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Modelowanie matematyczne, symulacja komputerowa i identyfikacja dynamiki przepływu cieczy w wannie szklarskiej

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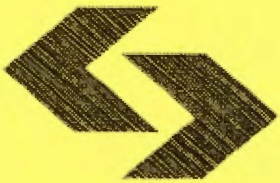

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W Raporcie przedstawiono dwa artykuły dotyczące modelowania matematycznego i komputerowej identyfikacji dynamiki przepływu masy szklanej w piecu wannowym do produkcji szkła okiennego. Artykuły te zostały zaprezentowane w postaci wykładów w Szkole Letniej nt. Zaawansowanych Problemów Mechaniki (Summer School on Advanced Problems in Mechanics – APM'2001), która była zorganizowana w lipcu br. w Repinie koło Petersburga (21-30.07.2001). Organizatorami Szkoły Letniej były: Instytut Problemów Inżynierii Mechanicznej Rosyjskiej Akademii Nauk z Petersburga (Institute for Problems in Mechanical Engineering of Russian Academy of Sciences) oraz niemieckie Towarzystwo Matematyki i mechaniki Stosowanej (Gesellschaft fuer Angewandte Mathematik und Mechanik – GAMM). Artykuły ukażą się w materiałach Szkoły Letniej w 2002 r. (Proceedings of the XXIX Summer School on Advanced Problems in Mechanics, St. Petersburg (Repino), IPME RAS 2002).

Spis treści

Identification of the glass mass flow dynamics in glass tank furnaces

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Three methods of modelling of the glass mass flow dynamics in a tank furnace are presented and analysed by the help of extensive computing results. The model description occurs with the partial or ordinary differential equations and the kind of the equations used makes the modelling methods considerably different.

Key Words - Mathematical modelling, computer simulation, parameter and structure estimation, dynamic system estimation.

The title shortened: *Glass mass flow identification*

Abstract. By the use of the computer simulation three methods of modelling of the glass mass flow dynamics in a glass tank furnace are investigated. In the first method the model is described by the partial differential equations and the static optimization is used to estimate the model parameters. In the second method the ordinary differential equations are used as a model description and an indirect identification method is applied for estimating the structure and the parameters of the models. In the third case a two-stage approach is proposed to develop the lumped parametr models of the glass mass flow dynamics while a PDE-model is used on the first stage of modelling.

1. Introduction

In the paper the problem of the mathematical modelling of the glass mass flow dynamics in glass tank furnaces is investigated. The aim of the investigation is to develop an numerical algorithm for setting up the models that are usuable for practical applications, i.e. the computer simulation of a tank furnace or the control of changing the chemical composition of the glass or the estimation of technological parameters of a tank etc.

To model the glass mass flow dynamics two kinds of mathematical description are usually applied, i.e. the distributed or lumped parameter equations (DPE or LPE-models). The distributed parameter models are very complicated and this makes the computer simulation and optimization of the models difficult. On the other side the lumped parameter models available nowdays are very simplified. They describe the models examined unaccurately and that is why their practical usefullness is rather small. This situation is caused by the lack of adequate identification methods.

To solve our task the possibilities of setting up both distributed and lumped parameter models are examined. In the first case the models are described by the quasilinear Navier-

Stokes and energy equations and by an equation added that describes the glass mass composition change in a tank furnace. The fitting of the models to an object occurs by estimating of some coefficients using the static optimization methods. In the second case the models are described by the linear ordinary differential equations or equivalently by the Laplace transfer function.To estimate the structure and the parameters of the models an indirect identification method is used. It has been developed especially for setting up continuous dynamic models of higher orders using the noisy impulse responce from the object. A third approach of modelling is also examined combining the cases mentioned. It consists in setting up a complex lumped parameter model by the help of a simplified distributed parameter model constructed on the first stage of modelling.

All computations are done using the data from a real glass tank furnace with conventional gas heating. The longitudinal section of the tank and the fundamental currents occuring in the glass melt are shown in Figure 1.1. The currents are the result of the characteristic temperature distribution on the glass melt surface (which induces the free convection in the melt) as well as of the permanent inserting of the raw materials *into* and pulling out of the glass sheet *from* the tank (which induce the forced convection in the melt). The currents are: the rotating current (that is caused by the free convection) and the withdrawal and surface currents (that are caused by the free and forced convections together). The rotating and withdrawal currents secure the rightness of the mixing, refining and homogenization processes in the melt and the withdrawal current transports the main share of the glass mass through the tank. The surface current is considered as an interference factor of the technological process.

Fig. 1.1.

In the glass tank furnace considered an active experiment was performed: A dose of an isotope was put with the raw materials charge into the tank and the radioactivity of the glass sheet pulled out was measured in fixed time intervals. The data obtained are shown in Figure 1.2 where the influence of the glass melt currents on the object output can be recognized explicitly. The rotating and withdrawal currents are responsible for the slow run of the data curve and its inertial character while the regular oscillations in the data are created by the surface current.

Fig. 1.2.

2. Development of the PDE-models with static optimization methods

2.1. Model formulation

To model the glass mass flow in a tank furnace by the partial differential equations the following general description is usually used:

$$\rho \frac{\mathrm{d}v}{\mathrm{d}t} = \mathrm{Div}S + \rho f \tag{2.1}$$

$$\rho \frac{\mathrm{d}}{\mathrm{d}t}\left(c_v T + \frac{1}{2}(v,v)\right) = \mathrm{div}(\lambda \mathrm{grad}T) + \mathrm{div}(Sv) + \rho(f,v) \tag{2.2}$$

$$\mathrm{div}v = 0 \tag{2.3}$$

where the parameters and symbols mean: ρ - density, $v=(v_1,v_2,v_3)$ - velocity vector with the longitudinal, vertical and cross velocities, t - time, $f=(f_1,f_2,f_3)$- mass forces vector, c_v - specific capacity, T- temperature, λ - thermal conductivity,

$$S = 2\mu T_d - \left(p + \frac{2}{3}\mu \mathrm{div} v\right)E \equiv (s_1, s_2, s_3) \tag{2.4}$$

$$S = \begin{bmatrix} \sigma_{x_1x_1} & \tau_{x_1x_2} & \tau_{x_1x_3} \\ \tau_{x_2x_1} & \sigma_{x_2x_2} & \tau_{x_2x_3} \\ \tau_{x_3x_1} & \tau_{x_3x_2} & \sigma_{x_3x_3} \end{bmatrix}, \quad T_d = \begin{bmatrix} \varepsilon_{x_1} & \frac{1}{2}\theta_{x_3} & \frac{1}{2}\theta_{x_2} \\ \frac{1}{2}\theta_{x_3} & \varepsilon_{x_2} & \frac{1}{2}\theta_{x_1} \\ \frac{1}{2}\theta_{x_2} & \frac{1}{2}\theta_{x_1} & \varepsilon_{x_3} \end{bmatrix}, \quad E = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

