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SUPPORT SYSTEMS FOR DECISION AND NEGOTIATION PROCESSES

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MULTICRITERIA DECISION MAKING IN FIRE EGRESS ANALYSIS

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Abstract: This paper presents a multicriteria dynamic approach to fire egress analysis. An algorithm for generating all nondominated egress paths for human occupants in a fire building is given. The approach uses the most recent theoretical developments in time dependent vector dynamic programming. An interesting application related to a fire in a North Carolina food plant is shown.

Keywords: vector dynamic programming, networks, egress models, multicriteria decision making.

1. Introduction

In this research the movements of human occupants of a building who are reacting to the presence of a fire in the building are considered. Such movement is generally known as egress and thus the present study is concerned with fire egress analysis. Our work seems to be the first to apply multi-objective programming models to this problem. The motivation to introduce more than one objective function to be minimized is quite natural in the fire environment. A building fire contains multiple hazards to be avoided by simultaneously minimizing travel time, distance of travel, amount of smoke inhaled, amounts of toxins encountered, etc. It is the goal of fire egress analysis to find all nondominated paths for each such occupant and then to make further study of the paths and their corresponding cost vectors. By obtaining such a detailed solution one obtains a deeper understanding of the building structure, the occupants and how they might best manage the risk of a fire.

A building fire and the reaction of the occupants to the fire form an inherently dynamic phenomenon. As such, the data to be considered in decision making is comprised of functions of time which may be derived either from actual fire measurements or from the output of mathematical models of fire and smoke dynamics. At the Building and Fire Research Laboratory of the National Institute of Standards and Technology in Gaithersburg, Maryland, research on the physics of fire and the mathematical modeling of fires in buildings is well established. Recently the Fire Hazards

Analysis group has published a computer package called HAZARD I, (Bukowski et al., 1989), which is comprised of two modules. First, a differential equations model of the spread of flames, chemicals and smoke resulting from a user defined fire in a building is solved by numerical integration. Next, a network is superimposed on the floor plan and the occupants behavior is simulated by a set of heuristic decision rules. The occupants move around in the network until they find an egress path by which they leave the building. The output data from the first module is available as input for the simulation of people-fire interaction.

The research of this paper is designed to provide additional insight into the people-fire interaction within a building. Theoretical foundation for this research work is given by vector dynamic programming (VDP). Hartley (1985), Corley and Moon (1985), and others considered vector routing problems in networks with constant vector costs on links and presented dynamic programming (DP) based algorithms for generating the entire set of nondominated (Pareto-optimal) paths in the network. Cooke and Halsey (1966) were the first who introduced a dynamic network assuming that travel times on links were general functions of time. They developed a DP based algorithm for finding the set of paths from every node to the destination node with shortest travel time.

Different applications have given rise to conducting research on path planning problems in dynamic networks. Orda and Rom (1990), motivated by related problems in computer communication networks, focused on the shortest path problem in the dynamic network with restricted or unrestricted departure time, and developed algorithms for finding an optimal path between two single nodes. Kaufman and Smith (1990) were interested in transporatation planning in congested road networks and specified conditions under which one can efficiently find the set of optimal paths from the origin node to every other node in the network. Kostreva, Wiecek, and Getachew (1991) applied several network models to fire egress analysis for residential buildings. Kostreva and Wiecek (1991) seem to be the first to introduce a network with vector time dependent costs and developed DP based algorithms for finding the set of nondominated paths. In this paper, as a continuation of the research effort reported above, this new approach will be applied to performing multicriteria decision making in fire egress analysis.

2. Time Dependent Vector Dynamic Programming

In the dynamic programming literature two problem formulations are considered: find all nondominated paths 1) from each node to the destination node or 2) from the origin node to every other node. The former formulation is based on the backward approach to DP while the latter uses the forward approach. An algorithm based on the forward technique will be presented in this paper and its applicability to fire egress analysis will be demonstrated.

