data for the assessment are examined. For solving this problem, parametric linear programming is used. This approach provides in a resemblant form an alternative device to solving the fuzzy linear programming based on interval analysis when diversified evaluation is introduced.

II. Two-layer Systems Analysis For International Conflict Solving

For solving the international conflicts among national interests, construction of international concords is intended as the results of multilateral negotiations. For effective formation of an international concord, an n-person cooperative game in characteristic function form is constructed.

The property of international conflict-solving is the lack of an authority as an effective mediator for guiding the conflicting national interests to an international compromise, while the effectiveness in alternative formation of international coalitions should be examined in a consistent way with the national interest. A game theoretic approach provides a device for evaluating the effectiveness in a Paretian sense. Each country which intends to participate in the formation of a coalition is treated as a player in a game and its efficiency is evaluated in terms of an n-person cooperative game. A characteristic function v(S) for the coalition S is defined on the total of incremental values of a pay-off value for each country l, $l \in S$, due to the formation of the coalition S:

$$v(S) = \frac{1}{s} \sum_{l \in S} (EG_S - EG_l). \qquad (1)$$

where $S \subsetneq N$, $N \triangleq \{1, ..., n\}$, and s is the number of players in a coalition S. EG_l and EG_S are the expected pay-off values for a country (player) l, $l \in S$, and for a coalition S, $S \subsetneq N$, respectively. The v(S) in Eq. (1) is a coalition value as a characteristic function which indicates a incremental value in the pay-off resulting from the formation of international cooperation.

A two-layer system is constructed for the international conflict solving. At the first layer, the expected pay-off value for each country EG_l is assessed independently of the consideration for other countries. The characteristic function (Eq. (1)) is assessed by the member country $l \in S$, with the equal right for the members as being assumed. EG_S is assessed on the EG_l-values and their changed values as predicted by an executive committee which is composed of the member countries.

At the second layer, an n-person cooperative game in characteristic function form is solved for assessing the effectiveness of alternative coalitions and compared with each other.

III. The Augmented Nucleolus For An N-person Cooperative Game For International Conflict Solving

A. Solution concepts of a cooperative game

The solution concepts of an n-person cooperative game in characteristic function form should be examined.

Define a payoff vector as $z \triangleq \{z_1, ..., z_n\}$, whose elements are the improved values of the pay-off for a player l, $l = 1, ..., n$, which is assessed in terms of the increased values of the EG_l due to the formation of a coalition.

As an extension of the nucleolus, a solution concept, define the concept of the augmented ε-core that is enlarged the ε-core with a parameter μ as

$$e_{\mu}(S, z) \triangleq v(S) - z(S) \leq \mu\varepsilon, \qquad (2)$$

along with $v(N) = z(N)$. The augmented nucleolus $N(\mu)$ is defined on the augmented ε-core (2), which is obtained as a solution by repetitively running a linear program. Consider the 0-normalization that is defined with $v(\{l\}) = 0$ for all $l \in N$. The augmented nucleolus $N(\mu)$ is obtained by solving the following linear programming problem and the $\xi_o(z)$-value (3) is found.

$$\xi_{\mu}(z) = \min_{z \in \chi} \max_{S \neq \phi, N} e_{\mu}(S, z) \qquad (3)$$

where χ is a set of preimputation that satisfies the collective rationality, $z(N) = v(N)$.

(P)

$$\begin{aligned} &\underset{z_l,\, \varepsilon}{\text{minimize}} \quad \varepsilon \\ &\text{subject to} \quad z_l \geq 0 \quad \text{for } l \in N \\ &\qquad v(S) - (z(S) + \mu\varepsilon) \leq 0 \quad \text{for all } S \subset N \\ &\qquad v(N) - z(N) = 0 \end{aligned} \qquad (4)$$

The interpretation of the linear problem (4) with the augmenting parameter μ is clear. When $\mu = 1$, the solution to (P) provides the nucleolus. When $\mu = s$, the weak nucleolus (Shapley and Shubik 1966) is obtained, where s denotes the number of players in a coalition S. When $\mu = v(S)$, the proportional nucleolus is obtained (Young, Okada and Hashimoto 1982).

B. Evaluation of an international concord

The assessment for conflict-solving is performed at the international level in the second layer of the two-layer hierarchical system. The characteristic function (Eq. (1)) of the cooperative game is constructed from the assessment of the EG_l values for each country at the first layer. The expected pay-off value for a country l obtained after the formation of the coalition N is shown as

$$w_l^{\dagger} = EG_l + z_l^{\dagger}, \quad l \in N, \tag{5}$$

where $w_l^{\dagger}$ represents a value of the expected pay-off assessed for each country l after the effective construction of an international concord. $z_l^{\dagger}$ is an incremental value of the pay-off for a country l obtained as a solution of the game.

Alternative solutions $z^{\dagger} \triangleq \{z_1^{\dagger}, ..., z_n^{\dagger}\}$ of the augmented nucleolus provide alternative payoff values for each player (country). These solutions depend on alternative criteria and are not unique for the final decision. Their property has been examined comparably in Seo and Sakawa (1990).

