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HANDLING LINGUISTIC INFORMATION IN DECISION MAKING PROCESSES

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Abstract

In this work we will introduce general issues and our own more recent researches in the field of modeling linguistically assessed decision making problems.

The basic hypotheses are a) the utilities are evaluated in a term set of labels and b) the information is supposed to be a "linguistic evidence", i.e. it is to be represented by a basic assignment of probability (in the sense of Dempster-Shafer) but taking its values on a term set of linguistic likelihoods.

We also present some results about the so called "evidential dispositions" which are incomplete pieces of probabilistic statements about gains.

Keywords: decision-making, linguistic-assessments

1. THE LINGUISTIC APPROACH

The graphical or analytical representation of properties and facts is not well captured by experts, who are accustomed to express and interchange information in a linguistic way ([2], [3], [4]). Thus to use labels or linguistic assessments to represent and model uncertainty or imprecise information seems adequate.

To formalize these ideas Zadeh [4] has introduced the concept of "linguistic variable" which roughly speaking may be defined as representing a given property in a certain referential set and whose values are words or statement (labels) belonging to a formal language (natural or artificial). In order to model and handle a practical situation, the appropriate linguistic variables and their values (with their syntax and semantic) are to be identified, as well as rules to aggregate, compose and compare labels are to be developed (see [2], [3], [4], [5], [6]).

Theoretically, the number of linguistic values may be increased to obtain more accurate assessments. However, from a practical point of view a limit exists for the number of terms: The decision-maker's ability of discriminating between two different ones. This limit is called **granularity**. Obviously, the granularity depends upon the syntactic rule.

The use of linguistic variables to model problems with vaguely known or assessed elements, constitutes the so called **linguistic approach to problems** (Zadeh (1975),[4]).

Before going further we want to remark two important points:

A) The semantic rule of any linguistic variable associates a fuzzy subset (of a given universe) to each label in the term set [4], and on its turn, every fuzzy set is characterized by means of a membership function. Our claim is that it makes no sense to use sophisticated shapes for such functions, taking into account that the linguistic assessments are just approximate assessments, given by the experts and accepted by the decision-makers because obtaining more accurate values is impossible or unnecessary. In fact, we consider that trapezoidal membership functions are good enough to capture the vagueness of linguistic assessments.

B) Two different approaches may be found for aggregation and comparison of linguistic values: **B.1)** direct computations on labels **B.2)** the use of the associated membership functions and the Extension Principle.

Most available techniques belong to the latter kind (see [15], [16] or [17]). Their main drawbacks are that the vagueness of results increases step by step, and that the shape of membership functions is in general not maintained. Thus the results are not labels in the original term set.

When the outputs of a model are not labels but unlabeled fuzzy sets, to fulfill the coherence condition of the outputs being in the same class of the inputs, the process known as **linguistic approximation** ([4],[6],[9]) is to be carried out.

In general, we claim that it makes no sense to develop combination or comparison methods with hard or sophisticated computations and from our own point of view, direct calculi on labels is the most reasonable tool to implement the linguistic approach. However, in general, it is to be developed yet and only a few papers about the topic may be found (see [7], [10], [15], [22], [27]). Additionally, the computations through membership functions are already available and thus they are to be borne in mind. A possible way to construct suitable algorithms is to build them like a "black-box" in which inputs and outputs will be labels. Inside the "black box" the computations are made on label membership functions and to obtain the output, a linguistic approximation must be carried out when needed.

2. A MODEL FOR DECISION PROCESSES WITH LINGUISTIC DATA

Generally speaking a decision making problem may be stated as follows: **To choose the best alternative between a set of feasible actions given some measure of the goodness of each**.

In most real situations, the results of any action are conditioned by external factors (the state of nature) and thus, when decision maker chooses an action ξ and the state of nature is ω, he receives some gain (in general sense) $g(\xi,\omega)$. The basic hypothesis is that there exists some lack of information about the true ω. The fitness of $g(\xi,\omega)$ with every decision maker's objective provides a performance measurement of ξ with respect to such criterion and the aggregation of all of them gives the global goodness of such action (when ω is supposed to be the true state of nature).