Consider a general network, not assumed to be acyclic, that consists of N nodes $\{1, 2, \dots, N\}$ and a set of links (i, j) connecting the nodes. Associated with each link is a vector cost $[c_{ij}(t)]$, where the cost functions (c_{ij}: R+ R^{m+}) are assumed to be positive vector valued functions of time. Let $[c_{ij}(t)]_1$ be the time to travel from node i to node j, given that travel starts at time t. The cost to traverse a path p between two nodes of the network is defined as $[c(p)] = \sum_{(i,j)\in p} [c_{ij}(t)]$. It is assumed that the cost of traveling along a link is a function only of the arrival time at the starting node at the link. This assumption, referred to as the frozen link model, allows for fixing a link cost not at the time of leaving the origin node, but later, at the time of arriving at the starting node of the link and, thus, for updating the link cost according to very recent data about the fire.

Let time t be a continuous variable, that is $t \geq 0$, and let the functions $[c_{ij}(t)]_1$ take any positive value. Let node 1 be the origin node. Assume that the departure time from the origin node is $t = 0$. Finally an assumption is introduced which allows the formulation of the principle of optimality for dynamic multiple objective networks. The assumption seems to be very realistic for any fire egress scenario: part a) states that evacuees may not pass each other during evacuation, and part b) refers to deterioration of evacuation conditions over time.

Assumption For any link (i, j) in the network and all $t_1, t_2 \geq 0$, if $t_1 \leq t_2$, then

a) $t_1 + [c_{ij}(t_1)]_1 \leq t_2 + [c_{ij}(t_2)]_1$, and

b) $[c_{ij}(t_1)]_r \leq [c_{ij}(t_2)]_r$ for all $r \in \{2, \dots, m\}$.

Theorem : Principle of Optimality for Dynamic Multiple Objective Networks

Under Assumption a) and b), a nondominated path p, that leaves the origin node at time $t = 0$ and arrives at node j at time t_j, has the property that for each node i lying on this path, a subpath p_1, that leaves the origin node at time $t = 0$ and arrives at node i at time t_i, $t_i \leq t_j$, is nondominated.

By Bellman's principle of optimality and the Theorem, we establish that for $t^\ell > 0$ and $t^n > 0$:

$$\{[G_j^\ell(t^\ell)],\ \ell = 1,\dots,N_j\} = \text{VMIN}\ \{\ [G_i^n(t^n)] + [c_{ij}(t^n)],\ n = 1,\dots,N_i\ \}, \qquad j=2,3,\dots, N,$$

$$\{[G_j^\ell(t^\ell)],\ \ell = 1\} = \{\ 0\ \},$$

where $[G_i^n(t^n)]$ is the vector cost of nondominated path n leaving the origin node at time $t = 0$ and arriving at node j at time t^n. Operation VMIN computes vector costs of nondominated paths in the set whose each element is a vector sum of the vector cost of the nondominated path n leaving the origin node at time 0 and arriving at node i at time t^n, and the cost vector of link (i, j) with the arrival time t^n at node i. For details about the computation see Kostreva and Wiecek (1991).

3. Application

The following is an illustrative example of the type of multi-objective analysis which the new theoretical dynamic programming development permits.

On Tuesday September 3, 1991 the worst industrial accident in the history of North Carolina occurred when a fire started in the Imperial Food Products plant in Hamlet, North Carolina, a small town 75 miles southeast of Charlotte. Twenty-five workers were killed in the fire and over 45 others were injured. Greenville (South Carolina) News reports stated that the plant had several blocked exits and lacked a formal evacuation plan. One of the exit doors was blocked by a truck which was subsequently moved forward by the driver to allow the escape of several employees. A floor plan of the plant was published in the Greenville News on September 5 and is depicted in Figure 1. The description given in the newspaper allowed us to construct the following example.

