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BARGAINING SYSTEM FOR ALLOCATION OF RESOURCES

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Abstract: As a tool to analyze resource allocation problems, an interactive system of bargaining game has been set up, which can be widely used in studies for development planning of regional economies and resource allocations of integrated corporations. The method to be used is the bargaining game for allocation of resources developed by Qu(1989,1990). A model, which consists of multiple I/O models with bargaining axiomatic characterization and high-productivity, is introduced as basis of the system. The main ideas and implementations of model generation and solution interaction are discussed in combination with the system.

Keywords: resource allocation problem; bargaining game; bargaining axioms; visual interactive system; model generation.

1. Introduction

The resource allocation for multiple Decision Makers(DMs) is an important decision problem in economic analyses, especially in development planning of regional economies and resource allocations of integrated corporations. In general, some methods of optimal allocation of resources are used to analyze this kind of problem, for example,Input-Output(I/O) model with mathematical programming. One of the major problems for the methods is that the distributions of the profits among the main bodies who are engaged in the economic activities are not paid much attention to, then the results of the analyses are lack of forecasting and operational qualities. The bargaining process, as a suitable way to be used, satisfies a set of rational bargaining axioms, so the bargaining result may be accepted by all of the DMs.

A visual interactive decision analysis system has been carried out in the light of bargaining game solution for allocation of resources(Qu 1989,1990) for the sake of offering a tool to users. The model of the system is based on n I/O models with bargaining axiomatic characterization and high-productivity for n DMs. The main ideas and implementations of model generation and solution interaction are discussed in combination with the system.

2. A Model of Bargaining Game for Allocation of Resources

The bargaining game, first proposed by Nash(1950) and developed by many scholars(see Roth and Malouf 1979; Roth 1979; Binmore and Dasgupta 1987), can be applied to analyze the resource allocation problem, but due to the oversimplification, the framework of the theory is limited for solving this problem. The theoretic framework of bargaining game is: let β_n denote the set of all n-person bargaining games, and $(E,d)\in\beta_n$ be a n-person bargaining game, where $E\subset\mathbb{R}^n$ is an attainable utility set of n DMs, any point on the set can be arrived so long as agreement is reached by DMs, assume that the E is a closed bounded convex set, there is a point $d\in E$ which is called "status quo point", and at least $u\in E$ exists, $u>d$. A bargaining solution is a map η: $\beta_n\longrightarrow\mathbb{R}^n$, such that $\eta(E,d)\in E$ for every $(E,d)\in\beta_n$ if all DMs reach an agreement. To select a unique feasible outcome η as the rational solution the $\eta(E,d)=(\eta_1(E,d),\cdots,\eta_n(E,d))$ must satisfies a set of bargaining axioms. Obviously it is difficult that the framework is used to solve such complicated resource allocation problem, the difficulties are how to determine the relationship between decision variables and E, and how to make E convex.

In order to overcome the shortcomings, we raised up a bargaining game for allocation of resources(Qu 1989, 1990) which extends the framework of bargaining game from a simple convex set of attainable utility to a multivariable description by which the features of the resource allocation problem are described directly no matter whether the attainable set is convex or not, it makes the theoretic framework of the bargaining game powerful to analyze the practical problems of resource allocations.

As an application of the bargaining game for allocation of resources(Qu 1989, 1990), we consider n production processes for n DMs, each DM has his own products for which some resources are needed. For any DMj, $j=1,2,\cdots,n$, let $x_j\in\mathbb{R}^{mj}$ be a set of decision variables (products or output values) of DMj. There are two classes of resources needed by the production of x_j. The first is sj kinds of resources which are independent on other DM's productions, we call the sj resources *non-share* resources. The second is r kinds of resources which are needed by all n productions for n DMs and called *share* resources. The problem is how to allocate the limited share resources in certain constraints among n DMs. To consider a model of mathematical programming, consisting of multiple I/O models, as follows(MLP):

$$\max \quad q_i(F_i(x_i), d_i, g_i) \tag{1}$$
$$\text{s.t.} \quad A_j x_j^T \leq b_j \tag{2}$$
$$B_j x_j^T - y_j \leq 0 \tag{3}$$
$$x_j \geq 0 \tag{4}$$
$$y_j^L < y_j \leq y_j^U \tag{5}$$
$$\textstyle\sum_{j=1}^{n} y_j \leq y \tag{6}$$
$$q_i(F_i(x_i), d_i, g_i) - q_j(F_j(x_j), d_j, g_j) = 0 \tag{7}$$