Despite the broad spectrum of classical Decision Theory (see [12]), there exist many real problems for which the classical approaches are inappropriate, concretely for problems in which some of the following events happen:

a) The available information about the true state of nature is some kind of evidence (in general sense).

b) The gains are assessed by means of linguistic terms instead of numerical (real) values.

c) Decision maker's objectives, criteria or preferences are vaguely established and they do not induce a crisp order relation.

For that kind of problems, the linguistic approach seems to be the most suitable tool. In the following we will illustrate its use on a general model.

Consider a single-criterion, single-decision-maker problem being characterized by the quintuple $(\mathbb{A},\mathbb{E},g,\mathbb{I},\ll)$ where:

- $\mathbb{A}$ is a finite set of feasible actions,
- $\mathbb{E}$ is a finite space state,
- g is the payoff function, i.e. $g:\mathbb{A}\times\mathbb{E} \longrightarrow \mathbb{U}$, $\mathbb{U}$ being a term set of linguistic assessments of utility,
- $\mathbb{I}$ represent the available information about the true state of nature. We suppose it is given by $\mathbb{I}:2^{\mathbb{E}} \longrightarrow \mathbb{L}$, where $\mathbb{L}$ is a term set of linguistic probabilities (see [3],[7]) so that, for any $E\subseteq\mathbb{E}$, $\mathbb{I}(E)$ linguistically assesses the likelihood of the event "the true state of nature belongs to E". We can see $\mathbb{I}$ as a linguistic evidence, because it has the same structure as a numerical one (see [24]) but assessed in a term set.
- $\ll$ denotes decision-maker's preference relation. It induces an order relation on $\mathbb{U}$ from which (through decision rules) the decision maker will obtain the ranking of the actions.

The semantics of labels in $\mathbb{U}$ will be defined by fuzzy sets on a utility interval $[u_0,u_1]$ whereas the terms of $\mathbb{L}$ will be associated with fuzzy subsets of [0,1]. According to our former argument A), we will only consider trapezoidal membership functions.

In the classical risk-environment models, the mathematical expectation is used to integrate information about states (probabilities) with values of gains (utilities), to provide a goodness measure for actions, and the key decision rule is to maximize it. In [24] these classical models are generalized by assuming that the information about states is a numerical evidence. Now the mathematical expectation changes into two operators (the so called lower and upper mathematical expectation) and the decision rules are based on an real interval instead of a real number.

The existing models for decision making problems in fuzzy environment (under risk) assume the information about the true state of nature is a linguistic probability ($P: E \longrightarrow L$) and thus the goodness measure for actions is assessed by the fuzzy expected utility (see [15], [17], [18]).

As our model generalizes that of [24] by considering a linguistic evidence (instead of a numerical one), it seems reasonable to assume the performance (thus the ranking of actions) to be measured by two ordered fuzzy numbers, each being a fuzzy expectation. Next section deals with developing such kind of decision rules. Let us remark we propose to consider any fuzzy expectations to be an (approximate) trapezoidal fuzzy number or its linguistic approximation.

3. SOME REMARKS ABOUT THE STRUCTURE OF U

In many cases, when a decision-maker assesses an expected result as "good", he does not reject the possibility of obtaining a "very good" result, but he is just rejecting a bad one. In the same way, when somebody claims that a "undesirable" result is unlikely, it is just establishing that only "desirable" results are to be expected. That is, the labels "good" and "bad" or "desirable" and "undesirable" include all of their quantifications or modifications (rather, fairly, more, less, etc.).

Additionally, finding a problem with several different coexistent granularity levels in payoff data is quite possible. Several reasons such as different information sources or different discrimination abilities may produce such effect.

These intuitive ideas, suggest to us that a more accurate model for decision making problems is obtained when a hierarchical structure for U is assumed. Thus we propose to consider U as a tree instead of a vector of labels. Every node will have an associated label which will represent a category of results containing (from a semantic and formal point of view) all linguistic values corresponding to its sons.

