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SUPPORT SYSTEMS FOR DECISION AND NEGOTIATION PROCESSES

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APPLICATION OF MULTICRITERIA AND MULTIPERSON DECISION MAKING TO THE PLANNING OF THE DEVELOPMENT OF RURAL AREAS.

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Abstract: The purpose of this paper is to study a special class of problems wherein systems are not driven by classical transfer functions or state equations but by an implicit relation between the state, the input and the output of the system. This relation is based upon a certain freedom left to subsystems to react according to their own objectives. We describe a decision supporting tool dealing with hierarchical multicriteria and multiperson problems. We use this original approach for a problem of planning of the development of rural areas.

Keywords: Hierarchical, multicriteria, multiperson problems; bi-level programming; regional development modeling.

1. Introduction

The planning of rural area development is characterized by the existence of many actors and many objectives. The actors may be farmers, banks and public authorities having objectives such as survival, profit and environmental protection (Albegov et al. (1982)). In this paper, we consider two main decision-makers: farmers and public authorities. These decision-makers are located at different hierarchical levels and each one has its specific objectives. The purpose of this paper is to discuss a decision support tool which could assist the public authorities, who are decision-makers at the upper level, and the farmers, who are the lower level decision-makers, in reaching an acceptable planning scheme for rural development. This approach differs from those previously presented (Bard (1983), Nijkamp and Rietveld (1981)) by allowing the user to deal with multiple upper decision-maker objectives and multiple lower decision-makers. It exploits the characteristics of real problems allowing one to transform, via the use of a particular data structure, the original implicit problem into one which can be solved using, with slight adaptations, well-known multiobjective optimization methods.

2. Problem Definition

The formulation that will be used is based on the general bi-level programming model :

$$\begin{aligned} &\text{Max}_y\ F(x_1^*,\dots,x_n^*,y) \\ &\text{s.t. :}\quad G(x_1^*,\dots,x_n^*,y) \le 0 \\ &\qquad x_i^* = \ \text{argmax}\ f_i(x_i,y) \qquad i=1,\dots,n \\ &\qquad \text{s.t. :}\ \ g_i(x_1,\dots,x_n,y) \le 0 \end{aligned}$$

where y is the decision variable vector of the upper decision-maker (the leader);
x^*_i is the decision variable vector of the i^{th} lower decision-maker (follower);
F and f_i are multiobjective function vectors;
G and g_i are resp. upper and lower problems constraint function vectors;
n is the number of followers.

Although omitted by numerous authors (see for ex. Bard and Falk (1982), Kornai and Liptak (1965)), the distinction between the constraints g_i and G is relevant and required in numerous models of real-life problems (Savard (1988)).

In this paper, we make the following restrictive hypotheses :

(H1) All constraints and objectives are linear.

(H2) The upper level variables (y) appear only in the lower problems constraints as linear right-hand-side terms.

(H3) The objectives of each follower may be aggregated into a single objective (existence of a utility function).

(H4) If the lower problem (P_i) admits more than one optimal solution, the solution chosen is that which maximizes the leader's objectives.

(H5) For each i= 1,..., n, the constraints (g_i) depends only on the decision variable x_i.

Hypotheses H1 and H2 are necessary for linearity. Hypothesis H3 is made because at this stage of investigation no pratically reliable ways exist to find the followers reactions through an interactive multiobjective procedure (Keeney and Raiffa (1976)). Hypothesis H4 guarantees that the optimal solution is well defined. These restrictions lead to the following model :

(P) $\quad \text{Min } \sum C_i x_i^* + C_0 y$

$\quad$ s.t. : $\quad \sum A_i x_i^* + A_0 y \le b_0$

(P_i) $\quad x_i^* = \text{ argmin } c_i x_i \quad i=1,\dots, n$

$\quad$ s.t. : $\quad B_i x_i \le b_i(y)$

where y is the leader's decision variable (the incentive);
x_i^* is the i^{th} follower's decision variable (the reaction);
C_i, $i=1\dots n$, and C_0 are matrices (leader's multiple objectives);
A_i, $i=1\dots n$, A_0 are matrices (leader's constraints);
c_i, $i=1\dots n$, are vectors (followers objectives);
B_i, $i=1\dots n$, are matrices (followers constraints);
b_0 and $b_i(y)$, $i=1\dots n$, are column vectors (right-hand-side terms);
n is the number of followers.