μ - dynamic viscosity, p - pressure, S, T_d and E - stress, distortion velocity and unit tensors, σ, τ - normal and tangential stresses, ε and θ - distortion velocities,

$$\varepsilon_{x_1} = \frac{\partial v_1}{\partial x_1}, \qquad \varepsilon_{x_2} = \frac{\partial v_2}{\partial x_2}, \qquad \varepsilon_{x_3} = \frac{\partial v_3}{\partial x_3}$$

$$\theta_{x_1} = \frac{\partial v_3}{\partial x_2} + \frac{\partial v_2}{\partial x_3}, \qquad \theta_{x_2} = \frac{\partial v_3}{\partial x_1} + \frac{\partial v_1}{\partial x_3}, \qquad \theta_{x_3} = \frac{\partial v_2}{\partial x_1} + \frac{\partial v_1}{\partial x_2}$$

$$\frac{\mathrm{d}}{\mathrm{d}t} = \frac{\partial}{\partial t} + v_1\frac{\partial}{\partial x_1} + v_2\frac{\partial}{\partial x_2} + v_3\frac{\partial}{\partial x_3}$$

$$\mathrm{div} v = \frac{\partial v_1}{\partial x_1} + \frac{\partial v_2}{\partial x_2} + \frac{\partial v_3}{\partial x_3}$$

$$\mathrm{Div} S = (\mathrm{div} s_1, \mathrm{div} s_2, \mathrm{div} s_3).$$

Equations (2.1), (2.2), (2.3) are known in the classical fluid mechanics as the Navier-Stokes (or motion), energy and continuity equations, respectively, and they are formulated on the base of the momentum, energy and mass conservation laws.

While setting up the equations several simplified assumptions are made that take into consideration the specific properties of the glass mass flow. The main assumptions will be explained in the following.

Equation (2.1) describes the motion of the glass mass. The fluid motion is caused in general by the inertial forces (appearing at the accelerated motion), the mass forces (appearing in an area with a temperature difference and deciding about the free convection in the liquid) and the surface ones (consisting of the normal, tangential and surface stresses). The first stresses represent the pressure and the second ones are caused by the viscosity. *The surface stresses are omitted* in the equation (2.1) after the assumption is made that the glass melt is a *homogeneous liquid*. The left side of equation (2.1) shows the influence of the inertial forces

on the fluid motion and the right side shows the influence of the surface forces (caused by the pressure and viscosity only) and the mass forces. The latter ones are the lift forces and they work only in the vertical direction, i.e. $f = (0, f_2, 0)$ and $\rho f = (0, \rho f_2, 0)$. The lift forces term ρf_2 describes the change of density ρ in dependance on the temperature differences using the following approximation:

$$\rho f_2 = g\Delta\rho = g\rho\beta\Delta T \tag{2.5}$$

with: g - gravitational acceleration, β - thermal expansion, $\Delta\rho = \rho(T) - \rho(T_0)$ and $\Delta T = T - T_0$, where T_0 is reference temperature. Using relation (2.5) *the change of density is replaced by the change of temperature* for a constant density value. Another assumption made is: The glass melt is *a real Newtonian liquid*. Then the relation (2.4) results that shows the linear dependance between the normal and tangential stresses and the distortion velocities in the homogeneous and viscous fluid.

Equation (2.2) describes the energy processes occuring in the glass melt. The changes of the inner energy (that is produced through the thermal motion of molecules) and of the kinetic energy in the glass melt (the changes are described by the left side of the equation) are transformed into the work executed by the mechanical forces (i.e. the mass and surface forces) and into the heat transmitted by the thermal conduction (the both objects are described by the right side of the equation). While setting up the equation *the heat transmitted by the radiation, the energy of the intramolecular processes and the heat produced by the chemical reactions are omitted.* Equation (2.2) can be divided into two equations:

$$\rho\frac{\mathrm{d}}{\mathrm{d}t}(c_v T) = \mathrm{div}(\lambda \mathrm{grad} T) + \Phi \tag{2.6}$$

$$\frac{1}{2}\rho\frac{\mathrm{d}}{\mathrm{d}t}(v, v) = v\mathrm{div}S + \rho(f, v) \tag{2.7}$$

that describe the changes of the inner energy and the kinetic one separately. The function $\Phi = \mathrm{div}(Sv) - v\mathrm{div}S$ means the energy dissipation.

The continuity equation (2.3) is formulated under the assumption that the glass melt is *an incompressible liquid* ($\rho = \text{const.}$). It means that the change of the density ρ in dependence on p is negligible.

The mathematical model of the glass mass flow described by equations (2.1, 2.2 and 2.3) is three-dimensional and instationary and in this fairly general form it is very difficult for numerical computation. Consequently farther simplifications are usually made: *The glass mass flow is two-dimensional* (the width of the tank furnace is omitted) *and stationary*; *the energy dissipation* (in equation (2.6)) *and the change of the kinetic energy* (equation (2.7)) *are neglected.* Subsequently a new model arises in the following scalar form (which is more clear for discussing):

$$\rho(v_1 D_1 v_1 + v_2 D_2 v_1) = D_1(2\mu D_1 v_1 - p) + D_2\big(\mu(D_1 v_2 + D_2 v_1)\big) \tag{2.8}$$

$$\rho(v_1 D_1 v_2 + v_2 D_2 v_2) = D_2(2\mu D_2 v_2 - p) + D_1\big(\mu(D_1 v_2 + D_2 v_1)\big) + \rho f_2 \tag{2.9}$$

$$\rho c_v(v_1 D_1 T + v_2 D_2 T) = D_1(\lambda D_1 T) + D_2(\lambda D_2 T) \tag{2.10}$$

$$D_1 v_1 + D_2 v_2 = 0 \tag{2.11}$$

with $D_1 \equiv \dfrac{\partial}{\partial x_1}$ and $D_2 \equiv \dfrac{\partial}{\partial x_2}$. One can see now explicitly the terms of the inertial, surface and mass forces in the motion equations (2.8, 2.9) (the terms decide about the liquid motion in a tank furnace) and either the terms of the inner energy and of the thermal conduction in the energy equation (2.10) (the terms decide about the heat transfer in the glass melt; the inner energy term describes the action of free convection).

Model (2.8 to 2.11) is not so sophisticated as the previous one but still uncomfortable for computer simulation of a real object. One of successive simplifications usually made is that *the spatial derivations of viscosity and thermal conductivity coefficients μ and λ are neglected.* The new model resulted is:

$$\begin{cases} \rho(v_1 D_1 v_1 + v_2 D_2 v_1) = \mu(D_1^2 v_1 + D_2^2 v_1) - D_1 p \\ \rho(v_1 D_1 v_2 + v_2 D_2 v_2) = \mu(D_1^2 v_2 + D_2^2 v_2) - D_2 p + \rho f_2 \\ \rho c_v(v_1 D_1 T + v_2 D_2 T) = \lambda(D_1^2 T) + \lambda(D_2^2 T) \\ D_1 v_1 + D_2 v_2 = 0 \end{cases} \tag{2.12}$$

Model (2.12) can be farther simplified in such the way *that the influence of the inertial forces on the liquid motion is omitted.* Finally one gets the model equations as follows:

$$\begin{cases} \mu(D_1^2 v_1 + D_2^2 v_1) = D_1 p \\ \mu(D_1^2 v_2 + D_2^2 v_2) = D_2 p - \rho f_2 \\ \lambda(D_1^2 T) + \lambda(D_2^2 T) = \rho c_v(v_1 D_1 T + v_2 D_2 T) \\ D_1 v_1 + D_2 v_2 = 0 \end{cases} \tag{2.13}$$

All assumptions shown above are made either as a result of specific features of the glass melt (to which small velocities of the glass mass flow and its greater variability relating to the length against the width of the tank count above all) or for merely numerical reasons.