The set labels in a level of the tree must constitute a (fuzzy) partition of the universe of results corresponding to a certain granularity (discrimination capacity level). On the other hand, they must be ordered according to the preferences of decision maker. That order must be compatible with the hierarchy, that is if $N \ll N'$ then $\Gamma(N) \ll N'$, where $\Gamma(N)$ stands for the set of descendants of N. M and N are, in general, indifferent when $M \in \Gamma(N)$, although a particular decision maker could rank them by using some additional semantic criteria

For models with hierarchical set of utility labels, an open question is to decide what will be the right level in the hierarchical U from which approximating labels must be taken out when a linguistic approximation is to be carried out. A possible (perhaps conservative) choice is to use the one where the coarsest data are contained.

4. RISK-INTERVAL-BASED DECISION RULES

With the above formulation, for any $a \in \mathbb{A}$ and $E \subseteq \mathbb{E}$, one obtains immediately

$$M(\xi,E) = \max \{g(\xi,\omega),\ \omega \in E\} \qquad m(\xi,E) = \min \{g(\xi,\omega),\ \omega \in E\},$$

which represent the best and the worst result that the decision maker can expect when he applies the action ξ and the true state of nature is in E.

From these values and by using either extended operations between fuzzy numbers or direct computation on labels, it is easy to obtain:

$$V(\xi) = \sum_E M(\xi,E) \odot I(E) \qquad v(\xi) = \sum_E m(\xi,E) \odot I(E)$$

Let us observe that v(.) and V(.) are respectively the linguistic generalization of the lower and upper expectation defined in [24] for numerical evidences.

From a practical point of view we need only consider those E for which a piece of knowledge is available (i.e. $\mathbb{I}(E) \neq$ impossible) which may be called focal set (like in classical Dempster-Shafer theory).

Thus the goodness or performance of any $\xi \in \mathbb{A}$ is assessed by the fuzzy interval $[v(\xi),V(\xi)]$, that we will call "risk interval" because of its intuitive meaning. It may be considered as a bidimensional (fuzzy) utility too. By means of these intervals, the decision maker must rank the alternatives in order to choose the best (the optimal decision). A general decision rule may be characterized by constructing some suitable value function $F: \mathbb{R}_T \times \mathbb{R}_T \longrightarrow \mathbb{R}_T$, ($\mathbb{R}_T$ stands for the label set were the problem is assessed) and then ξ^* (the opimal decision) will be obtained through maximizing $F(v(\xi),V(\xi))$ (the maximum is taken according to the order induced by $\ll$) over $\mathbb{A}$. Depending on decision's maker attitude against the risk, the following basic decision rules exist

pessimistic: ξ^* is the action for which $v(\xi^*) = \max \{v(\xi),\ \xi \in \mathbb{A}\}$.

optimistic: ξ^* is defined by $V(\xi^*) = \max \{V(\xi),\ \xi \in \mathbb{A}\}$.

Like the case of numerical evidences (see [24]), now these basic rules admit variants in several ways. For instance, a lexicographic order (starting by $v(\xi)$ or $V(\xi)$ according to decision-maker's attitude) may be used.

5.- A MAXIMIN METHOD BY DIRECT CALCULUS ON LABELS

Let consider $m(\xi,E) = \min \{g(\xi,\omega),\ \omega \in E\}$ which is the worst result when ξ is applied and the true state of nature is in E.

Now, for any $\xi \in \mathbb{A}$ the set $\{[m(\xi,E),\mathbb{I}(E)] : E \subseteq \mathbb{E}\}$ is to be seen as a generalized

linguistic lottery (see [18]). It is obvious that such a linguistic lottery is a subset of $\mathbb{U}\times\mathbb{L}$, and additionally it has been shown that $\mathbb{U}\times\mathbb{L}$ may be ordered (Delgado et al. 1988). Thus

$$\forall\ \xi\in\mathbb{A}\ \exists u(\xi) = \inf\ \{[m(\xi,E),\mathbb{I}(E)]:E\subseteq\mathbb{E}\}\in\mathbb{U}\times\mathbb{L}$$

is a pessimistic goodness measure and thus the optimal decision is obtained from

$$v(\xi^*) = \sup\ \{u(\xi):\xi\in\mathbb{A}\}$$

For a more detailed description of this method see [10].