Hence in this problem, n followers react to an incentive (y) according to their own objectives (c_i). This incentive influences only the right-hand-side terms of the follower's constraints. The leader has to choose the incentive vector which optimize his own objectives, while taking into account the follower's reactions and his own constraints. The n followers react independently but they share limited resources and the repartitioning of these resources is under the leader's control.

3. Proposed Solution Method

3.1 Basic principles

For a fixed value of the incentive (y_0), we can solve the linear problems (P_i) for i=1,..., n and find an optimal basis for each of them. If we change the value of y so that the current optimal bases remain unchanged, the value of the optimal solution x* will depend linearly on y. Similarly the contribution of this optimal solution to the leader's objectives ($\sum C_i x_i^*$) is linear in y. Thus, it is possible to get a linear explicit expression of the leader's objectives in terms of the incentive. This expression remains valid around y_0, as long as the current optimal bases are unchanged. If we partition the set of feasible incentives into regions where the optimal bases are constant, then in each of these regions we can compute a linear expression of the leader's objectives in terms of the incentive. Assembling these expressions, we obtain an explicit piecewise linear expression of the leader's objective which is valid on the whole set of feasible incentives. Hence we can rewrite problem (P) in the following way :

(P*) $\quad \text{Min } \sum C_i x_i^*(y) + C_0 y$

$\quad \text{s.t. : } \sum A_i x_i^*(y) + A_0 y \le b_0$

Hence this procedure allows us to replace the implicit problem (P) by an equivalent explicit problem (P*). We can then solve this explicit problem using an adapted multiobjective optimization procedure. As the algorithm is based upon a regions generation phase followed by an optimization phase, these two phases are strongly related and can be combined. Specifically :

- The data structure generated in the first phase must be adapted to its use in the second phase. As described below, the operations executed in the second phase are row manipulations, dual optimization, pivoting, etc... Hence the suitable data structure that will be used for this kind of operations is similar to the one used in the simplex formalism.
- A preprocessing of the data structure in view of the optimization phase should be performed during the generation phase. This preprocessing will eliminate *a priori* suboptimal or non-feasible regions, according to the leader's objectives and constraints.
- During the generation phase we can identify critical data that may be useful during the optimization phase (ideal point, nadir point, etc...).

3.2. Algorithm

The first phase of the algorithm is the conversion of the implicit original problem (P) into the explicit equivalent one (P*). As an illustration, with a fixed value y_0 we compute the optimal simplex array corresponding to one of the followers (B stands for the basic variables, N for the out-of-the-basis variables and I for the identity matrix, the index i being omitted):

$$\text{Min } c_N x_N$$
$$\text{s.t. : } \quad A_N x_N + I x_B = b_B(y_0)$$

The constraints $b_B(y) \geq 0$ define the region where the current optimal basis B is unchanged. In this region, the value of the optimal solution is $x_B = b_B(y_0)$ and $x_N = 0$ and the contribution to the leader's objectives is $C_B b_B(y)$. Suppose one of these regions ($b_B(y) \geq 0$) is known; we can compute a new region, adjacent to the previous one in the following manner:

- choose a constraint $b^j_B(y) \geq 0$ among those describing the known region;
- check whether this constraint is a facet (not a redundant constraint); if not, choose another one and check again, until a facet is found. Let $b^j_B(y) \geq 0$ be this facet.
- consider the opposite constraint $-b^j_B(y) \geq 0$ and perform a dual pivot on the line j of the array ;
- if the current basis is not feasible, perform a dual simplex algorithm on the array generating a new optimal simplex array, with a new optimal basis B' and a new right-hand-side term $b'_{B'}(y)$.