Model (2.13) as well as the more complex models (2.12) and (2.8 to 2.11) and (2.1 to 2.3) describe the distributions of the temperature and the glass melt velocities in a tank furnace. The models are not the operating ones i.e. they can not be used for control and simulation of the production process. To get these possibilities the models must be extended by some new equations that contain the relations between the decision variables (which realize the control) and the state ones (which are controlled).

In the following we take model (2.13) for computing the temperature and velocities distributions in the glass melt. A new equation added to the model is developed on the basis of the mass balance and it has the form:

$$\frac{\partial z}{\partial t} + e_1 v_1 \frac{\partial z}{\partial x_1} + e_2 v_2 \frac{\partial z}{\partial x_2} = D(e_3 \frac{\partial^2 z}{\partial x_1^2} + e_4 \frac{\partial^2 z}{\partial x_2^2}) \qquad (2.14)$$

with: z - chemical composition of the melt, t - time, e_1 to e_4 - some fitting coefficients (to fit the model to an object), D - diffusion coeficient. Equation (2.14) describes the glass mass composition changes induced by the convection currents (the changes are described on the left side of the equation) and the diffusion (described on the right side). The diffusion processes are very slow and their influence on the arising of the glass mass flows can be neglected while formulating the model (2.13) but they are important when the glass mass composition is calculated. While developing equation (2.14) the assumption is made that *the spatial derivations of the diffusion coefficient D are neglected.* Analogous assumptions were made for the viscosity and thermal conductivity coefficients μ and λ while developing equations (2.13).

2.1. Model simulation

Equations (2.13 and 2.14) make together a parameter distributed model of the glass mass flow in a tank furnace. After some boundary conditions are taken and the temperature and velocities values are calculated from equations (2.13) one can calculate the glass melt composition in each point of the tank by solving equation (2.14). To get the numerical solution of the model equations the finite difference method is proposed.

On the first step the model (2.13) is solved. A theoretical analysis of the numerical solvability of the model is made in (Studzinski, 1987). The boundary conditions defined for the model are (see Fig. 2.1): $v_1 = 0$ and $v_2 = 0$ on the bottom and on the side walls of the tank furnace; $v_1 = v_1(x_1)$ and $v_2 = v_2(x_1)$ on the free surface of the glass mass with v_2 beeing quadratic in sections 1 and 3 of the tank; $T=T(x)$ on all boundaries and T is a linear function on the side tank walls and it is cubic elsewhere.

Fig. 2.1.

The boundary conditions for the function p are unknown and this makes necessary to transform (2.13). It is advisable to replace the velocities v_1, v_2 by the current function ψ that fulfills the relations:

$$v_1 = a_0 \frac{\partial \psi}{\partial x_2}, \quad v_2 = -a_0 \frac{\partial \psi}{\partial x_1} \qquad (2.15)$$

with $a_0 = \frac{\lambda_0}{\rho c_v}$ and $\lambda_0 = \lambda(T_0)$. Then we get equations (2.13) in a new form:

$$\frac{\partial^4 \psi}{\partial x_1^4} + 2\frac{\partial^4 \psi}{\partial x_1^2 \partial x_2^2} + \frac{\partial^4 \psi}{\partial x_2^4} - \frac{\rho g \beta}{\mu a_o}\frac{\partial T}{\partial x_1} = 0 \qquad (2.16)$$

$$\frac{\partial^2 T}{\partial x_1^2} + \frac{\partial^2 T}{\partial x_2^2} - \frac{\lambda_o}{\lambda}\left(\frac{\partial \psi}{\partial x_2}\frac{\partial T}{\partial x_1} - \frac{\partial \psi}{\partial x_1}\frac{\partial T}{\partial x_2}\right) = 0 \qquad (2.17)$$

that consist of only two equations in the contrary to the four ones in (2.13). The reduction of the number of equations causes in general a better convergence when the model is solved numerically. On the other hand the higher order of the motion equation (2.16) leads usually to a worse stability of difference quotients of the finite difference method.

The boundary conditions for equations (2.16 and 2.17) defined more exactly are:

$$\begin{cases}
\psi = 0, \quad \dfrac{\partial \psi}{\partial x_2} = 0 & \text{for } x_2 = 0 \\
\psi = 0, \quad \dfrac{\partial \psi}{\partial x_1} = 0 & \text{for } x_1 = 0 \text{ and } x_1 = L \\
\psi = \psi_1(x_1), \quad \dfrac{\partial \psi}{\partial x_2} = 0 & \text{for } x_2 = 0 \text{ and } 0 \le x_1 \le l_1 \\
\psi = \psi_H, \quad \dfrac{\partial^2 \psi}{\partial x_2^2} = 0 & \text{for } x_2 = H \text{ and } l_1 \le x_1 \le L - l_2 \\
\psi = \psi_2(x_1), \quad \dfrac{\partial \psi}{\partial x_2} = 0 & \text{for } x_2 = 0 \text{ and } L - l_2 \le x_1 \le L \\
T = T_1(x_1) = a_1 x_1^3 + b_1 x_1^2 + c_1 x_1 = 0 & \text{for } x_2 = 0 \\
T = T_2(x_2) = a_2 x_2 + b_2 & \text{for } x_1 = 0 \\
T = T_H & \text{for } x_2 = H \text{ and } 0 \le x_1 \le l_1 \\
T = T_3(x_1) = a_3(x_1 - l_1)^3 + b_3(x_1 - l_1)^2 + c_3(x_1 - l_1) + d_3 & \\
\qquad \text{for } x_2 = 0 \text{ and } L - l_2 \le x_1 \le L & \\
T = T_4(x_2) = a_4 x_2 + b_4 & \text{for } x_1 = L
\end{cases} \qquad (2.18)$$

where L and H mean the length and hight of the tank furnace and l_1, l_2 indicate three sections on the surface of the melt (see Fig. 2.1). In the first section ($0 \le x_1 \le l_1$) the raw materials are put into the tank and there $v_1 = 0$ holds and a quadratic function for v_2 is assumed. In the second section ($l_1 \le x_1 \le L - l_2$) $v_2 = 0$ and $D_2 v_1 = 0$ hold and then $\psi_H = \text{const}$ results. In the third section ($L - l_2 \le x_1 \le L$) the glass sheet is drawn out of the tank and there $v_1 = 0$ holds and a quadratic function for v_2 is assumed. The temperature is constant ($T_H = \text{const}$)

and equivalent to the melting point of the glass in the first section and it changes according to a third order function in the region $l_1 \le x_1 \le L$ consisting of sections 2 and 3. These relations related to the walls and the bottom of the tank result from the boundary conditions concerning the velocities v_1 and v_2 and the temperature T as given generally with the equations (2.13) (Mase and Sasagava, 1973).

All parameters in equations (2.16 and 2.17) are constant with the exception of μ and λ which are described as follows:

$$\begin{cases} \mu = \exp(A_1 + B_1 / (T - C_1)) \\ \lambda = A_2 \exp(B_2 (T - C_2)) \end{cases} \tag{2.19}$$

A discrete approximation of the model equations occurs by the help of classical difference quotients. They lead, however, in the case of high order derivations to a bad stability at the edge of the difference equations. This is explained by an inaccurate approximation at the edge of the knotted grid. Becuase of that some new central difference quotients were developed for the fourth order derivations of ψ (Studzinski, 1991):

$$\begin{cases} \dfrac{\partial^4 \psi}{\partial x_1^4} = -16 \dfrac{\psi_{i+1j} - 2\psi_{ij} + \psi_{i-1j}}{h_1^4} \\ \dfrac{\partial^4 \psi}{\partial x_2^4} = -16 \dfrac{\psi_{ij+1} - 2\psi_{ij} + \psi_{i-1j}}{h_2^4} \end{cases} \tag{2.20}$$

They contain a smaller number of the grid knotted points than the classical difference quotients. This allows to approximate more appropriate the edge region of the grid and to choose greater discretization steps while calculating the equations.