6.- EVIDENTIAL DISPOSITIONS. DECISION MODELS

It is well known, in the setting of classical Probability Theory, that the sentence *the event* A *has probability* p,(*), contains the same knowledge (gives the same information) as *the event* ¬A *has probability* 1-p. Moreover, using any of them as an isolated sentence is not formally correct, as the model of Probability Theory requires establishing the probabilities of the whole set of possible results.

These formal properties are intuitively taken into account when one uses a sentence like the above ones in order to express the relevant or available information about the result of an experiment or action. In fact the meaning one assigns to it is, *the event* A *is to be expected with probability* p *and there is a probability* 1-p *of obtaining any other thing*.

In real risk-environment decision making problems, it is usual to give and use statements like: *a* good *result is* likely (**), *an* undesirable *result* may-be (**). These sentences are similar to (*), but now there is a linguistic probability assessing the decision-maker's belief (in general sense) about the value of a result which, is in turn linguistically established.

Let us assume the linguistic probabilities are labels of a term set with 2n+1 elements $\mathbb{L}=\{p_1,p_2,\ldots,p_{2n+1}\}$, p_i and $p_{2(n+1)-i}$, i=1,2,..,n being symmetrical with regard to p_{n+1} and that p_1 = *impossible* and $p_{2(n+1)}$ = *certain*. Thus p_i and $p_{2(n+1)-1}$ are "complementary" probabilities (i.e. the linguistic equivalent to the numerical p and 1-p) for any i=1,2,...,n. The semantics of the elements in $\mathbb{L}$ is given by a family of fuzzy subsets of [0,1] , (see [2],[3]).

According to our former comments and the models developed in section 2, any sentence like (**) may be modeled as the assignment of a linguistic evidence mass to a node in the hierarchy of problem results. This may be represented by

$$(\mathbf{p},\mathbf{N}),\ \mathbf{p}\in\mathbb{L},\ \mathbf{N}\in\mathbb{U} \qquad (***)$$

Taking into account the formal properties of the probability assignments we have that this is equivalent to establish a whole linguistic evidence on $\mathbb{U}$ in the form

$$\{(\mathbf{p},\mathbf{N});(\neg\mathbf{p},\neg\mathbf{N}),\mathbf{p}\in L,\mathbf{N}\in\mathbb{L}\} \qquad (***)$$

where $\neg\mathbf{p}$ denotes the complementary linguistic probability of $\mathbf{p}$ and $\neg\mathbf{N}$ the set complement of **N** in its level of the hierarchy.

Zadeh called "dispositions" those quantified sentences in which the quantifier is not explicitly included because it is usually inferred in common language. From this idea we propose to call **evidential dispositions** the sentences like (**) (formally $(\mathbf{p},\mathbf{N}), \mathbf{p}\in\mathbb{L}, \mathbf{N}\in\mathbb{U})$ in which there is an untold (but perfectly understood) information about a set of results. In the following we will denote $\mathbb{DE}(\mathbb{L},\mathbb{U})$ the set of evidential dispositions we can construct from $\mathbb{L}$ and $\mathbb{U}$.

In real decision making situations it is quite usual to assess any action by means of an evidential disposition and thus, to choose the "best" action, (the final goal) the evidential dispositions must be ranked. In the following our developments about this topic are presented.

