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A TIME SEQUENCES NONLINEAR MODEL AND ALGORITHMS OF SOCIAL DILEMMA

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Abstract

In social dilemma a short-term or individual need is pitted against its long-term or group need. Many scientists study social dilemma and model of social dilemma.So far,the time sequence model of social dilemma is linear. In this paper, a time sequence nonlinear model is proposed to describe and analysis a typical social dilemma. A recurrence formula is also given for time sequence dilemma for any time interval. The reinforcement mechanism of the 'social traps' has been described quantitatively. Two algorithms of social dilemma are proposed to describe the general situation of social dilemma. Second algorithm provides some possible pathes and methods for breaking out or extricating of trap. Some more discussions and significant results are given.

KEY WORDS: TIME SEQUENCE, NONLINEAR MODEL, SOCIAL DILEMMAS

1.Introduction

In the article of Social Traps(Platt,1973), Platt described the social dilemma with Skinnerian theory of reinforcement. According to his theory, social dilemma could be divided into three categories. One of these is one-person trap,i.e. short-term goods will result to long term bads.This kind of dilemma with nonlinear model is expressed in this paper, including extricable trap and drugs trap are also analyzed. Two algorithms for a time sequence dilemma are given.

2.Nonlinear model of a time sequence dilemma

Time sequence social dilemma is that each optimal choice will result to long-term bads. For example, the smoking cigarettes or taking drugs,the short-term benefit can result to long-term bad results. One begins to smoke cigarettes for curiosity or need of sociality. Gradually, changes taking in his body and this addiction is reinforced again and again, finally a serious damage is found in his body . So time sequence dilemma have two properties :(1). there is a strategy which yielded person the best profit in each step.(2).choosing the strategy in all steps results in deficient outcome.

The nonlinear model of time sequence social dilemma is established as follows:

One must choose two strategies C and D in every step of time sequence. D expresses behavior such as defecting, smoking, taking drugs and so on. On the contrary C expresses cooperating, no-smoking or giving up smoking, no-taking drugs or giving up drugs and so on. Ui expresses the choice at the ith step. Ui=1 means choosing D strategy, and Ui=0 choosing C strategy.

Assuming that the personal profit function at ith step is Jk,

$$Jk= \begin{cases} \alpha/(Pk+Lc) & Uk=0 \quad (C) \\ \alpha/(Pk+Ld) & Uk=1 \quad (D) \end{cases} \tag{1}$$

where Lc, Ld>0, Lc-Ld>1, $\alpha > Lc^2$, $Pk=\sum_{i=0}^{k} Ui$, i.e. Pk is the number of steps chosen D.

Baby is no-smoking and no-taking drugs. At the beginning, one does not want to take drugs. So first step choice is cooperating, i.e. U0=0, and P0=0. therefore, the initial value of the profit function $J0=\alpha/Lc$ at the cooperating profit curve.

Now let us take the ith step to see what will happen.

Case 1. Chosen cooperating C at last step, i.e. $Uk_{-1}=0$ (see Fig.1 at point A) If C is chosen, i.e. Uk=0, the value of personal profit function will not be changed; if D is chosen, i.e. Uk=1, we obtain the increment of $\alpha/(Pk+Ld) - \alpha/(Pk_{-1}+Lc)$ of the value of personal profit function. (at point B)
So, when $Uk_{-1}=0$, we have

$$Jk= \begin{cases} Jk_{-1} & Uk=0 \quad (C) \\ Jk_{-1} + \alpha/(Pk+Ld) - \alpha/(Pk_{-1}+Lc) & Uk=1 \quad (D) \end{cases} \tag{2}$$

or

$$Jk= Jk_{-1} + [\alpha/(Pk+Ld) - \alpha/(Pk_{-1}+Lc)]\, Uk \tag{3}$$

Case 2. Chosen defecting D at last step, i.e. $Uk_{-1}=1$ (see Fig.1 at point E) If C is chosen, i.e. Uk=0, the value of personal profit function will be decreased by $\alpha/(Pk_{-1}+Ld) - \alpha/(Pk_{-1}+Lc)$ (at point A). If D is chosen, i.e. Uk=1, the value of personal profit function will be decreased by $\alpha/(Pk_{-1}+Ld) - \alpha/(Pk+Ld)$ (at point B).
So, when Uk = 1, we have

$$Jk= \begin{cases} Jk_{-1} - \alpha/(Pk_{-1}+Ld) + \alpha/(Pk_{-1}+Lc) & Uk=0 \quad (C) \\ Jk_{-1} + \alpha/(Pk+Ld) - \alpha/(Pk_{-1}+Ld) & Uk=1 \quad (D) \end{cases} \tag{4}$$

or

$$Jk= Jk_{-1} + [\alpha/(Pk_{-1}+Lc) - \alpha/(Pk_{-1}+Ld)](1-Uk) + [\alpha/(Pk+Ld) - \alpha/(Pk_{-1}+Ld)]Uk \tag{5}$$

