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# SUPPORT SYSTEMS FOR DECISION AND NEGOTIATION PROCESSES

*Preprints of the IFAC/IFORS/IIASA/TIMS Workshop*

*Warsaw, Poland*

*June 24-26, 1992*

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**VOLUME 1:**

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A THEORY OF SUCCESS  
WITH APPLICATION TO DECISION SUPPORT

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1. Introduction

The human life can be viewed as a sequence of actions (which can be grouped into activities) taking place in time and space. For each activity a specific time interval, out of the given (limited) individual time resources, is chosen. Each activity requires as inputs (in addition to time) such resources as capital, energy, land, water etc. The outcome or result of each activity, called also output, can take the form of a material good (in case of productive activities), services (education, recreation etc.), or a nonmaterial good (e.g. authority, friendship, political power etc.).

In the model of activities studied here the individual takes into account the prices of inputs and outputs and evaluates each activity according to a given goal or utility function, concerned with output and input costs, or profit. He is called efficient when he is able to derive (and implement) for each activity and the given input time the best endowments of input resources .

Efficiency is regarded here as a measure of decision maker's performance. In many cases the good performance requires, in addition, that decision maker be able to allocate (optimally) the resources, in the presence of competition. As an example consider the travelling salesman who is visiting several markets in order to sell goods priced  $p$ . Due to competition from other salesmen he is facing the problem of access to markets and it is reasonable to assume that only a part of his goods will be sold at price  $p$ . That

part depends first of all on the demand (market capacity) and on the number of suppliers (supplied goods). The competition may also increase the salesman's input costs (travel, accommodation, marketing etc.) and it results in a decrease of utility. Due to competition the access induced conflicts may also develop. It is therefore important to take into account the utilities of all the agents who compete at the markets and find the time allocation strategies which are Pareto optimal. The efficient decision makers who are, in addition, able to follow (and implement) such strategies can be called effective. Effectiveness is understood here as an ultimate standard against which the individual performance can be evaluated. Effectiveness is also a prerequisite to achieve maximum of utility, which is regarded as success. The theory of success, described in the present paper, is concerned mainly with the strategies of allocation of time, capital and the like resources in order to achieve success.

The individual activity model can be extended to deal with organization (which is a voluntary collective of individuals with a chosen leader, who is making decisions in the name of the collective).

The models presented employ some concepts of G.S.Becker [1], and M.D.Intrilligator [2,3] and are based on a number of assumptions concerned with existence of utilities. It is shown for each model that a unique effective strategy exists and can be explicitly derived. The basic principles of success, i.e. achieving maximum of utility, are also formulated. Since the theory of success is both descriptive and normative, it can be used to derive the strategies for decision support. Different aspects of that theory were developed and already applied to a number of practical problems in Refs.[4+8].

## 2. Models of Individual Activities

Consider an individual who performs an activity using as inputs: time interval  $T$  (which is a given part of planning horizon) and other resources (capital, land, energy, water etc.) denoted by  $x_k$ ,  $k=1...K$ . The output  $X$  can be written in the form of a production function:

$$X = F(T, x_1, \dots, x_K),$$

where  $F$  is increasing, differentiable, strictly concave and homogeneous (constant return to scale) so one can write the monetary value of output:  $Y = pX$ , in the form

$$Y = pTf(u_1, \dots, u_K), \quad u_k = x_k/T, \quad \forall k, \quad (1)$$

where  $u_k$  are called factor endowments,  $p$  - output price.

The property of "constant return" makes it impossible to generate output by simple change of units of measurements (e.g. by changing 1\$ to 100 cents).

The monetary value of the cost of activity

$$C = T \sum_{k=0}^K \omega_k u_k, \quad u_0 = 1, \quad (2)$$

$\omega_k$  - prices of factors  $x_k$ ,  $\forall k$ .

One of the objectives of a decision maker is to find  $u_k \hat{=} \hat{u}_k$ ,  $\forall k$ ; such that  $Y$  is maximum, subject to input cost constraint, or - alternatively - maximum profit  $\Pi = Y - C$ .

Since  $\Pi$  is strictly concave a unique optimum vector  $u \hat{=} \hat{u}$  exists and it can be derived by solving the eqs.  $\Pi'_{u_k}(u) = 0$ ,  $\forall k$ . Then

the output (1) can be written

$$Y = pbT,$$

where  $b = f(u_1, \dots, u_K)$  can be called optimum productivity of time.

