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FUZZY RELATIONS AS A TOOL FOR NON-FUZZY DECISION-MAKING

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Abstract: We discuss some problems connected with modelling preferences by fuzzy relations, especially in multi-criteria situation. A method of evaluating fuzzy preferences by aggregating multi-criteria comparisons is proposed.

Keywords: Fuzzy preferences, Multi-criteria decision analysis.

Our information about surrounding world is imprecise, uncertain and vague. Yet, we should constantly choose and decide. If we consider a typical decision situation and if we assume that the set of decision alternatives is fixed and finite (even such simple assumption may be unrealistic - often we don't even know what are our possibilities!), then our decision problem is to choose one (or two, or three, or ...) of them. Our act of choosing should be, by no means, precise and non-vague. To choose the "best" alternative a decision-maker should compare all the possible alternatives and this is done on the basis of his preferences.

Classical decision models use "crisp" preference relations to model DM's preferences, but, if we want a DM to describe exactly his preferences and he (she) has, for any two alternatives, say A and B, only two (three) possibilities: A is better than B, B is better than A (A is equivalent with B), then he, in some sense, may feel forced to make a difficult choice before his real choice could be done.

One possibility to obtain an adequate model of DM's preferences is to use fuzzy relations, i.e. to attach to every pair (A,B) of alternatives a number P(A,B) from interval [0,1] (another possibility we could think about is to connect with a pair (A,B) some fuzzy number (fuzzy interval) P(A,B)).

There may be some reasons for which fuzzy preferences may be more adequate in a decision situation then crisp preferences. The main reasons seem to be:

1. Uncertainty of DM as to his (her) preferences (hesitation of DM).

2. Existence of different opinions (in the problem of group choice).

3. Existence of many criteria.

4. Uncertainty of consequences of alternatives.

5. Lack of information.

In the existing literature on fuzzy preferences the main attention is paid to situations 1 and 2 (see Orlovsky(1978), Nurmi(1981), Tanino(1984)). There are also some papers in which situation 3 is considered (Barrett, Pattanaik(1985), Chanas,Florkiewicz(1987), Zhukovin et al.(1987)). Modelling of preferences in situations 4 and 5 would be (as I think) a good theme for future research in this area. Notice that in real situation we often have 1,2,3,4 and 5 simultaneously.

I would like to say some words about situation 3. The first problem is how to model the preferences of DM in this situation properly.

Assume that we have n criteria $K_1,K_2,\dots,K_n$ and we can compare every pair of alternatives A and B with respect to criterion K_i, obtaining fuzzy relation $P_i(A,B)$. We could construct a global fuzzy relation P(A,B) by

$$P(A,B) = \frac{1}{n}\sum_{i=1}^{n} P_i(A,B)$$

or

$$P(A,B) = \min_i P_i(A,B)$$

(see Zhukovin et al.(1987).

Such an approach has two disadvantages:

1. Numbers $P_i(A,B)$ are bounded by 1, this condition may be too restrictive, there can be situation when A is definitely preferred over B with respect to K_i, B is definitely preferred to A with respect to K_j (hence $P_i(A,B) = P_j(B,A) = 1$) and yet the alternative A is better than B with respect to K_i in greater "degree" than B is better than A with respect to K_j.

2. The Pareto condition:

$P_i(A,B) \geq 0.5$ for all i, and $P_i(A,B) > 0.5$ for some i,

implies $P(A,B) = 1$,

may not be satisfied.

To avoid the above disadvantages I propose to use for global preferences the relation

$$P(A,B) = \frac{\Sigma^+ P_i(A,B)}{\Sigma^+ P_i(A,B) - \Sigma^- P_i(A,B)},$$

where $P_i(A,B)$ are numbers from $(-\infty,+\infty)$ (we don't assume that there are bounds for DM preferences with respect to K_i), and Σ^+ means summing over all positive $P_i(A,B)$, Σ^- - summing over all negative $P_i(A,B)$ ($P_i(A,B) < 0$ means that B is better than A with degree $-P_i(A,B)$, so $P_i(A,B) + P_i(B,A) = 0$).

The above index can be used in a situation when the DM is certain about his preferences with respect to every K_i and the number $P_i(A,B)$ measures only "intensity" of his preference (see Tanino (1984), for example when alternatives are characterized by some criterial function f_i and $f_i(A) > f_i(B)$, then the DM may be quite certain that A is better than B, independently of the difference $f_i(B) - f_i(A)$, but his "intensity" or "strenght" of preference will, in general, hardly depend on this difference.

The second problem is how to choose the "best" alternative on the basis of fuzzy preferences $P(A,B)$. There can be defined many different "choice functions" associated with a given fuzzy relation (see Barrett, Pattanaik(1985), Roubens(1989), Switalski (1988)). But we should be very careful when applying one of them. Consider the following operators defined on the set of all alternatives:

$$B_1(A_i) = \min_{j \neq i} P(A_i, A_j) ,$$

$$B_2(A_i) = \frac{1}{n} \sum_{j \neq i} P(A_i, A_j) .$$

The numbers $B_1(A_i)$, $B_2(A_i)$ may be treated as some kind of degrees of "bestness" for alternative A_i. The number $B_1(A_i)$ is minimal degree with which A_i is better than the others A_j. From the point of view of many-valued (fuzzy) logic $B_1(A_i)$ may be interpreted as truth-value of the sentence "A_i is preferred over all A_j", if $P(A_i, A_j)$ are truth-values of the sentences "A_i is preferred over A_j". The number $B_2(A_i)$ is the degree with which A_i is better than A_j "on average". After computing B_1 (or B_2) we should choose an alternative with the greatest number $B_1(A_i)$ (or $B_2(A_i)$). See that $B_1(A_i)$ may be not quite good indicator of "bestness" in some situations. For example, if for one j, $P(A_i, A_j) = 0.5$, and for all other j, $P(A_i, A_j) = 1$, then $B_1(A_i) = 0.5$, although A_i is almost best in degree 1. If we use the second operator we could be unsatisfied if for example $P(A_i, A_j) = 0$ for some A_j, and $P(A_i A_j)$ are near 1 for many other j, and as a result we obtain $B_2(A_i)$ near 1, although there is an alternative - A_j, almost definitely preferred over A_i. To avoid such disadvantages we could use some operator (say B_3) for which $B_3(A_i) = 0$ if there is A_j such that $P(A_i, A_j) =0$ and $B_3(A_i) = B_2(A_i)$ if $P(A_i, A_j) \geq 0.5$ for all j. An example of such operator could be the following:

$$B_3(A_i) = \begin{cases} B_2(A_i), & \text{if } B_1(A_i) \geq 0.5, \\ [1 - 2B_1(A_i)]B_1(A_i) + 2B_1(A_i)B_2(A_i), & \text{otherwise.} \end{cases}$$

The operator B_3 combines advantages of the operators B_1 and B_2 (and has no their drawbacks). First, observe that if

$B_1(A_i) = 0$, then $B_3(A_i)$ is also 0. Secondly, if $0 < B_1(A_i) \le 0.5$, then $B_3(A_i)$ is weighted average of $B_1(A_i)$ and $B_2(A_i)$ - with weights $1 - 2B_1(A_i)$ and $2B_1(A_i)$, and if $B_1(A_i) \ge 0.5$, then $B_3(A_i) = B_2(A_i)$. Hence, if some of $P(A_i, A_j)$ are near 0, then also $B_3(A_i)$ is near 0, and if almost all $P(A_i, A_j)$ are near 1 (and some of them are near 0.5), then $B_3(A_i)$ is near 1.

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