For all $j \in N$, for any $i \in N$, $i \neq j$

We call model (MLP) Bargaining Game for Allocation of Resources (BGAR). Where $b_j \in \mathbb{R}^{sj}$ is a total amount vector of sj non-share resources possessed by DMj; $A_j = [a_{ik}^j]_{sj \times mj}$ is a sj×mj dimensional constant matrix, a_{ik}^j is the consumption coefficient of i^{th} non-share resource produced k^{th} unit product for DMj, i=1,2,···,sj; k=1,2, ···,mj. The equation (2) represents that the production of DMj is restricted by his sj non-share resources. $y_j \in \mathbb{R}^r$ is a variable vector of r share resources obtained by DMj; $B_j = [b_{ik}^j]_{r \times mj}$ is a r×mj dimensional constant matrix, b_{ik}^j is the consumption coefficient of i^{th} share resource produced k^{th} unit product for DMj, i=1,2,···,r; k=1,2,···,mj. The equation (3) gives the constraint of share resources for the production. The upper bound y_j^U and lower bound y_j^L of the y_j are constant vectors, which respectively are known as expected ideal point and necessary lowest point for the share resources, they can be determined according to the practical situations. The equation (5) draws the line at allocating the share resources, which is consistent with practical situations. The constraint (6) means that the allocations of r share resources among n DMs must not exceed the total amount $y \in \mathbb{R}^r$ that the n DMs have. The constraint (7) is the definition equation for the solution of bargaining game for allocation of resources(Qu 1989,1990). $F_i(x_i) = C_i x_i^T$ can be a single objective function or a compromise objective function integrated by addition of multiple linear objectives with weights. The other mathematical notations in the model are defined as $q_j(F_j(x_j),d_j, g_j)=(F_j(x_j)-d_j)/(g_j-d_j)$, $d_j=\max_{x_j\in\Omega_j^L} F_j(x_j)$, $g_j=\max_{x_j\in\Omega_j^U} F_j(x_j)$, $\Omega_j^L=\{x_j\in\mathbb{R}^{mj} \mid A_j x_j^T\le b_j, B_j x_j^T-y_j^L\le 0, x_j\ge 0\}$, and $\Omega_j^U=\{x_j\in\mathbb{R}^{mj} \mid A_j x_j^T\le b_j, B_j x_j^T-y_j^U \le 0, x_j\ge 0\}$, $j=1,2,\cdots,n$.

Let's define that $\Omega=\{x=(x_1,\cdots,x_n) \mid x_j\in\mathbb{R}^{mj}, A_j x_j^T\le b_j, B_j x_j^T-y_j\le 0, x_j\ge 0, y_j^L<y_j\le y_j^U, \sum_{j=1}^n y_j\le y, q_i(F_i(x_i),d_i,g_i)-q_j(F_j(x_j),d_j,g_j)=0$, for some $i\in\{1,\cdots,n\}$, $j=\{1,\cdots,n\}$ $\}$, $d=(d_1,\cdots,d_n)$, $g=(g_1,\cdots,g_n)$ and the solution of (MLP) is $\eta(E,d,g)$. A set of bargaining axioms to be used are:

AXIOM 1(strong individual rationality): $\eta(\Omega,d,g)>d$.

AXIOM 2(weak Pareto optimality): $\eta(\Omega,d,g)\in\Omega_w$. $\Omega_w\in\Omega$ is the weak Pareto optimal set on Ω.