Consider, as an example, the Cobb-Douglas production function

$$Y = pAT^\alpha K^{1-\alpha}, \quad 0 < \alpha < 1,$$

$K$  - capital,  $A$ ,  $\alpha$  - given positive coefficients

and the cost constraint

$$\omega_T T + \omega_K K \leq C,$$

$\omega_T$ ,  $\omega_K$  labour and capital unitary costs.

It is possible to show that (see e.g. [8])

$$u_1 = \frac{\hat{K}}{\hat{T}} = \frac{\alpha}{1-\alpha} \frac{\omega_T}{\omega_K}, \quad b = A \left( \frac{\alpha}{1-\alpha} \frac{\omega_T}{\omega_K} \right)^\alpha.$$

Consider now a decision maker who is concerned with  $m$  alternative activities  $A_j$ ,  $j = 1, \dots, m$ , each described by the function

$$Y_j = p_j b_j x_j^{\alpha_j}, \quad \forall j, \quad x_j - \text{time of activity } A_j,$$

in such a way that

$$\sum_{j=1}^m x_j = T.$$

Among  $A_j$  one does not take into account the routine activities, which do not require decision making.

Assuming the individual's choice probabilities  $a_j = x_j/T, \forall j$ ,  $\sum_{j=1}^m a_j = 1$ ; to be known one can derive the expected outcomes

$$\bar{V}_j = p_j b_j a_j T = \bar{B}_j T, \quad \forall j \quad (3)$$

where  $\bar{B}_j$  can be called the motivation index.

Each probability  $a_j$  represents the internal attractiveness of  $A_j$  for the individual and, generally, it depends on a number  $L$  of criteria or values. Among these values one can list: nourishment, lodging, health, recreation, security, authority, religious and political values etc. Following the model of probabilistic individual choice, introduced by Intrilligator (see Ref. [2,3]), one can write

$$a_j = \sum_{l=1}^L w_l \alpha_{lj}$$

where  $w_l$  - given weights, attached to criteria  $l$ ,  $w_l > 0, \sum_l w_l = 1$ ;

$\alpha_{lj}$  - probability that individual will choose  $A_j$  taking into account solely criterion  $l$ .

In the present model of alternative activities the decision maker finds out that the accomplishment of an outcome is equivalent to gambling at a lottery. The lottery can be imagined as a circle with unit circumference subdivided into arcs of lengths  $a_1 \dots a_m$  and a fair pointer which spins around. When it comes to a stop in the arc of length  $a_j$ , the prize  $\bar{V}_j = p_j b_j T$  is the outcome. Using the lottery model to study the risky actions one is using it only once and the outcome indicates a simple alternative chosen. Repeating the process of gambling one arrives at an allocation of total time  $T$  among the  $m$  alternatives, such that

$$T_j = a_j T, \quad \forall j.$$

Following Von Neumann's and Morgenstern's axiomatic theory of utility (see e.g. [9]), one can write the expected utility of the lottery

$$U(a_1 \bar{V}_1, a_2 \bar{V}_2, \dots, a_m \bar{V}_m) = \sum_{j=1}^m a_j U(\bar{V}_j).$$

Thus, whenever the assumptions of that theory hold, there exists a utility function, preserving order ( $\bar{v}_1 \geq \bar{v}_2 \geq \dots \geq \bar{v}_m$ ) and satisfying the expectation principle: The utility of a lottery equals expected utility of its outcome.

In the present paper it is assumed that utility depends on two factors:

- a. Expected financial outcome  $\bar{v}_j$
- b. Financial award (e.g. savings or profit)

$$s_j = \omega x_j, \quad v_j \quad (6)$$

where

$$\omega = \frac{1}{T} \sum_j (\bar{v}_j - c_j) = \sum_{j=1}^m (\bar{B}_j - \sum_{k=1}^K \omega_k u_{kj}), \quad (7)$$

$u_{kj}$  - factor endowment for the activity  $A_j, v_j$ ;

is the average expected profit per unit of time.

According to the experimental evidence it is assumed that for an alternative  $A_j$  the utility function  $\phi[s_j, \bar{v}_j]$  increases in both monetary variables  $s_j, \bar{v}_j$  and is strictly concave and homogeneous (called also "risk averse").

