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SUPPORT SYSTEMS FOR DECISION AND NEGOTIATION PROCESSES

Preprints of the IFAC/IFORS/IIASA/TIMS Workshop

Warsaw, Poland

June 24-26, 1992

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VOLUME 1:

Names of first authors: **A-K**

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Preprints, IFAC/IFORS/IIASA/TIMS Workshop, June 24-26, 1992, Warsaw, Poland

DECISION SUPPORT SYSTEMS BASED ON RELATIVE LOGICAL INFERENCE

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Abstract: There is presented a concept of using the principles of relative logic in computer aided decision making systems. A version of relative logic /based on a general concept of topological logic given by C.G. Hempel in 1937/ has been proposed by the author in 1986. It accepts fully the classical definition of logical implication and, thus, makes easier using the relative logic as a basis of logical inference in decision making. A decision supporting system should contain also a mechanism of detection and elimination of inconsistency between the statements as it has been mentioned in the paper.

Keywords: artificial intelligence, decision making, logical inference, relative logic, topological logic.

1. Introduction

A decision supporting system /DSS/ is a system recommending to the users decisions adequate to the given circumstances and satisfying some preassumed quality criteria. In modern DSS a justification of the proposed decisions is also required. Computer aided decision making is one of the most important problems of artificial intelligence. However, most of the DSSs, expert systems etc. used in practice are based on the inference rules of classical logic. In particular, the modus ponens syllogism:

assumption: if A then B
premise: A

conclusion: B

where A and B are some statements, is widely used. However, in many applications /say, in management, medicine, human relations etc./ a less rigid "natural" rule of inference is used:

assumption: usually, if A then B

premise: it seems that A

conclusion: it looks that B.

Therefore, in designing a user friendly DSS we have to choose between two extremes: of classical inference rules, exact but rigid and thus not quite adequate to real situations, and the other ones, more natural but formally incontrollable. In the last decades many attempts have been made to find a satisfactory compromise between the above-mentioned two extremes. The concepts of using to this purpose multi-valued, modal and/or probabilistic logics, fuzzy sets, rough sets etc. belong to this area of investigations. Most of the concepts are based on the assumption that a scale of logical values, of probabilities, of belonging to a set etc. is given. This leads to the problem of evaluation of the corresponding parameters or weights, which in many cases has not a satisfactory solution. In relative /topological/ logic the statements are not logically evaluated but with respect to some other statements. The aim of this paper is to show how this general concept can be used in a computer aided DSS.

2. Inference rules in relative logic

We shall start with defining an elementary statement as a syntactically correct asserting phrase containing no logical operator. If there are given some elementary statements then using standard logical operators of negation /$\neg$/, disjunction /$\vee$/ and conjunction /$\wedge$/ we can define statements according to the following rules:

1° each elementary statement A is a statement,

2° each negation $\neg A$ of a statement A is a statement,

3° each disjunction $A \vee B$ and each conjunction $A \wedge B$ of statements A, B is a statement.

The above-given rules can be easily used to generate statements as any finite logical combinations of elementary statements.

Let us denote by T a primary set of elementary statements. One of possible ways to generate T is defining a non-empty set X of parameters, taking into account a predicate g/x/, $x \in X$, and putting

$$T = \{g/x/: x \in X\}. \qquad /1/$$

However, logical values will be assigned to the statements in the way that will be described below.

Let us remark that any finite logical combination of elementary statements can be presented in a reduced form if the associative and commutative properties of logical operators as well as the reducing rules: $A \vee A \equiv A$, $A \wedge A \equiv A$, are used. For any given T we shall denote by T^* a set of all statements that can be defined as reduced finite logical combinations of the elementary statements of T.

We shall define on T^* the following bi-argumental relations:

i. a similarity /reflexive, symmetrical and transitive/ relation ev /read: "is logically equivalued to"/,

ii. a strong semi-ordering /irreflexive, antisymmetrical and transitive/ relation lv /read: "is logically less valuable than"/ such that for any $A, B \in T^*$ there is:

$$/A \vee B/ \text{ ev } \begin{cases} A \text{ for } B \text{ lv } A, \\ A \text{ /or } B/ \text{ for } A \text{ ev } B, \\ B \text{ for } A \text{ lv } B \end{cases} \qquad /2/$$

and

$$/A \wedge B/ \text{ ev } \begin{cases} A \text{ for } A \text{ lv } B, \\ A \text{ /or } B/ \text{ for } A \text{ ev } B, \\ B \text{ for } B \text{ lv } A. \end{cases} \qquad /3/$$

Taking a set-algebraic sum of the above-defined relations we obtain a new relation

$$\text{lev} = \text{ev} \cup \text{lv} \qquad /4/$$

reflexive, asymmetrical and transitive. Therefore, lev is a semi-ordering relation described in T^* ; it can be read as "is logically equi- or less valuable then".