VI. Robustness Of The Solutions Of A Game

We shall examine the robustness of the optimal solutions, $z_l^{\dagger}$ and $\varepsilon^{\dagger}$, to the linear program (P) for obtaining the nucleolus and the augmented nucleoli of the game. For this purpose, we shall construct a post-optimization program as a parametric program. In the linear programming formulation (P), the characteristic functions of the game, v(S) and v(N), are given as data, which are treated as a constant vector in the minimization problem (Eq. (4)). We will construct a parametric problem in which the data in the original program (P) are varied in a continuous manner and variations of the optimal solutions are observed as functions of the varied value of data . A parametric change of the constant vector, $V \triangleq \{ v(S), v(N) \}$, is performed by varying these values as a linear function of a parameter θ,

$$V = V_0 + \theta\delta \tag{6}$$

where $V_0 \triangleq \{v_0(S), v_0(N)\}$, and δ are fixed value vectors.

The linear programming problem for obtaining the nucleolus and the augmented nucleoli is rewritten in a form of the parametric program to check the robustness of solutions in the parametric change of the coalition values (Eq. (6)).

(P')

$$\begin{aligned} &\underset{z_l,\,\varepsilon}{\text{minimize}} \quad \varepsilon \\ &\text{subject to} \quad z_l \geq 0 \quad \text{for } l \in N \\ &\qquad z(S) + \mu\varepsilon \geq v_0(S) + \theta\delta \quad \text{for all } S \subset N \\ &\qquad z(N) = v_0(N) + \theta\delta \end{aligned} \tag{7}$$

For a known optimal solution, it is possible to set as $\theta = \theta_0 = 0$ by a change of origin on θ. Let θ increase (or decrease) from 0 through positive (negative) values. The problem for checking the robustness of the solution of a game is to examine the existence of a critical value $\theta = \theta_1$ beyond which the values of the solutions $z_l^{\dagger}$ loose the current structure composed of the nucleolus and augmented nucleolus.

From the theoretical point of view, the general characteristic in the parametrization of the requirement vector in a linear program has been known.
Let $\bar{x}_0^B \triangleq \{ z, \varepsilon \}$ be an optimal basic solution vector to a linear programming problem. The basic solution to a parametric program is varied with a parameter θ and described as

$$\begin{aligned} x^B = \bar{x}^B &\triangleq B^{-1}(V_0 + \theta\delta) \\ &= B^{-1}V_0 + \theta B^{-1}\delta \\ &\triangleq \bar{x}_0^B + \theta\xi, \end{aligned} \tag{8}$$

where B denotes a basis of a linear program and $\xi \triangleq B^{-1}\delta$ is a vector. Then it is induced that (i) if $\xi \geq 0$, the basic solution x^B remains an optimal solution for every value of $\theta \geq 0$; (ii) if any $\xi_i < 0$, there exists a critical value θ_1 beyond which the basic solution $\bar{x}^B$ is no more assured to be the optimal solution, at least any one component of $\bar{x}^B$ becoming negative (Simonnard 1966).

For examining the robustness of the game, the post-optimization problems are considered. A parametric program (Eq.(7)) is formulated with the discrete modifications of the given data for the requirement vector v(S), $S \subseteq N$, and solved in several cases.

Case 1. The requirement vector v(S) varies continuously as a linear function of a parameter θ. Set

$$v(S) = v_0(S) + \theta\delta, \quad S \subseteq N, \tag{9}$$

where δ is a fixed vector and changes independently of the v_0 (S)-values fixed as data.

Case 2. The requirement vector v(S) varies in proportion to the fixed values of $v_0(S)$ with a parameter θ as

$$v(S) = v_0(S) + \theta\delta' v_0(S), \qquad S \subseteq N \tag{10}$$

Set $\delta' v_0(S) = \delta$. Then Eq. (10) is equivalent to Eq. (9).

Case 3. A variant of Case 2 is constructed with

$$v(S) = v_0(S) + \theta\delta' v_0(S), \; S \ni i, \; S \subseteq N. \tag{11}$$

The solutions are examined and evaluated for robustness of the proposed game.

V. Similarity To Fuzzy Linear Programming With Interval Analysis And Its Interpretation

The parametric programming problem (P') as a post-optimization program can be converted as a similar form to a fuzzy linear programming problem with fuzzy constraints based on interval analysis. In both formulation, the characteristic function can be treated as including the variant assessment for the v(S)-values, which may be a diversification as a result of the existence of internal group decision making. This interpretation will be particularly meaningful in examing the robustness of solutions in diversified value environments.

REFERENCES

Seo, F. and Sakawa, M. " A game Theoretic Approach with Risk Assessment for International conflict solving", *IEEE Transactions on Systems, Man, and cybernetics* **20**(1), pp. 141-148, 1990.

Shapley, L. S. and Shubik, M. "Quasi-Cores in a Monetary Economy with Nonconvex Preferences, " *Econometrica*, **34**(4), pp. 805-827, 1966.

Simonnard, M. *Linear Programming*, translated by W. S. Jewell, Prentice-Hall, N. J. 1966.

Young, H.P., Okada, N. and Hashimoto, T. "Cost Allocation in Water Resources Development," *Water Resources Research*, **18**(3), pp.463-475, 1982.