In other words the utility can be written in the form

$$U = \sum_{j=1}^m \phi[s_j, \bar{v}_j] = \sum_{j=1}^m \bar{v}_j \phi\left(\frac{s_j}{\bar{v}_j}\right) = \sum_{j=1}^m \bar{B}_j T \phi\left(\frac{\omega x_j}{\bar{B}_j T}\right), \quad (8)$$

where  $\phi$  is strictly concave and increasing ( $\phi'(\cdot) > 0$ ).

The problem which faces the efficient decision maker can be formulated as follows: find the vector  $x = (x_1, \dots, x_m) \in \hat{\Omega}$ , which maximizes  $U(x)$ ; i.e.

$$U(\hat{\Omega}) = \max_{x \in \hat{\Omega}} U(x)$$

where

$$\hat{\Omega} = \{ x \mid \sum_j x_j \leq T, x_j \geq 0, v_j \}.$$

Since  $\phi$  is strictly concave and, according to assumptions, there is no stationary point of  $U(x)$  within  $\hat{\Omega}$  it must belong to the border line of  $\hat{\Omega}$ , or - in other words - the constraint  $\sum_j x_j \leq T$  is active while  $x_j \geq 0, v_j$ , are not active (one can show this by checking Kuhn-Tucker conditions). Then the first order optimality conditions become:

$$U'_{x_j} = \varphi' \left( \frac{\omega x_j}{\bar{B}_j T} \right) \omega = \lambda = \text{const}, \quad \forall j$$

$$\sum_j x_j = T.$$

It can be easily shown (see Ref.[7]) that the optimum strategy becomes

$$\hat{x}_j = \frac{\bar{B}_j}{\bar{B}} T, \quad \bar{B} = \sum_{j=1}^m \bar{B}_j, \quad \forall j \quad (9)$$

and

$$U(\hat{x}) = \bar{B} T \varphi(\omega/\bar{B}) = \Phi(\omega T, \bar{B} T) \quad (10)$$

When  $b_j p_j = \text{const}, \forall j$ , the optimum strategy (9) becomes  $\hat{x}_j = a_j T, \forall j$ , i.e. it takes into account the "internal" factors only. In the general case (9)  $\hat{x}_j$  is modified to take into account, as well, the "external" factors  $b_j p_j, \forall j$ . When  $a_j = \text{const}, \forall j$ , the strategy is motivated by "external" factors only.

Observe also that  $U$  is expressed in monetary units, linear in  $T$  and strictly concave (increasing) in award  $\omega T$  and output  $\bar{B} T$ . It is possible to show that for linear  $\Phi$  all the time resources  $T$  should be allotted to  $A_{j_0}$  with the largest performance index  $B_{j_0} = \max B_j$ .

An advantage of the theory is that the resulting utility (10) is an increasing function of aggregated motivation index  $\bar{B}$  and award rate  $\omega$  only. However, the substitution of these two factors is possible. For example, the young, ambitious entrepreneurs may care less for award than for the outcome of activity (because it gives them prestige and an image of success), while the older businessmen may care for award mostly.

For a fixed level  $U = \bar{U}$  and  $\Phi$  given explicitly one can derive  $\omega \triangleq g(B)$ , which is a solution of  $\bar{U} = \Phi(\omega, B) T$ . For example, in the case of Cobb-Douglas utility function

$$U = c \omega^\beta B^{1-\beta} T, \quad 0 < \beta < 1, \quad c = \text{const},$$

one gets

$$\omega = \left\{ \frac{\bar{U}}{c T} B^{\beta-1} \right\}^{1/\beta}.$$

The marginal rate of substitution  $s = \frac{\partial U}{\partial \omega} : \frac{\partial U}{\partial B} = \frac{1-\beta}{\beta} \omega$ , where  $\omega = \omega/B$  is the award rate.

The elasticity of substitution,  $E_{S^u} = \frac{ds}{s} : \frac{du}{u}$ , for Cobb-Douglas function is equal unity. It corresponds to the situation when one substitutes outcome by award proportionally to already achieved  $u$ . Such property of utility function may be regarded as too restrictive. As an alternative one can assume, as a model of utility, the constant elasticity of substitution function, where  $E_{S^u} = \nu + 1$ ,  $\nu \in [-1, 0]$ ,  $s = \frac{1-\nu}{\nu} \vartheta^{\nu+1}$ ,  $\vartheta \in [0, 1]$ , and  $\nu, \vartheta$  - given parameters.