After the discrete approximation is done the following difference schemes:

$$\begin{aligned} \psi_{ij} = {} & (16d^2(\psi_{i+1j} + \psi_{i-1j}) + 16\frac{1}{d^2}(\psi_{ij+1} + \psi_{ij-1}) - \\ & - 2(\psi_{i+1j+1} - 2\psi_{ij+1} + \psi_{i-1j+1} - 2\psi_{i+1j} - 2\psi_{i-1j} + \psi_{i+1j-1} - 2\psi_{ij-1} + \psi_{i-1j-1}) + \\ & + \frac{\rho g \beta \hat{T} \hat{L} \hat{H}^2 h_1 h_2}{2\mu a_o}(T_{i+1j} - T_{i-1j}))/(8(4d^2 + 1 + 4\frac{1}{d^2})) \end{aligned} \tag{2.21}$$

$$\begin{aligned} T_{ij} = {} & (d(T_{i+1j} + T_{i-1j}) + \frac{1}{d}(T_{ij+1} + T_{ij-1}) + \\ & + \frac{\lambda_o}{4\lambda}((T_{ij+1} - T_{ij-1})(\psi_{i+1j} - \psi_{i-1j}) - (T_{i+1j} - T_{i-1j})(\psi_{ij+1} - \psi_{ij-1})))/(2(d + \frac{1}{d})) \end{aligned} \tag{2.22}$$

result from equations (2.13 and 2.14) where: i=1,2,...,M and j=1,2,...,N; $\hat{T}$, $\hat{L}$, $\hat{H}$ are standarization constants, $d = \hat{H}h_2 / \hat{L}h_1$; h_1 and h_2 are discretization steps with: $h_1 = L/(M+1)$ and $h_2 = H/(N+1)$.

The schemes (2.21 and 2.22) are solved by means of the relaxation method using the following iterative algorithm:

$$\psi_{ij}^{n+1} = (1-\omega_1)\psi_{ij}^{n} + \omega_1 \psi_{ij} \tag{2.23}$$

$$T_{ij}^{n+1} = (1-\omega_2)T_{ij}^{n} + \omega_2 T_{ij} \tag{2.24}$$

where ω_1, ω_2 are relaxation coefficients and T_{ij}, ψ_{ij} are calculated from the equations (2.21 and 2.22) in each iteration n=0,1,2,...

For the numerical calculation the values of the physical coefficients and of the space dimensions of the model were chosen according to those of a real tank furnace. The convergence of the iterative algorithm was relatively fast with highly satisfactory accuracy of the calculation. Some examplary results of the temperature and current fields are shown in Fig. 2.2. They were obtained for the grid of 600 modes with the accuracy of calculation 10^{-6} for T and 10^{-5} for ψ. The computation was stopped after the total number of 2900 iterations for both the temperature and current schemes. The iteration number for the temperature scheme was 800 and for the current scheme was 2100.

Fig. 2.2. One can see in Fig. 2.2 that only the rotating and withdrawal currents but not the surface current are determined after the simulation of model (2.13) was made. This current can not be obtained with a PDE-model that is only two-dimensional.

The numerical solution of equation (2.14) occurs on the second step of modelling. To approximate the derivations the central difference quotients of the finite difference method are used and the diffusion coefficient D is described as follows:

$$D = A_3 \exp(B_3 + C_3 / T) \tag{2.25}$$

The difference scheme resulted from (2.14) has the form:

$$z_{ij}^{k+1} = z_{ij}^{k} - e_1 E_1 (\psi_{ij+1} - \psi_{ij-1})(z_{i+1j}^{k} - z_{i-1j}^{k}) + e_2 E_2 (\psi_{i+1j} - \psi_{i-1j})(z_{ij+1}^{k} - z_{ij-1}^{k}) - \\ - D(e_3 E_3 (z_{i+1j}^{k} - 2z_{ij}^{k} + z_{i-1j}^{k}) + e_4 E_4 (z_{ij+1}^{k} - 2z_{ij}^{k} + z_{ij-1j}^{k})) \tag{2.26}$$

with: $E_1 = a_o \hat{t} h_3 / (4\hat{L}\hat{H}h_1 h_2)$, $E_2 = \hat{t} h_3$, $E_3 = 1/(\hat{L}^2 h_1^2)$, $E_4 = 1/(\hat{H}^2 h_2^2)$ where: k=0,1,2,...; h_3 is temporal standarization step and $\hat{t}$ is standarization constant.

2.2. Model identification

The glass tank model described by equations (2.13 and 2.14) constitutes an approximation of a real object. This approximation is usually not exact although the parameters and dimensions of the model correspond to those ones of the tank. The possible inaccuraties occur while the model equations and boundary conditions are formulated and the parameter values are determined. Also the numerous simplifications made during the setting up the model are responsible for many inaccuraties and this is practically unavoidable. The fitting of the model to the object can be realized by the help of equation (2.14) and the data obtained from the isotope experiment (see Fig. 1.2) using a numerical identification approach.

One can formulate the following identification problem (Studzinski and Straubel, 1989):

$$\min_{e_i} Q(e_i) = \min_{e_i} \sum_{k=0}^{K} (\hat{z}^k - z^k)^2 \qquad (2.27)$$

where: i=1,2,3,4; $\hat{z}^k$ and z^k mean the measured data and the model output that is calculated by solving equation (2.26) (the glass composition z is here considered as the radioactivity of the melt).

To solve problem (2.27) a static non-gradient optimization method is used. The isotope data are very noisy and they had to be smoothed before the identification. The number of the fitting coeffitients (which are to be estimated) is small and equals to only 4 but the number of the measurements is relatively big (K=255) and in the consequence the criterion function $Q(e_i)$ is strong nonlinear relating to e_i. Because of that the start points for the optimization runs had to be chosen very carefully and close enough to the optimum. The model output obtained from the calculation is shown in Fig. 1.2.

One can see that the ouput fits well to the data in the farther section of the curve where the influence of the rotating and withdrawal currents on the glass mass flow is the strongest. The approximation of the data with the model output in the initial section of the curve is much worse but there the surface current determines the data which is noticeable through the high oscillations of the curve. This situation can be explained through the omission of the surface current in the PDE-model. This current could be considered in a three-dimensional PDE-model but such a model would be hardly possible to identify because of its complexity.

3. Development of the ODE-models with an indirect identification method

3.1. Indirect identification method

The glass mass flow in a tank furnace can be also described using ODE models. Their parameters have no physical meaning and this makes up the interpretative troubles when comparing the models and objects. On the other side the setting up such the models is easier than PDE models regarding the work complexity and the computing time needed for

simulation and identification. Usually the nonlinear regression methods are used for developing the lumped parameter models. These methods are generally successful if the models of lower orders have to be set up (the model structure is simple and the number of the parameters to be estimated is small) but they are not effective in more complicated cases. The main problems then are connected with the choice of an adequate model structure and the fixing of an appropriate start point (possibly closely to the optimum) for an optimization approach. The methods of nonlinear regresion either do not converge at all or they converge to the local optimal points if the start points are not good (Nahorski et al., 1988).