Let $(\mathbf{p},\mathbf{N})$ be an evidential disposition belonging to $\mathbb{DE}(\mathbb{L},\mathbb{U})$. Because it determines a linguistic evidence on a certain level of $\mathbb{U}$, the afore described risk interval decision rules may be directly used. Let denote $W = \min \neg\mathbf{N}$ and $B = \max \neg\mathbf{N}$, the worse and the best result in $\neg\mathbf{N}$. Then (if $\odot$, $\oplus$ respectively stand for some product and addition of labels)

$$v(\mathbf{p},\mathbf{N}) = \mathbf{p}\odot\mathbf{N}\oplus\neg\mathbf{p}\odot W \qquad\qquad V(\mathbf{p},\mathbf{N}) = \mathbf{p}\odot\mathbf{N}\oplus\neg\mathbf{p}\odot B$$

are the lower and upper expected values, associated to the evidential disposition $(\mathbf{p},\mathbf{N})$. Thus the risk-interval $[v(\mathbf{p},\mathbf{N}),V(\mathbf{p},\mathbf{N})]$ is to be used to rank evidential dispositions (through the decision rules we mentioned in section 4).

To implement $\odot$ and $\oplus$, either direct computation on labels or extended arithmetic operation between fuzzy numbers may be used. In the following we will analyze the first approach for this particular case.

Both, $v(\mathbf{p},\mathbf{N})$ and $V(\mathbf{p},\mathbf{N})$ may be seen as the linguistic convex combination (with coefficients $\mathbf{p}$ and $\neg\mathbf{p}$) of two utility labels belonging to a certain level of $\mathbb{U}$. From the classical properties of the two-terms-convex-combination, we generally define its linguistic version as an application $\mathbb{C}:\mathbb{L}\times\mathbb{R}\times\mathbb{R} \longrightarrow \mathbb{R}$, $\mathbb{L}$ being a term set of linguistic probabilities and $\mathbb{R}$ an ordered set of labels, such that

- **a)** $p\odot A\oplus\neg p\odot B=\neg p\odot B\oplus p\odot A$, $\forall p\in\mathbb{L}$, $\forall A,B\in\mathbb{L}$, and $p\odot A\oplus\neg p\odot B\in[B,A]$ if $A\geq B$, $\forall p\in\mathbb{L}$,
- **b)** $p\odot A\oplus\neg p\odot A = A; \varnothing\odot A\oplus\neg\varnothing\odot B=B$, $\forall p\in\mathbb{L}$, $\forall A,B\in\mathbb{L}$, $\varnothing$ being the label *impossible*.
- **c)** If $A\geq B$ and $p\geq(\leq)p'$ then $p\odot A\oplus\neg p\odot B\geq(\leq)p'\odot A\oplus\neg p'\odot B$,
- **d)** If $A\geq B\geq C$ then $p\odot A\oplus\neg p\odot C\geq p\odot B\oplus\neg p\odot C$ and $p\odot A\oplus\neg p\odot B\geq p\odot B\oplus\neg p\odot C$.

By a suitable constraint propagation algorithm, $\mathbb{C}$ may be obtained and represented as a table (because of the discreteness of the term sets). Annex I presents a general version for this constraint propagation algorithm.

Let us remark that properties a) to c) do not univocally define $\mathbb{C}$. In some examples we have found that the number of possible $\mathbb{C}$'s may be very large. The choice of a concrete one will depend on decision maker's requests and the problem context (see comments about the procedure SELECT in Annex I).

In dealing with evidential dispositions, it is obvious that we may identify ℝ in each case with the level of 𝕌 where the evidential disposition is defined and then to compute (totally or partially) ℂ in order to obtain v(**p**,N) and V(**p**,N). According to our former developments, these expected values are labels belonging to the current 𝕌-level and v(**p**,N)≤V(**p**,N). Thus a linguistic risk interval is associated to any evidential disposition.

7. FURTHER REMARKS

In this paper we have restricted ourselves to single objective decision making problems. To construct models for multi objective ones, tools to aggregate several goodness measures (one for each objective) are needed. On the other hand, models for group decision problems with vague data are to be developed. These problems will be dealt with in forthcoming papers.

We have assumed here that the evidential dispositions are defined on a certain level of a hierarchy of results, that is all information about gains have the same granularity level. Problems with different granularity levels may be easily faced, but for them, an open question is to decide what will be the right level in the hierarchical 𝕌 in which the evidential dispositions must be taken out. A possible choice is to use the one where the coarsest granular data was contained, but we are conscious this is a conservative attitude which may become wrong depending on the data.