To find the numerical value of optimum  $u = \hat{u}$ , one can reason as follows. It is necessary to attach certain weights, say  $w_1, w_2$ , to the award and output respectively. These weights depend on the age, ambition, accepted standard of living, postponed consumption (savings) and external parameters (taxes) etc. Then the problem is to maximize  $\Phi(\omega T, BT)$  subject to  $\omega T w_1 + B T w_2 = C_0$ , where  $C_0$  - a constant.

The necessary and sufficient (due to strict concavity of  $\Phi$ ) conditions of optimality become:

$$\Psi'_\omega(\cdot) + \lambda w_1 T = \Psi'_B(\cdot) + \lambda w_2 T = 0$$

where  $\Psi = \Phi + \lambda C_0$ ,  $\lambda$  - Lagrange multiplier.

These conditions can be used to derive the optimum award rate, which for Cobb-Douglas function becomes:

$$\hat{u} = \frac{\beta}{1-\beta} \frac{w_2}{w_1}$$

When  $\hat{u}$  (which can be called the "equitable reward rate") is known and fixed one can reduce the general utility maximization problem to the simple discrete optimization problem:

$$\max_{x \in \Omega} \sum_{j=1}^M \alpha_j(x) \beta_j, \quad \alpha_j(x) = \frac{a_j x_j}{\sum_{k=1}^M a_k x_k}, \quad \beta_j = b_j p_j, \quad \forall j, \quad x_j = 0 \text{ or } 1 \quad (11)$$

subject to the constraint  $\omega/B \geq \hat{u}$ ,

where

$$\omega = \sum_{j=1}^M \alpha_j(x) [\beta_j - c_j], \quad B = \sum_{j=1}^M \alpha_j(x) \beta_j,$$

$c_j$  = cost of  $j$ -th activity per unit of time

The admissible set  $\Omega$  becomes

$$\Omega = \left\{ x : \sum_{j=1}^M \alpha_j(x) [(1 - \hat{u}) \beta_j - c_j] \geq 0, \quad x \in [0, 1]^M \right\}$$



In that problem  $M$  is the number of potentially possible activities, out of which a number  $m \leq M$  of preferable activities is chosen (i.e. derived). In the simpler situation, when  $M$  is small and constraint is not active, one can try to maximize  $B$  by accepting activities with large  $\alpha_j \beta_j$  value and rejecting those with small  $\alpha_j \beta_j$ ,  $\forall j$ . Such a process can be also called "matching" (of the intrinsic preferences vector  $\alpha \triangleq \{\alpha_1, \dots, \alpha_m\}$  to the vector of extrinsic opportunities  $\beta \triangleq \{\beta_1, \dots, \beta_m\}$ ).

In other words a success minded entrepreneur should scan the field of potential activities in order to find the most preferable (maximizing utility) set of  $m$  activities subject to the constraint that his reward rate is not less  $\hat{\alpha}$ .

Assume therefore, that  $n$  decision makers ( $D_i, i=1, \dots, n$ ) (e.g. salesmen) having  $T_i$  time resources,  $\forall i$ , compete at the same markets ( $A_j, j=1, \dots, m$ ) so the expected outcomes become

$$\bar{v}_j = \bar{B}_j Q_j \frac{x_{ij}}{\sum_{\nu=1}^n x_{\nu j}}, \quad \forall j$$

where  $Q_j$  - given market shares,  $x_{ij}$  - part of  $T_i$  resource allocated by  $D_i$  to  $A_j$ ,  $\forall i, j$ .

It is convenient to write  $Q_j = q_j Q$ , where  $Q$  is the total demand,  $\sum_j q_j = 1$ ,  $q_j > 0$ ,  $\forall j$  (it is assumed that  $Q$  is less the total demand  $\sum_{\nu} T_{\nu}$ ).