Therefore, instead of assigning exact logical values to the statements, taken from a binary /"truth" or "false"/ or a multi-valued logical scale, we have introduced a relation of logical semiordering into T^*. It becomes evident that a disjunction of all elementary statements of T and of their negations has a maximum logical value while a conjunction of those statements has a minimum logical value in T^*.

If T^* is finite or countable the relation lv in T^* can be described by a following construction.

Each statement A from T^* when introduced to a knowledge-base of a DSS will be represented there in the form of an ordered pair

$$D = [A, q] \qquad /5/$$

where $q \in Q$ and Q is a finite set of symbols called logical weights /it is important that logical weights are not numerical/. Then we shall construct a contourless directed graph

$$G = [Q, L, h] \qquad /6/$$

whose nodes are the logical weights, L is a set of arcs and h is a relation that assignes an arc $l_{ij} \in L$ to an ordered pair of nodes $[q_i, q_j]$ if and only if the corresponding statements $A_i, A_j \in T^*$ satisfy the relation A_i lv A_j. Let us remark that in general the assignment of logical weights to the statements is not reversive and a fixed weight q may correspond to a subset of statements being a similarity class in the sense of the relation ev.

The relationships of the form A_i lv A_j can be established in two possible ways: 1^o some of them can be deduced from the basic properties of the relation lv and from basic logical formulae, like /2/ or /3/, 2^o the other ones are indicated as the result of primary logical evaluation of some pairs of statements. Any such evaluation generates, as a consequence, a series of the relationships of the first type, deduced from the logical rules.

Each statement A /describing a fact or a rule/ being to be stored in a knowledge base of the DSS, first of all, should be logically evaluated with respect to its negation; there is no reason in storing A if A lv $\neg$A /in such case $\neg$A should be stored/.

Let us assume that a statement

$A \Rightarrow B$ /"if A then B"/

where A, B are some other statements is stored in the knowledge base. A logical weight q_0 has been assigned to this statement. Then we observe that A and we assign a logical weight q_A to this fact. It arises the question: what logical weight q_B shoul be assigned to the conclusion B?

According to the classical definition of implication we have $/A \Rightarrow B/ \equiv B \vee \neg A$ and as a consequence in relative logic

$$/A \Rightarrow B/ \text{ ev } \begin{cases} B \text{ for } \neg A \text{ lv } B, \\ B \text{ /or } \neg A/ \text{ for } \neg A \text{ ev } B, \\ \neg A \text{ for } B \text{ lv } \neg A. \end{cases} \qquad /7/$$

A geometrical illustration of this formula has been given by Kulikowski /1986/. It follows from it that a relative logical evaluation is not always possible. However, an analysis of /7/ leads to the conclusion that

$$B \text{ ev } /A \Rightarrow B/ \text{ for } \begin{cases} A \text{ lv } \; A \text{ lv } /A \Rightarrow B/, \\ /A \Rightarrow B/ \text{ lv } \neg /A \Rightarrow B/ \text{ lv } A \\ \neg /A \Rightarrow B/ \text{ lv } /A \Rightarrow B/ \text{ and } \neg A \text{ lv } B. \end{cases} \qquad /8/$$

Therefore, in the last case /the only one accepted in the knowledge base/ we obtain:

$$q_B \text{ ev } q_0 \text{ independently on } q_A \qquad /9/$$

which is a basis of logical inference. It follows from it that in a series of implications: $A \Rightarrow B$, $B \Rightarrow C$, ... , $D \Rightarrow E$ in which A and all implications are logically more valuable then their negations the last conclusion E is logically equivalued to the last implication.

3. References

Hempel, C.G. /1937/. A purely topological form of non-Aristotelian logic. Journ. Symb. Logic, vol. 2, no. 3.

Kulikowski, J.L. /1986/. Decision making in a modified version of topological logic. Prace IBS PAN, no.134, Warsaw, 24-44.