The new set of constraints $b'_{B'}(y) \geq 0$ define a region, adjacent to the previous one, where the basis B' is optimal. As the number of regions is finite and the feasible set is connected, this procedure allows us to decompose the whole feasible set into such regions. The crucial point of this procedure consists in determining whether a constraint $b^j_B(y) \geq 0$ is a facet. If such a constraint is not a facet, then the feasible set defined by $\{b_B(y) \geq 0, b^j_B(y) \leq 0\}$ is empty. As a consequence, the dual of the problem whose constraints are $b_B(y) \geq 0$ and $b^j_B(y) \leq 0$ is unbounded. If the dimension of the incentive vector (y) is small, then it is easy to check whether such a dual problem is unbounded.

The second phase of the algorithm consists in solving the explicit problem (P*) computed in the first phase. As this problem is not classical, we have to choose a suitable optimization procedure and adapt it to the particular data structure generated in the first phase of the algorithm. We have chosen an interactive method (Dong and Installé (1990)), easy to perform with the particular data structure considered here. At each iteration, maximum and minimum aspiration levels are given by the user and a "good" feasible solution is computed. The user may accept this solution or request another one based on new minimum and maximum levels of aspiration for some of his objectives. The algorithm terminates when an acceptable solution (if one exists) is reached. At each iteration, some of the regions previously generated are eliminated, others remain unchanged and the remaining ones have to be (easily) modified.

4. Application

We consider a simplified land-use planning problem in which a public authority (the leader) tries to promote the production of export crops while minimizing the costs involved and keeping soil erosion under a fixed tolerance level. The incentives are technical assistance and subsidies available for export crop production. The farmers (the followers) produce export and subsistence crops on a limited land using limited manpower. They wish to maximize their profit while incurring a limited risk. The model is the following one :

Upper level problem

Objectives :	Max $\sum CE_i$ and Min $\sum M_i + \sum L_i$	(export crops and costs)
Constraints :	$e_E \sum CE_i + e_S \sum CS_i \leq ertol$	(soil erosion)

where CS_i and CE_i are areas of subsistence and export crops wich are also solutions of the i^{th} lower problem;

M_i and L_i are technical assistance and subsidies provided to the i^{th} producer;

e_E and e_S are erosion rates of export and subsistence crops;

ertol is the maximal soil erosion rate.

Lower level problem (i^{th} producer)

Objective :	Max $p_E CE_i - p_N N_i$	(profit)
Constraints :	$t_S CS_i + t_E CE_i \leq P_i + M_i$	(manpower)
	$CS_i + CE_i \leq S_i$	(land)
	$CS_i + N_i \geq bP_i$	(subsistence)
	$p_N N_i - p_E CE_i \leq A_i + L_i - p_M M_i$	(liquid assets)
	$r_S CS_i + (r_E - p_E) CE_i + p_N N_i + p_M M_i - L_i \leq A_i$	(risk)
	$L_i - cCE_i \leq 0$	(subsidy)

where
N_i : amount of food bought on external markets.
P_i and S_i : Available manpower and land.
A_i : available cash-flow.
t_S and t_E : manpower rate needed for subsistence and export crops.
r_S and r_E : risk rate of subsistence and export crops.
b : food requirement rate for subsistence.
c : subsidies rate of export crops.
p_M and p_N : prices of technical assistance and food.
p_E : selling price of export crops.

Numerical results for this problem are currently investigated and will be presented and discussed at the conference.

5. Conclusions and Aknowledgements

In this paper a method has been presented to solve a linear multiobjective multiperson hierarchical optimization problem. The original implicit problem is converted into an explicit one with an appropriate resulting data structure. This method is then illustrated through a simplified land-use planning problem. This work is currently supported by a research contract with the European Community, program "Science and Technology for Development".

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