AXIOM 3(invariance under linear utility transformations): $T[\eta(\Omega,d,g)] = \eta(T[\Omega],T[d],T[g])$. T is a linear utility transformation: $\forall i\in\{1,\cdots,n\}$, $a_i\in\mathbb{R}_{++}$, $b_i\in\mathbb{R}$, $T[y]=(a_i y_i+b_i)_{i\in\{1,\cdots,n\}}$, $y\in\mathbb{R}^n$, $y=(y_1, y_2,\cdots,y_n)$.

AXIOM 4(symmetry): If (Ω,d,g) is symmetric, that is, $\forall i,j\in\{1,\cdots,n\}$, $d_i=d_j$, $g_i=g_j$, π is a permutation on $\{1,\cdots,n\}$, π^* denotes the corresponding transformation on $\mathbb{R}^n$, for all $v\in\Omega$, $v^*=\pi^* v$, then $v^*\in\Omega$, therefore, $\forall i,j\in\{1,\cdots,n\}$, $\eta_i(\Omega,d,g)=\eta_j(\Omega,d,g)$.

AXIOM 5(independence of irrelevant alternatives other than ideal point):There are two bargaining game (Ω,d,g) and (Ω',d,g), if $\Omega'\subset\Omega$, and $\eta(\Omega,d,g)\in\Omega'$, then $\eta(\Omega',d,g)=\eta(\Omega,d,g)$.

In the light of bargaining game for allocation of resources(Qu 1989, 1990), we have following theorem:

THEOREM There exists a unique solution for (MLP) which satisfies axiom 1-5.

In the other hand, the (MLP) model is built up with the rule of high-productivity system of Zeleny(1982). This means that it is a rational solution in the sence of not only share resource allocation among n DMs, but also inner allocation of share resources after obtaining his share resources for each DM.

3. Framework of BGRA System and Model Generation

In applications of mathematical models, users in general understand practical problems but not mathematical models, it is necessary for us to build up a tool as a bridge between the models and the users. In addition, the processes of building a model and obtaining a satisfactory solution for a practical problem will be a man-machine interactive course also. The BGAR system provides following features: a BGAR model will be automatically generated so long as problem description with natural language is given by users through man-machine interface of computer; an interactive course can be done if users want to modify their definition of a problem; both graphics and languages are used in the interface for supporting the whole courses of problem definition and solution, the users can insert their experience and preference to the solution process.

The kernel module of the BGAR system is model generation. In general the models of the problems consist of entities, relations and qualities. In our system, the entities include the sets of DMs, decision variables, share and non-share resources and so on, their structures are arranged according to specific functions in a real problem. There are specific relations among the entities, for example, in BGAR system, the number of the products(decision variables) and the structure of the resource allocation are determined by DMs, and the relations of the products and the resources exist objectively. For the sake of describing the relations among the entities, it is necessary for us to describe and define the qualities of the entities, such as aspiration levels, expected objectives and preferences of DMs, restraint of products and resources in environment. Therefore, frameworks of structures and relations for entities can be formed.

At first every user gives the description of the problem with natural language through computer interface seperately. All the operations above-mentioned are guided by the menu selections. After finishing the users' descriptions, a set of algorithms and control variables of model structures are used to fulfil the transformation between descriptions of natural language and symbolic language, and generate the final model. The transformation process of natural language $\longrightarrow$ symbolic language $\longrightarrow$ model generation will automatically be carried out by computer after module of natural language has been built up.

The structure of the BGAR model is determined so long as the control parameters are produced. In other words, any modification of the problem description from users will cause the changes of the control parameters and the model structure from computer. For instance,a BGAR problem description with natural language, including names of DMs, decision variables, share and non-share resources etc., has been put into the model base through the interface, so the model structure is also determined. If an user, such as DMj, wants to add a decision variable to the model, he can input the name of the new decision variable with the interface, the node of DMj in the network of the model base for entities will be found and a leaf relative to the node will be produced automatically, consequently the control parameters and the model structure are changed also. Similarly the operation process of data is just like that. A case study for industry development of a county has been carried out by use of the system, but for the limitation of the paper, the contents for the case study are omitted.

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