In the following an indirect identification method is proposed to constitute an multistage approach for modelling the glass mass flow dynamics. The method was developed to estimate the structure and the parameters of continuous time models from the sampled impulse responses of dynamic objects and it makes possible to set up the models of higher orders relativly easy (Nahorski et al., 1985). The indirect identification method is as folows:

The mathematical description of a linear dynamic object has the form of a Laplace transfer function:

$$G(p) = \frac{b_{R-1}p^{R-1} + b_{R-2}p^{R-2} + \ldots + b_1 p + b_0}{p^R + a_{R-1}p^{R-1} + \ldots + a_1 p + a_0} \tag{3.1}$$

where R is the object order and p is the complex variable of the Laplace transformation. For function (3.1) the following equation (in the p region):

$$Z(p) = G(p)\, U(p) \tag{3.2}$$

holds with: $Z(p)$, $U(p)$ - the Laplace transforms for the output and input of the object, respectively. The data for identification are the object response on the input being the Dirac function, i.e. $u(t) = \delta(t)$ and consequently $U(p) = 1$ holds. Then the equation (3.2) can be written down in an equivalent form of the homogeneous ordinary differential equation:

$$\frac{d^R z}{dt^R} + a_{R-1}\frac{d^{R-1} z}{dt^{R-1}} + \ldots + a_0 z = 0 \tag{3.3}$$

with the non-zero initial conditions added:

$$\begin{cases} z(0) = b_{R-1} \\ z^{(1)}(0) = b_{R-2} - a_{R-1}z(0) \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ z^{(R-1)}(0) = b_0 - a_1 z(0) - \ldots - a_{R-1} z^{(R-2)} \end{cases} \tag{3.4}$$

The analytical solution of equation (3.3) has the general form:

$$z(t) = \sum_{j=1}^{J} \sum_{l=0}^{m_j-1} t^l \exp(\alpha_j t)\left(c_{jl} \cos(\varphi_j t) + d_{jl} \sin(\varphi_j t)\right) \qquad (3.5)$$

where $\sum_{j=1}^{J} m_j = R$ and $m_j > 0$. The coefficients c_{jl} and d_{jl} appear linearly in (3.5) („linear" coefficients) and their values depend on the parameters of the transfer function numerator and on the initial conditions (3.4). The coefficients α_j and φ_j appear nonlinearly in (3.5) („nonlinear" coefficients) and their values are the roots of the following algebraical equation (that is characteristic for equation (3.3)):

$$r^R + a_{R-1} r^{R-1} + \ldots + a_1 r + a_0 = (r - \gamma_1)^{m_1} \ldots (r - \gamma_J)^{m_J} = 0 \qquad (3.6)$$

with $\gamma_j = \alpha_j + i\varphi_j$.

The continuous equation (3.3) can be approximated by the following discrete equation:

$$z_k + s_{R-1} z_{k-1} + \ldots + s_0 z_{k-R} = 0 \qquad (3.7)$$

with: $z_k = z(k\Delta t)$, k=1,2,...,K and Δt - discretization step. The analytical solution of equation (3.7) has the general form:

$$z_k = \sum_{j=1}^{J} \sum_{l=0}^{m_j-1} k^l \sigma_j^k \left(f_{jl} \cos(\psi_j k) + g_{jl} \sin(\psi_j k)\right) \qquad (3.8)$$

Function (3.8) has got the linear appearing coefficients f_{jl} and g_{jl} (whose values depend on its initial conditions) and the nonlinear appearing coefficients σ_j and ψ_j. The values of the „nonlinear" coefficients are the roots of the following algebraical equation (that is characteristic for equation (3.7)):

$$w^R + s_{R-1} w^{R-1} + \ldots + s_1 w + s_0 = (w - \eta_1)^{m_1} \ldots (w - \eta_J)^{m_J} = 0 \qquad (3.9)$$

where $\eta_j = \nu_j + i\varsigma_j$ holds with $\sigma_j = (\nu_j^2 + \varsigma_j^2)^{1/2}$ and $\psi_j = \operatorname{arctg}(\varsigma_j / \nu_j)$.

For the „linear" and „nonlinear" coefficients in functions (3.5 and 3.8) the following formulas:

$$\alpha_j = \frac{\ln \sigma_j}{\Delta t}, \qquad \varphi_j = \frac{\psi_j}{\Delta t}, \qquad c_{jl} = \frac{f_{jl}}{\Delta t^l}, \qquad d_{jl} = \frac{g_{jl}}{\Delta t^l} \qquad (3.10)$$

hold that result by making the comparison of both functions.

Fig. 3.1. The numerical algorithm realizing the indirect identification method on a computer (in 5 steps) is as follows (see Figure 3.1):

1. Fitting the difference equation (3.7) to the impulse response obtained from the object (the model order R and the parameters s_j are here estimated using an optimization method, e.g. the Clarke generalized square method).
2. Calculation of the roots of the equation (3.9) (as a result the „nonlinear" coefficients σ_j and ψ_j in (3.8) are computed).
3. Estimation of the „linear" coefficients f_{jl} and g_{jl} in the time discrete function (3.8) (using the coefficients σ_j and ψ_j and an optimization method, e.g. the linear regression method).
4. Calculation of the „linear" and „nonlinear" parameters c_{jl}, d_{jl} and α_j, φ_j in the time continuous function (3.5) using formulas (3.10).
5. Calculation of the parameters of the transfer function (3.1) (the parameters a_j are obtained from the equation (3.6); the parameters b_j are calculated from the equations (3.4) in which the values of the initial conditions $z(0),...,z^{R-1}(0)$ are to be estimated from equation (3.5)).

The main idea of the indirect identification method is that at first a discrete model is found and afterwards it is converted into the time continuous one. In this way the search for a continuous model is realized „indirectly", i.e. using a discrete model that is much easier to develop from the numerical point of view.

3.2. Modelling approach

In the case of complex objects it is well-adwised to devide the modelling process into several stages at which the submodels with different dynamics are constructed and put together to one overall model afterwords. On each stage the different data sequences must be used for identification and they are to be isolated from the original measurements. The currents distribution occuring in the glass melt (see Figure 1.1) suggests that the features of the melt mixing dynamics in a tank furnace depend in a different way on the character and velocities of the currents. The slow-running withdrawal current decides on the dynamics of the slow-varying inertial character and the fast-running surface current as well as the rotating currents decide on the dynamics of the different-varying oscillatory characters. Also the isotope data for identification display both the inertial and oscillatory characters (see Figure 1.2).