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ANNEX I

Constraint propagation algorithm (a general version)

We will write

$$\mathbb{L}=\{p_1,p_2,\ldots,p_{2n+1}\} = \{1,2,3,\ldots,2n+1\} \quad \mathbb{R}=\{u_1,u_2,\ldots u_q\} = \{1,2,3,\ldots,q\}$$

for the sake of simplicity in describing the algorithm.

program Convex-combination

```
const q=   , n=    ;
var C = array[1..2*n+1,1..q,1..q], i,j,k,l = integer;
begin
{initial values seting}
      for i=1 to 2*n+1 do
      for j=1 to q do
      for k=1 to q do
      C[i,j,k] := 0;


{the remaining sentences constitute the core of the constraint propagation algorithm}
      for i=1 to 2*n+1 do
      for j=1 to q do
      for k=j to q do
{the three next sentences are to satisfy constraint b)}
      if j=k then C[i,j,k]:=j else
      if i=1 then C[i,j,k]:=k else
      if i=2*n+1 then C[i,j,k]:=k else
           begin
{next procedure is to select a tentative value, the label associated to the index l,
for the convex combination of jth and kth utility values with respect to the ith
likelihood}
           select(l,i,j,k);
{next procedure is to check if constraints c) and d) hold}
           checking(l,i,j,k);
{after checking the value is assigned}
           C[i,j,k]=l;
           end;
```

{this final set of sentences completes the C-table by using the symmetry constraint}

```
        for i=1 to 2*n+1 do
        for j=1 to q do
        for k=1 to j-1 do
              C[i,j,k]:=C[2*n+1-i,k,j];
end.
```

Let us remark some points about the procedures select(....) and checking(....).

A) select(l,i,j,k,) chooses l as a tentative value for C[i,j,k]. A great part of the efficiency of the whole algorithm relies on it.

The most basic constraint to be fulfilled is j≤l≤k but it is not enough to guarantee coherence with the more exigent constraints c) and d)

Some heuristic (inspired on the continuous real convex combination properties) may be used to improve the performance of this procedure. A very simple one may be stated as follow: *the smaller (greater) i the nearer l to k (to j).*

B) checking(l,i,j,k) is a procedure to check and eventually to change the previously selected l in order to achieve constraints c) and d). The main difficulty arises because in some cases changing a previously assigned C[...] will be needed. In that situation a call for checking this new assignation is to be made and so this procedure is basically a recursive one. In the following we present a possible version of it.

```
Procedure checking(l,i,j,k)
var r,j1,k1:integer;
begin
{the next two loops check constraint c), previously assigned values may be changed}
        for r=1 to i-1 do
              if C[r,j,k]<l then
                                  if l>j then
                                              begin
                                              l=l-1;
                                              checking[l,i,j,k];
                                              end;
                                        else
                                              begin
                                              z=C[r,j,k]+1;
                                              checking[z,r,j,k];
                                              end;
        for r=i+1 to 2*n+1 do
              if C[r,j,k]<>0 then
                    if C[r,j,k]>l then
                                        if l<k then
                                                    begin
                                                    l=l+1;
                                                    checking[l,i,j,k];
                                                    end;
                                              else
                                                    begin
                                                    z=C[r,j,k]-1;
                                                    checking[z,r,j,k];
                                                    end;
```

```
{next loops check constraint d); no previously  assigned  value  is changed}
      for k1=1 to k-1 do
           if C[i,j,k1]>l then
                         begin
                         l=l-1;
                         checking[l,i,j,k];
                         end;

      for k1=k+1 to q do
           if C[i,j,k1]<>0 then
                         if l>C[i,j,k1] then
                                        begin
                                        l=l-1;
                                        checking[l,i,j,k];
                                        end
      for j1=1 to j-1 do
           if C[i,j1,k]>l then
                         begin
                         l=l+1;
                         checking[l,i,j,k];
                         end:
      for j1=j+1 to q do
           if C[i,j1,k]<>0 then
                         if l>C[i,j1,k] then
                                        begin
                                        l=l-1;
                                        checking[l,i,j,k];
                                        end;
end.
```