Then the utilities of  $D_i$  (taking into account (8)) become

$$\bar{U}_i(x_i) = \sum_{j=1}^m \frac{B_j Q x_{ij}}{\sum_{\nu} x_{\nu j}} \varphi_i \left( \frac{\omega \sum_{\nu} x_{\nu j}}{B_j Q} \right), \quad B_j = q_j \bar{B}_j, \quad \forall i, j. \quad (12)$$

Since

$$\begin{aligned} \frac{d\bar{U}_i}{dx_{ij}} &= B_j Q \left[ \frac{\sum_{\nu=1}^m x_{\nu j}}{\left( \sum_{\nu} x_{\nu j} \right)^2} \varphi_i \left( \frac{\omega \sum_{\nu} x_{\nu j}}{B_j Q} \right) + \frac{x_{ij}}{\sum_{\nu} x_{\nu j}} \varphi_i' \left( \frac{\omega \sum_{\nu} x_{\nu j}}{B_j Q} \right) \frac{\omega}{B_j Q} \right] = \\ &= \text{const}, \quad \forall j \end{aligned}$$

$$\text{for } \hat{\alpha}_{ij} = \frac{B_j}{B} T_i, \quad B = \sum_j B_j, \quad \forall i, j \quad (13)$$

it is possible to prove that the strategy (13) is unique and Pareto optimal, while (Ref.[7]):

$$\bar{U}_i(\hat{x}_i) = BQ \frac{T_i}{\sum_p T_p} \varphi_i \left( \frac{\omega \sum_p T_p}{BQ} \right), \quad \forall i \quad (14)$$

It should be observed that for growing  $Q_j$  the access probability  $Q_j / \sum_p T_p \rightarrow 1$  and the expected outcomes  $\bar{Y}_j(\hat{x}_i) \rightarrow \bar{Y}_j(\hat{x}_i)$ ,  $\forall j$  while  $\bar{U}_i(\hat{x}_i) \rightarrow U_i(\hat{x}_i)$ ,  $\forall i$ , so the model with competition can be regarded as an extension of (8) + (10) model.

On the other hand, introducing given access probabilities  $\bar{q}_j, \forall j$ , one can express alternatively (instead of (3)) the expected outcomes

$$\bar{Y}_j = \bar{q}_j \bar{B}_j T_i \triangleq \bar{B}_j T_i, \quad \forall j,$$

and derive  $x_{ij} \triangleq \bar{x}_{ij}$ ,  $\forall i, j$ , maximizing (8) by (9), i.e.

$$\bar{x}_{ij} = \frac{\bar{B}_j}{\bar{B}} T_i, \quad \forall i, j, \quad \bar{B} = \sum_j \bar{B}_j, \quad (15)$$

That strategy is equivalent to (13) when  $\bar{q}_j = q_j Q / \sum_p T_p$ ,  $\forall j$

and  $\bar{U}_i = U_i(\bar{x}_i) = \bar{U}_i(\hat{x}_i) = \bar{U}_i$ ,  $\forall i$ .

It should be noted that in the process of proving (13), (14) it is essential that there is no stationary point within  $\Omega_i$ ,  $\forall i$  (i.e.  $\text{grad } U_i(x_i) > 0$ ). If e.g.  $\varphi$  is losing concavity (the individual is risk fond) unstable strategies (see Ref. [6]) or bifurcations follow.

According to the definition given in paper [7] a risk averse decision maker, who maximizes utility  $U_i$  (by allocating time and other resources for each activity  $\lambda_j$ ), is called efficient. When, in addition, he is able to cope with the access problem (competition) using  $\bar{U}_i$  and Pareto optimality approach (13) he is called effective.

As already mentioned, in order to be effective one has to know the access situation, i.e. demand to supply ratio  $Q_j / \sum_p T_p$ . Otherwise he has to use the efficient strategy (with access probability  $\bar{q}_j$ ), but a loss of utility ( $U_i < \bar{U}_i$ ) may follow. If e.g. one makes decision to stay at a hotel, based on  $\bar{q}_j$  rather than inquiry and reservation, he may find the hotel occupied and a loss of utility follows. The loss is here the price paid for imperfect

information. Generally, to achieve effectiveness the efficient decision makers should engage in an exchange of information and negotiations, if necessary, which may end up in a market sharing process.

The main problem of the theory of success is to plan a strategy of success, i.e. an allocation of resources among activities in such a way that maximum of expected utility follows.

The main results of that theory can be summarized in the form of the following basic principles:

Basic principles of success:

1. Choose the best award rate  $\hat{a}$ .
2. Choose the best (i.e. maximally motivated) subset of activities, by matching  $a, b, p, q$  vectors and observing award rate  $\hat{a}$ .
3. Assign resources to activities in proportion to motivation indices  $B_j, v_j$ .
4. When in doubt consult the computerized success support system (triple S) which solves the problem (11).