Fig. 3.2. The above remarks justify the formulation of the multistage approach (consisting of 3 stages) for modelling glass tank furnaces. The approach bases on the indirect identification method and it is as follows (see Figures 3.2):

1. Preparation of the data for identification of the slow-varying inertial submodell. The preparation consists of smoothing the original isotope data to eliminate the noise components and the oscillations.
2. Setting up the „slow" inertial submodel using the indirect identification method (stage 1).
3. Preparation of the data for identification of the slow-varying oscillatory submodell. In order to get the data the output of the inertial submodell is substracted from the original noisy data and then the results (that contain the „slow" and „fast" oscillations) are smoothed to eliminate the noise components and the signals of higher frequencies.
4. Setting up the „slow" oscillatory submodell using the indirect identification method (stage 2).
5. Preparation of the data for identification of the fast-varying oscillatory submodell. In order to get the data the output of the „slow" oscillatory submodell is substracted from the noisy data (that contain the „slow" and „fast" oscillations) and then the results are smoothed to eliminate the noise components.
6. Setting-up the „fast" oscillatory submodell using the indirect identification method (stage 3).
7. Putting together all submodells into one overall modell and the subsequent estimation of its „linear" parameters using the linear regression method. To improve the results all parameters of the overall model can be estimated additionally using the nonlinear regression method.

The multistage approach proposed is used next to identificate the glass tank furnace modelled in paragraph 2. The approach is realized in the one-stage, two-stage and three-stage Fig. 3.3. version (see Figure 3.3). The choice of the best („optimal") submodels as well as of the best overall model occurs by means of the residual sums. The best models are chosen from the quantity of the models of different quality which are obtained while using the indirect identification method (stages 1, 2 and 3) or the the linear and nonlinear regression method (step 7). The quality judgement of the rightness of a model is also realized through the visual valuation of the similarity between the model output and the measurements data used for the identification. The greater the similarity the better the object approximation by the model. Also the statistical tests called FPE (Final Prediction Error) and AIC (An Information Criterion) were taken into account to estimate the models quality while using the indirect identification method but they played an auxiliary role only.

The best models obtained as the results of the one-stage or two-stage or three-stage Fig. 3.4. identification (model 1, model 2 and model 3, respectively) are shown in Figure 3.4 and their parameters are shown in Table 3.1. All models fit well to the farther part of the data curve (where the „slow" dynamics of the object dominate) but their adaptation to the initial phase of Tab. 3.1. the curve (where the oscillatory components dominate) is much worse. The slow-varying character of the isotope data is the result of the withdrawal and rotating currents in the glass mass and the oscillations of the data are produced through the surface current. Model 1 of second order has only the inertial character and it can not approximate the oscillations. Model 2 of fourth order (with one oscillatory component of second order added) approximates the first pick of the measurements only but it does not approximate the other numerous picks. Model 3 of sixth order (with two oscillatory second order components added) is the best one. It fits to the first pick as well as to the farther ones but the approximation is not quite exact

especially at the very beginning of the data. The modelling of this data section depends considerably on the division of the whole data sequence into the components which are used for setting up the submodels. This makes the main trouble when using the multistage modelling approach. Since the runs of the data components are not known from the beginning, they can be guessed only in general and the appropriate data curves are obtained using various smoothing algorithms. This leads, however, to great inaccuracies of the proceeding.

4. Combined approach for development of the complex ODE-models

A third approach for modelling glass tank furnaces is presented here (Studzinski, 1992). The final models obtained by means of this approach are described by ordinary differential equations but a distributed parameter model is used at the first level of the modelling either. The conception of the combining modelling resulted from the experience which was gathered after the models with distributed and lumped parameters were developed separately (see paragraphs 2. and 3). In the latter case the modelling of the oscillations appearing in the isotope data (and caused by the surface current) is not exact. The only use of smoothing algorithms does not allow to determine exactly the initial run of the data curve which is used later to set up the „slow" inertial submodel and because of that there is not possible to get the right data for farther stages of the modelling. On the other hand these difficulties can be surmounted by help of the PDE model obtained in paragraph 2. The model allows to isolate correctly the surface current component of the data from the component which is responsible in the main for the glass mass transport in a tank furnace. The latter data component is caused by the withdrawal and rotating currents and it is approximated by the PDE model.

Fig. 4.1. The two-level modelling approach is as follows (see Figures 4.1 and 4.2):

Fig. 4.2.

1. Formulation of the partial differential equations describing the distributed parameter model.
2. Computer simulation of the PDE model.
3. Identification of the PDE model using the isotope data (i.e. estimation of some fittings coefficients by means of an optimization method).
4. Developing of the slow-varying ODE submodel using the output of the PDE model as the data for the indirect identification method.
5. Preparation of the data for setting up the fast-varying ODE submodel using smoothing algorithms (the submodel will describe the contribution of the surface current in the isotope data).
6. Developing of the fast-varying ODE submodel using the indirect identification method.
7. Putting together the submodells into one overall modell and the subsequent estimation of its parameters using the linear and possibly the nonlinear regression methods.

Steps 1 to 3 create the first level of the approach and the remaining steps create the second level.

The complex ODE model of the glass mass flow dynamics was finally set up using the combined approach. The model has the eleventh order and it consists of two submodels of sixth and fifth orders, respectively. The 6th order submodel has the inertial-oscillatory character and owns two real and four complex roots in its transfer function. It fits very well to

Fig. 4.3. Tab. 4.1. the output of the PDE model. The 5th order submodel has the inertial-oscillatory character too and it has one real and four complex roots in its transfer function (see table 4.1). It fits very well to the oscillations caused by the surface current. The overall ODE model fits well to the original measurements and it approximates exactly the oscillations occuring in the initial section of the data (see Figure 4.3).

5. Conclusions

The problem of mathematical modelling of the glass mass flow dynamics in a glass tank furnace is solved and three numerical approaches of modelling are presented, tested and discussed. The first approach (see paragraph 1) develops two-dimensional PDE models that describe the slow-varying dynamics of the glass tanks in which the withdrawal and rotating currents occur and no surface current appears. The second approach (see paragraph 2) allows to develop ODE models of the relativ small order that do not describe exactly the complex dynamics of the objects in which all kinds of the currents occur. The troubles arise while modelling by means of this approach the initial section of the isotope data where the simultaneous effects of the slow and fast varying currents are particulary strong. There is no effective algorithm to divide the data curve into the components for there is not known *a priori* in which way the individual currents influence the meausrements. The third approach is a combination of the former ones and it makes possible to develop complex ODE models of the high order that have got the inertial-oscillator features and very differentiated parameter values. The models computed by help of this approach decribe very well the dynamics of the glass mass flow and all the same they are simple and convenient enough for numerical treatment. They can be used for the development of control or stabilization algorithms with reference to the chemical composition of the glass (Studzinski and Straubel, 1991b) as well as for the calculation of the technological parameters of glass tank furnaces (Studzinski et al., 1991).

The essential properties of the third approach which decide about its efficiency are:

- The division of the modelling procedure into some stages on which the ODE submodels with different frequency characterisctics (the „slow" and „fast" submodels) are set up.
- The use of a PDE model for isolating the data components which are used to develop the submodels.
- The use of the indirect identification method for setting up the individual submodels. This method allows to get the continuous-time models whereas the most calculations are made using the discrete-time models.
- The use of the nonlinear regression method for the final estimation of the overall model parameters. This method works right while performing the calculation for the structure of the model and the start points for the estimation are determined well at the former steps of the modelling.