Combining case 1 with case 2, we get the recursive form as follows:

$$Jk= Jk_{-1} + [\alpha/(Pk+Ld) - \alpha/(Pk_{-1}+Lc)]Uk + [\alpha/(Pk_{-1}+Lc) - \alpha/(Pk_{-1}+Ld)]Uk_{-1} \tag{6}$$

where $k>=1$.
therefore

$$Jk|_{U_k=0} = Jk_{-1} + [\ \alpha/(Pk_{-1}+Lc) - \alpha/(Pk_{-1}+Ld)\ ]Uk_{-1} \qquad (7)$$

$$Jk|_{U_k=1} = Jk_{-1} + [\ \alpha/(Pk+Ld) - \alpha/(Pk_{-1}+Lc)\ ] + [\ \alpha/(Pk_{-1}+Lc) - /(Pk_{-1}+Ld)\ ]Uk_{-1} \qquad (8)$$

Definition 1. Define Platt's reinforcement as follows:

$$Jk|_{U_k=1} - Jk|_{U_k=0} = \alpha/(Pk+Ld) - \alpha/(Pk_{-1}+Lc) \qquad (9)$$

When $\alpha/(Pk+Ld) > \alpha/(Pk_{-1}+Lc)$, choosing D is better than choosing C, but personal profit function is decreased as the number of D increases. Choosing D is always better than choosing C so long as $Lc-Ld>1$.

Definition 2. When $Lc-Ld>1$, form (1) or (6) is said to be one-person trap.

Of course, we can think of "one-person" as one-group.

Property 1. In one-person trap, the result of choosing strategy D is better than choosing strategy C at every single step.

Property 2. In one-person trap, the value of one-person profit function decrease as the number of choosing strategy D increases. Gradually, the curve D become easy curve.

Property 3. In one-person trap, the more the number of choosing strategy D, the less the reduction value of one-person profit function. (i.e. the less

$$\alpha/(Ld+\sum_{i=1}^{k} Ui) - \alpha/(Ld+\sum_{i=1}^{k+1} Ui)).$$

The value of one-person profit finally tend to be zero.

We can explain these properties as follows:
In one-person trap, we have $Lc - Ld>1$. One is effected by the reinforcement of $[\ \alpha/(Pk+Ld) - \alpha/(Pk+Lc)\ ]$ at every step choosing, forced him to choose strategy D and get into trap. The more the number of choosing strategy D, the less the reduction value of one-person profit function. 1001 boxes of cigarettes do damage to body as though 1000 boxes of cigarettes do. The addiction is reinforced, finally a serious damage is found in his body.

The point $(\sqrt{\alpha}-Ld, \sqrt{\alpha})$ of curve D is the vertex of the hyperbola D. When $0<Pk<\sqrt{\alpha}-Ld$, the curve of one-person profit function makes a fast descent. When $\sqrt{\alpha}-Ld<Pk$, the curve makes a slow descent. Gradually, one be unconsciously addicted to drugs. We have asummed $\alpha>Lc^2$, So we have

Definition 3. We call the point $(\sqrt{\alpha}-Ld, \sqrt{\alpha})$ at the curve D as a point of being addicted.

The more large Ld, the more easy to be addicted; and the little α,the more easy to be addicted.But $\sqrt{\alpha}$ >Ld,i.e. Ld/$\sqrt{\alpha}$ <1. there are different and Ld for different drugs.It is easy to get property 4.

Property 4. The more large the Ld/$\sqrt{\alpha}$ and approach 1, the more easy one to be addicted.

Definition 4. We call Ld/$\sqrt{\alpha}$ as addiction index.