In agreement with these principles young people are trying to match their inborn abilities, tastes or preferences ( $a$ ), (and acquire skills ( $b$ ) by education) to the expected job opportunities ( $p$ ), while the older are looking for jobs matching their skills and tastes.

3. Models of Organizations

The theory of effective activities can be extended to deal (besides individuals) as well - with organizations.

By an organization one understands here a voluntary collective of individuals with a chosen leader who is making decisions in the name of collective. The leader is also organizing (directing and checking implementation of decisions) and awarding individuals (out of organization income). In the model studied here  $n$  organizations are given with  $N_i, i=1, \dots, n$  individuals each.

The utility of the individuals are of the form (12) i.e.

$$U_{i1} = \sum_{j=1}^m C_{j1} \frac{x_{i1j}}{\sum_v x_{v1j}} \varphi_{i1} \left( \frac{\omega \sum_v x_{v1j}}{C_{j1}} \right), C_{j1} = a_{j1} b_{j1} p_j q, \quad (16)$$

$$\forall j, l, i \quad l=1, \dots, N_i$$

Each individual has an admissible set of activities

$$\Omega_{il} = \left\{ x_{ilj} \mid \sum_j x_{ilj} \leq T_i, x_{ilj} \geq 0, \forall i, l, j \right\}, \quad (17)$$

It is assumed that the leader (as an individual) possesses the utility of the form (16) but instead of personal he uses the collective aggregated resources and is motivated by the aggregated preferences. Constructing the model of organization, in such a way, one avoids a difficult problem of assigning utilities to organization.

The leader's aggregated time resources  $T_i$  and preferences  $C_j$  are assumed to be

$$T_i = N_i T_i, \quad \forall i \quad (18)$$

$$C_j = \frac{1}{N_i} \sum_l \frac{C_{jl}}{C_1}, \quad C_1 = \sum_j C_{jl}, \quad \forall j, l \quad (19)$$

It should be mentioned that (18), (19) require a democratic form of management (the leader should be equally sensitive to individual preferences  $C_{jl}/C_1$ ,  $\forall j, l$ , within organization).

When  $b_j p_j q_j = \text{const} \forall j$  one gets by (19):

$$\bar{a}_j = \frac{1}{N_i} \sum_{l=1}^{N_i} a_{jl}, \quad \forall j \quad (20)$$

As shown in Ref. [7] the values of  $\bar{a}_j$ , derived by averaging individual preferences ( $a_{jl}$ ), can be regarded (under assumptions of: 1. existence, 2. unanimity for a loser, i.e. when all individuals reject an alternative so does society, and 3. strict and equal sensitivity to individual probabilities) as the social choice.

Extending the notion of efficiency, by assuming

$$x_{ij} = \sum_{l=1}^{N_i} x_{ilj}, \quad \forall i, j,$$

and of effectiveness (for leaders of organizations) one can prove (see Ref. [7]) that the unique sets of effective strategies for leaders

$$\hat{x}_{ij} = \frac{C_j}{C} T_i, \quad C = \sum_j C_j, \quad \forall i, j \quad (21)$$

and employees

$$\hat{x}_{ilj} = \frac{C_{jl}}{C_1} T_i, \quad C_1 = \sum_j C_{jl}, \quad \forall i, l, j \quad (22)$$

exist, and

$$u_{i1}(\hat{x}_{i1}) = c_1 \frac{T_i}{\sum_v T_v} \varphi_{i1} \left( \frac{\omega \sum_v T_v}{c_1} \right) \quad v_{i,1} \quad (23)$$

$$u_i(\hat{x}_i) = c \frac{T_i}{\sum_v T_v} \varphi_i \left( \frac{\omega \sum_v T_v}{c} \right), \quad v_i \quad (24)$$

It should be observed that organizations' (i.e. leaders') preference structure  $\bar{a} \hat{=}$   $(\bar{a}_1, \dots, \bar{a}_m)$  is not fixed and can be changed by proper employment and reorganization policy. By employing people with proper preferences, or aspiration vector  $(a_1)$ , and skills  $(b_1)$ , one can get  $\bar{a}$ ,  $\bar{b}$  - vectors which match well the market opportunities i.e.  $(p, q)$  vectors. For that reason a mathematician may work effectively within large business organization, which employs mostly economists and managers being, at the same time, ineffective in making individual business.