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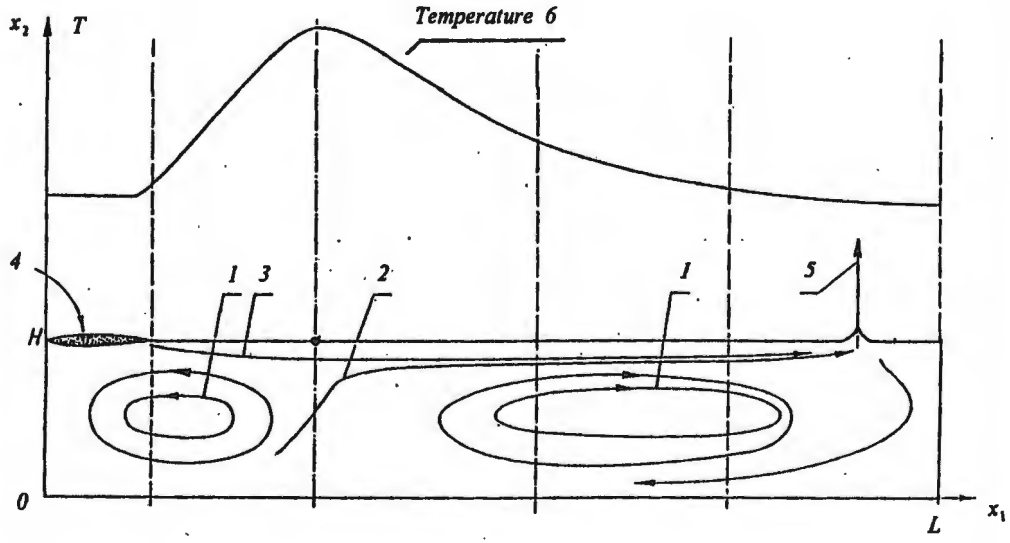


Figure 1.1.

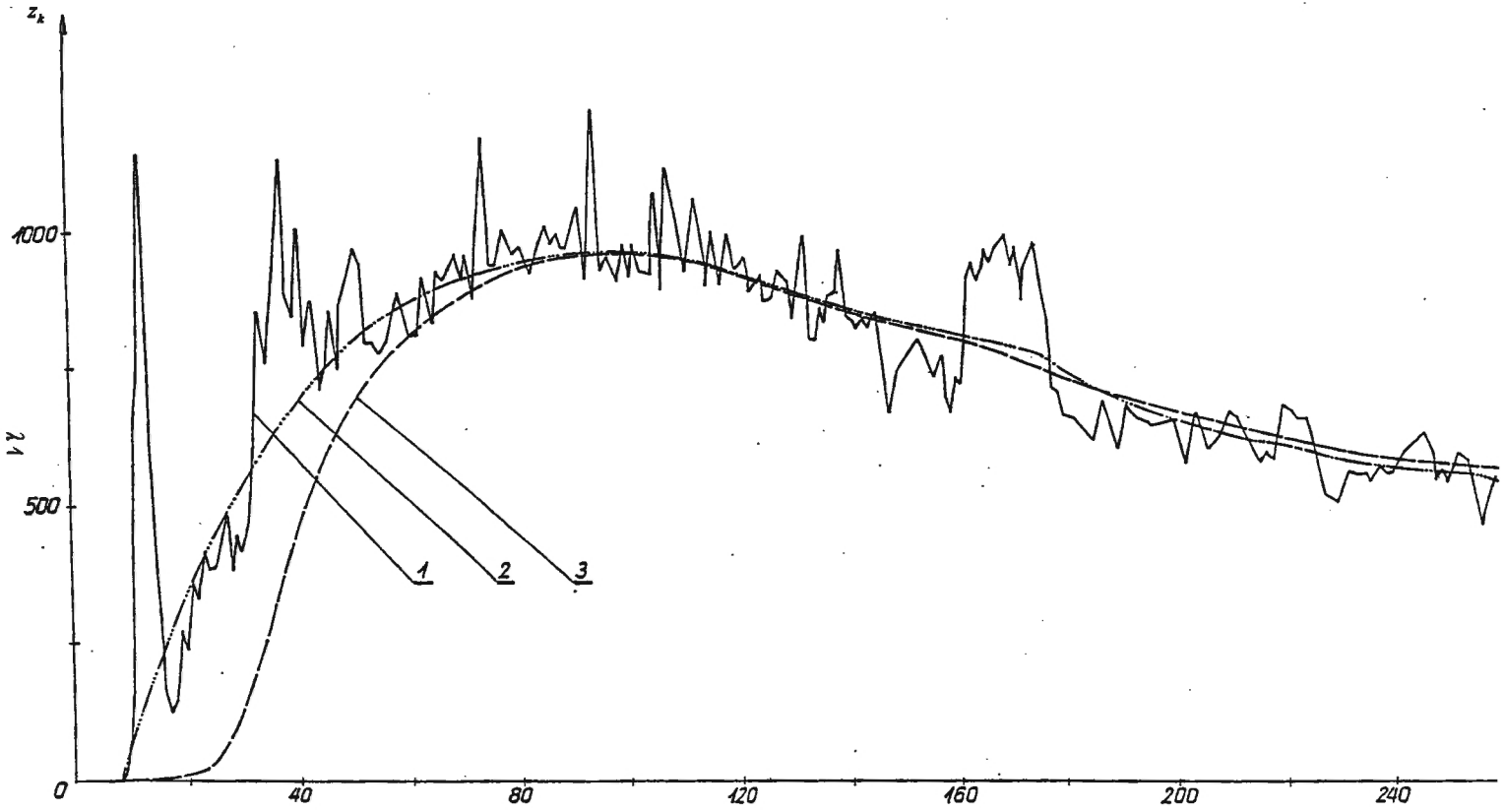


Figure 1.2.