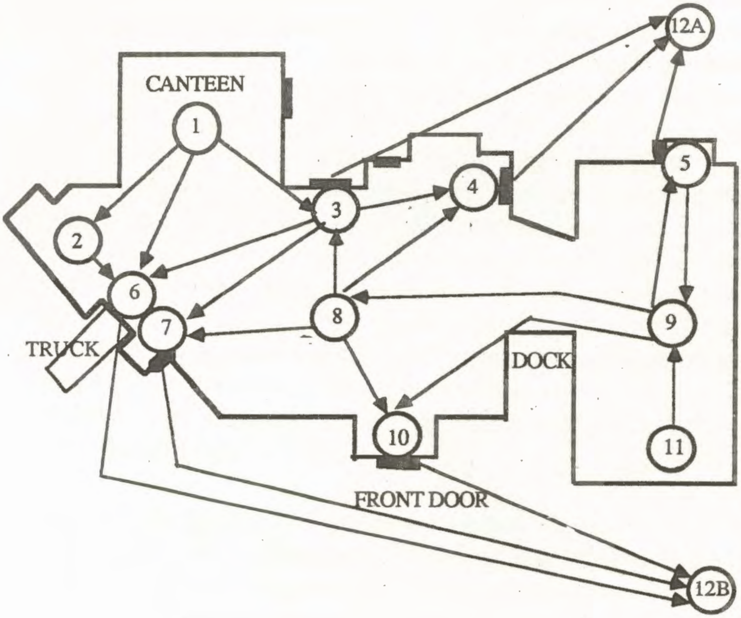

Figure 1. The plant floor plan and related network.

Consider that a fire occurred at the location of node 8 (in the center of the building) and the fire spread rapidly toward node 3. At time $t = 3$, the associated rooms and passageways become blocked by smoke. Two objectives are considered, travel time and distance of travel. Nodes 12A and 12B represent the same outside location. Assume constant vector costs on the following links: $c_{12} = (2, 2)$, $c_{16} = (3, 2)$, $c_{26} = (1, 1)$ $c_{34} = (1, 1)$, $c_{36} = (3, 2)$, $c_{37} = (2, 2)$, $c_{3,12A} = (2, 2)$, $c_{4,12A} = (2, 2)$, $c_{59} = (1, 1)$, $c_{5,12A} = (1, 1)$, $c_{67} = (1, 1)$, $c_{6,12B} = (2, 2)$, $c_{7,12B} = (2, 2)$, $c_{87} = (1, 1)$, $c_{8,10} = (1, 1)$, $c_{95} = (1, 1)$, $c_{98} = (3, 3)$, $c_{9,10} = (3, 3)$, $c_{10,12B} = (2, 2)$, $c_{11,9} = (1, 1)$, and monotone increasing step functions on the links leading to node 3:

$$c_{13} = \begin{cases} (2, 2), & t < 3 \\ (10, 15), & t \geq 3 \end{cases} \quad \text{and} \quad c_{83} = \begin{cases} (1, 1), & t < 3 \\ (10, 15), & t \geq 3 . \end{cases}$$

Applying the forward approach for every node i in the network, $i = 1, \ldots, 12$, we can find the set of all nondominated paths leaving that node at time $t = 0$ and arriving at node j, $j \neq i$, that is, we find all nondominated paths from node i, $i = 1, \ldots, 11$, to the destination node 12. Nondominated evacuation paths from nodes 1 and 9 are of special interest due to these nodes' locations. There are two nondominated paths from node 1 to node 12, namely path $\{(1,3), (3,12A)\}$ that has the total cost of (4, 5), and path $\{(1,6), (6,12B)\}$ with the total cost of (5, 4). Path $\{(9,5), (5,12A)\}$ is the only nondominated path leading from node 9 to node 12 with the total cost of (2, 2).

From these calculations we do not wish to conclude anything in particular about the North Carolina fire. It is simply of interest to know about nondominated evacuation paths in order to prepare the occupants of a building for any emergency which might arise.

4. Conclusions

In this short paper we have introduced a dynamic programming application which has two unusual and important characteristics: multiple criteria on each link of the related network and time dependent cost functions. A principle of optimality is given which allows the computation of the set of all nondominated paths from an origin node to any other node in the network. An illustrative example related to fire egress analysis has been included in which some costs are time dependent discontinuous functions. For this example, all nondominated paths are computed leading from two of the nodes(rooms of a building) to the destination node(outside of the building). The technique makes an assumption which is realistic and quite natural for fire egress analysis and thus it is of general applicability for these problems.

5. Acknowledgement

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