#### 4. Extensions and applications

It should be observed that though some concepts underlying the theory of success were already formulated by household economists, noticeably G. Becker [1], or social choice economists e.g. M.D. Intrilligator [2,3], this theory is not just another branch of economic sciences.

The economic formulation of outcome, costs and profit, based on prices as units of measurement is not indispensable.

There are situations where prices as measure of output are generally unknown, as e.g. in science and education. It is, however, possible to describe output of a university in terms of paper published or students graduated. The award of a teacher is the number of students who have passed the examination with good marks and which contribute to the teacher's reputation (the teacher's salary is usually correlated with the reputation). In order to compare the output with the cost of teaching (and derive award) one can take into account the time foregone for preparation of lectures, administrative work etc. and convert this into the number of students foregone.

When the teacher is specializing in several subjects (such as

mathematics, physics, chemistry etc.) with preferences  $a_1, \dots, a_m$  and his skills are  $b_1, \dots, b_m$  respectively, he can derive the optimum strategy to allocate his time  $T$  to teaching activities by (9), assuming motivation indices  $B_j = a_j b_j$ ,  $\forall j$ . When, in addition, the competition from other teachers, characterized by  $q_j$ ,  $\forall j$ , is taking place, his strategy should be corrected by using  $B_j = a_j b_j q_j$ ,  $\forall j$  indices.

Obviously, it is possible to give more examples of applications of the theory of success for individuals as well as organizations, e.g. political parties trying to win election, research institutes and consulting firms trying to win contracts, firms dealing with services, including the so called "public goods" etc. (see Ref. [4+8]).

So far, in the models studied, the time-resource owner was allocating time among a number of given alternatives, while the rest of resources, e.g. capital, was borrowed or rented. In similar way the capital owner (e.g. the banker) can start with allocation of his capital and find the necessary labour and the rest of resources. Such a model was described in Ref. [7].

When time and capital owners try to allocate their resources in the form of  $m$  given joint ventures their strategies may, generally, differ and a negotiation process is required to arrive at the common consensus strategy. Such a situation may happen, for example, when a number of research organizations is competing in order to get government support (contracts). As shown in Ref. [8] the effective strategies and a consensus strategy, which is effective for both sides, exists and can be explicitly derived.

It should be also noted that the success theory presented enables one to understand better the process of decision making when alternative activities are planned as well as enables him to avoid possible pitfalls or fallacies. First of all it is necessary to notice that success is defined in the paper in the individual, i.e. subjective, form (as maximum of expected utility attainable for different strategies out of a set of alternative activities). Some care must be here taken to avoid a possible fallacy. The popular belief that a universal strategy exists among the set of possible strategies, ensuring success, is a fallacy. In other words,

the fallacy is the assumption that an optimum strategy for one person is also optimum for another person. According to the success theory two persons with, generally, different utility functions will possess also different optimum award rates  $\hat{\phi}_i, i=1,2$  and  $a_j$  coefficients. These persons, when confronted with large set of alternative activities, will choose different subsets i.e. such combinations of activities, which make  $\omega_i/B_i$  closer to  $\hat{\phi}_i$  and maximize  $B_i$ . One can not maintain, however, that when both persons achieve their optimum strategies they would be equally happy (it is due to the fact that their utility functions are, generally, different).

Another fallacy is that a concrete person possesses an universal constant success strategy. As already demonstrated the optimum award rate  $\hat{\phi}$  may change with individual's age, social status, taxes, business opportunities and the optimum strategy changes as well. That property explains the known fact that some people become unhappy though seemingly nothing has changed in their everyday activities. Since one deals here with expected utilities the successfulness is not an inherent virtue of an individual but rather his statistical advantage.

Another fallacy is expressing opinions on other individual's success judging by one's own standards.

The theory of success can be also used to explain behaviour of organizations, when they are confronted with sudden changes of market opportunities, tax changes, competition, as well as the change of organization leadership.

Besides being descriptive the theory is also normative. It enables construction of a decision support system. The system operates by exchanging information with decision maker by:

- a) asking for information regarding  $a_j, \forall j$  and  $\hat{\phi}$  coefficients
- b) analyzing market opportunities and finding optimum combination of coefficients
- c) suggesting to the decision maker the best strategies for allocation of resources.

It should be noted that the support system does not need information regarding decision maker's utility function and consequently it does not bother him with utility identification problem.

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