Above mentioned two curves C and D do not intersect. We can also obtain two curves intersecting. (see Fig.2 and Fig.3)

$$Jk=\begin{cases} \alpha c/(Pk+Lc)+\beta c & Uk=0 \quad (C) \quad \alpha c, Lc, \beta c>0 \\ \alpha d/(Pk+Ld) & Uk=1 \quad (D) \quad \alpha d, Ld>0 \end{cases} \tag{10}$$

and

$$Jk=\begin{cases} \alpha c/(Pk+Lc) & Uk=0 \quad (C) \quad \alpha c, Lc>0 \\ \alpha d/(Pk+Ld)+\beta d & Uk=1 \quad (D) \quad \alpha d, Ld, \beta d>0 \end{cases} \tag{11}$$

The expression (10) forms the extricable trap,the expression (11) forms the drugs trap.(Wu,Lu and Zheng, 1990)

The reinforcement tendes to be βd as the number of choosing strategy D increases.

3. Discuss algorithms

Now we study the general case. Assume that fc and fd are cooperation profit function and defecting profit function, respectively. They are nonincreasing function (linear or nonlinear).

We have

$$Jk=\begin{cases} fc(Pk) & Uk=0 \\ fd(Pk) & Uk=1 \end{cases} \tag{12}$$

$$Pk=\sum_{i=0}^{k} Ui$$

Algorithm 1.

Step 1: k=0; U0=0; P0=0; J0=fc(0);
Step 2: k=k+1;
Step 3: If C is chosen then Uk=0 else Uk=1;
Step 4: Pk=Pk+Uk;
Step 5: If Uk=0 then Jk=fc(Pk) else Jk=fd(Pk);
Step 6: return to step 2.

Of course, algorithm 1 is died cycle.We can limit the number of k to stop.If chosen cooperating C at last step , we choose cooperating C again at this step, the value of personal profit does not increase. In the following algorithm 2 we make the value of personal profit increase.If one chooses cooperating C again , he will be rewarded with increasing personal profit.

Algorithm 2.

Step 1: k=0; U0=0; P0=0; J0=fc(0);
Step 2: k=k+1;
Step 3: If D is chosen then Uk=1 and go to step 5 else go to step 4;
Step 4: (a) If chosen cooperating C at last step and $P_{k-1} = \eta$ than Uk=0; (η is defined afterwards)
(b) If chosen cooperating C at last step and $P_{k-1} \neq \eta$ then Uk=-1;
If P_{k-1} =0 then Uk=0;
(c) If chosen defecting D then Uk=0;
Step 5: Pk=Pk+Uk;
Step 6: If Uk=1 then Jk=fd(Pk) else Jk=fc(Pk);
Step 7: return to step 2.

In the step 4(b) of algorithm 2 we set that if $Pk \neq \eta$ then Uk=-1 in the case of continued cooperating, there are two different subcases : $Pk < \eta$, and $Pk > \eta \neq 0$.

For illustration (see Fig.1), one consider the fishing strategy. Assume that at the begining one choose cooperating C, U0=0 , and obtain α/Lc of fish. It is not an overfish, so remains of fish produce future generations. If one chooses coopeating C again at next step,From step 4(b) of algorithm 2, he has U1=0,and from step 5, get P1=P0+U1=0 , one obtain α/Lc of fish. If one choose defecting, U1=1, one obtains more fish than one chooses cooperating . Because choosing strategy D is better,one chooses D again ,again ,again. Pk will reach η and after that Pk will greater than η .At that time one chooses cooperating C for some reason,Pk moves to left because of step 4(b) of algorithm 2 . If one chooses cooperation again, again. Pk will reach η again, but Pk cannot move because of step 4(a). This means that fish resources are destroyed. Amount of fish cannot recover original value α/Lc.

So long as $Pk < \eta$, amount of fish can recover original value α/Lc.Thus η is defined as recovery threshold.

4. Conclusion

In this paper, we describe time sequence social dilemma with nonlinear model and analyzed several situation . A recurrence formula is given for time sequence dilemma for any time interval. Two aigorithms of social dilemma are proposed to describe the general situation . The algorithm 2 provides some possible pathes and methods for breaking out or extricating of trap. A time sequence nonlinear model might be much better , but it is a matter under study.

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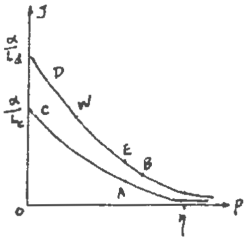

Fig. 1

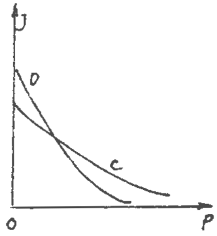

Fig. 2

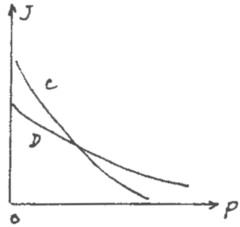

Fig. 3