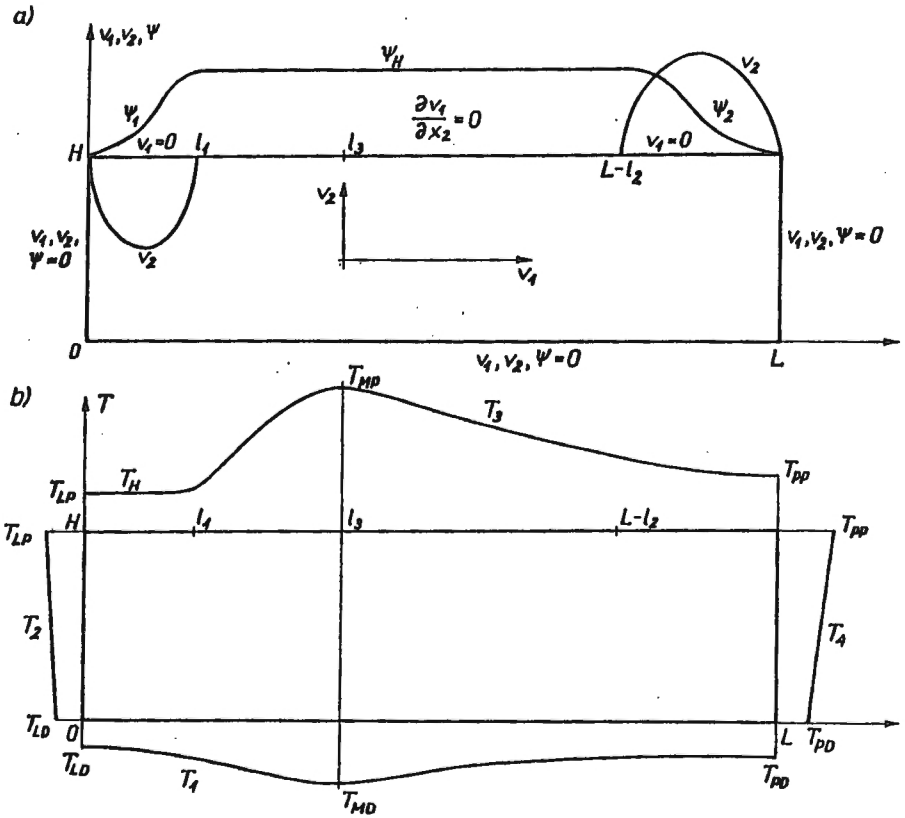

a)
v_1, v_2, ψ
ψ_H
ψ_1
ψ_2
v_2
$\frac{\partial v_1}{\partial x_2} = 0$
H
$v_1 = 0$
l_1
l_3
$v_1 = 0$
$L - l_2$
v_2
v_1
$v_1, v_2, \psi = 0$
$v_1, v_2, \psi = 0$
v_2
0
$v_1, v_2, \psi = 0$
L
b)
T
T_{MP}
T_3
T_{LP}
T_H
T_{PP}
T_{LP}
H
l_1
l_3
$L - l_2$
T_{PP}
T_2
T_4
T_{LD}
0
L
T_{PD}
T_{LD}
T_1
T_{PD}
T_{MD}

Figure 2.1.

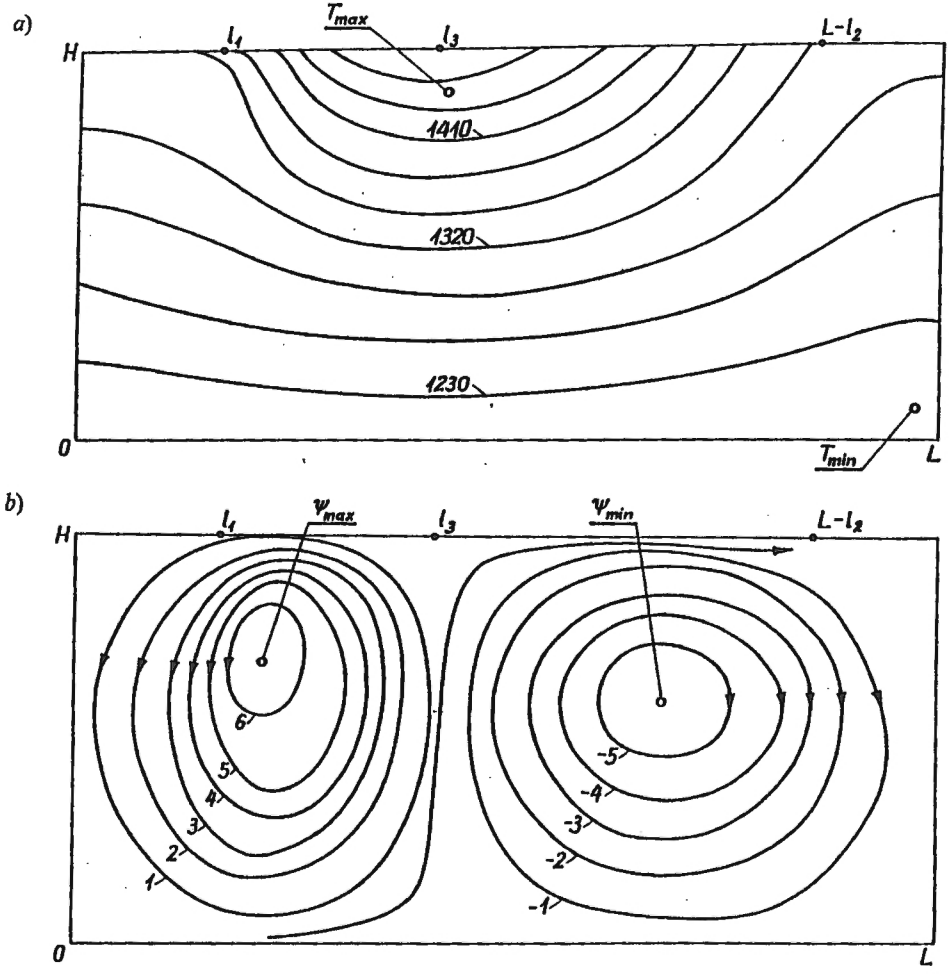

a)
H
l_1
T_{max}
l_3
$L-l_2$
1410
1320
1230
0
T_{min}
L
b)
H
l_1
ψ_{max}
l_3
ψ_{min}
$L-l_2$
6
5
4
3
2
1
-5
-4
-3
-2
-1
0
L

Figure 2.2.

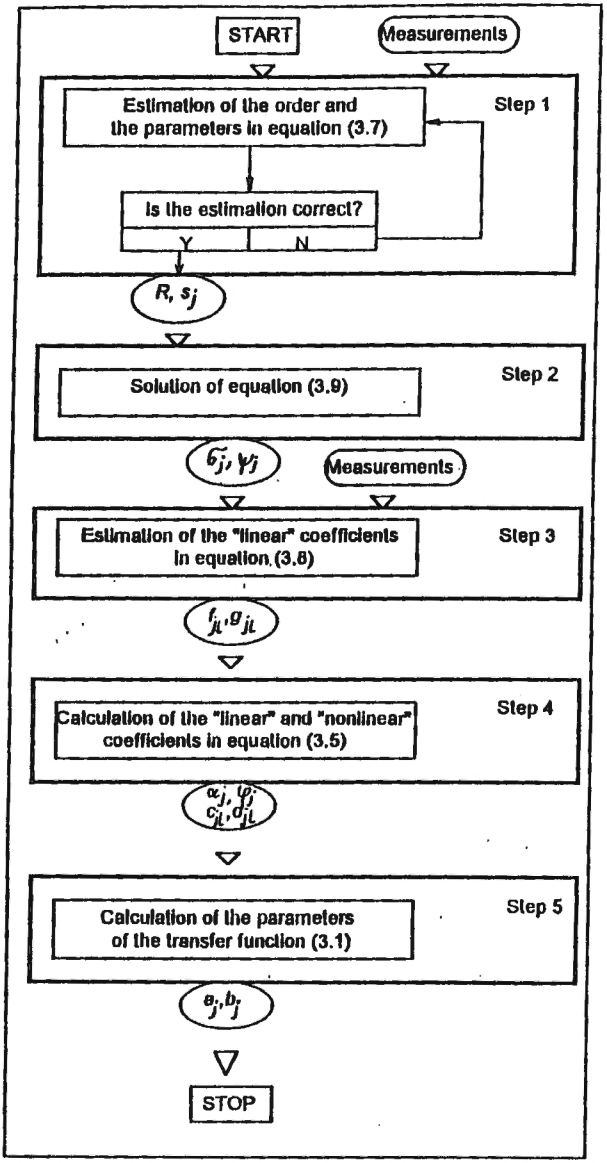

START
Measurements
Estimation of the order and the parameters in equation (3.7)
Step 1
Is the estimation correct?
Y
N
R, s_j
Solution of equation (3.9)
Step 2
σ_j, ψ_j
Measurements
Estimation of the "linear" coefficients in equation (3.8)
Step 3
f_{jl}, g_{jl}
Calculation of the "linear" and "nonlinear" coefficients in equation (3.5)
Step 4
α_j, β_j
c_{jl}, d_{jl}
Calculation of the parameters of the transfer function (3.1)
Step 5
a_j, b_j
STOP

Figure 3.1.

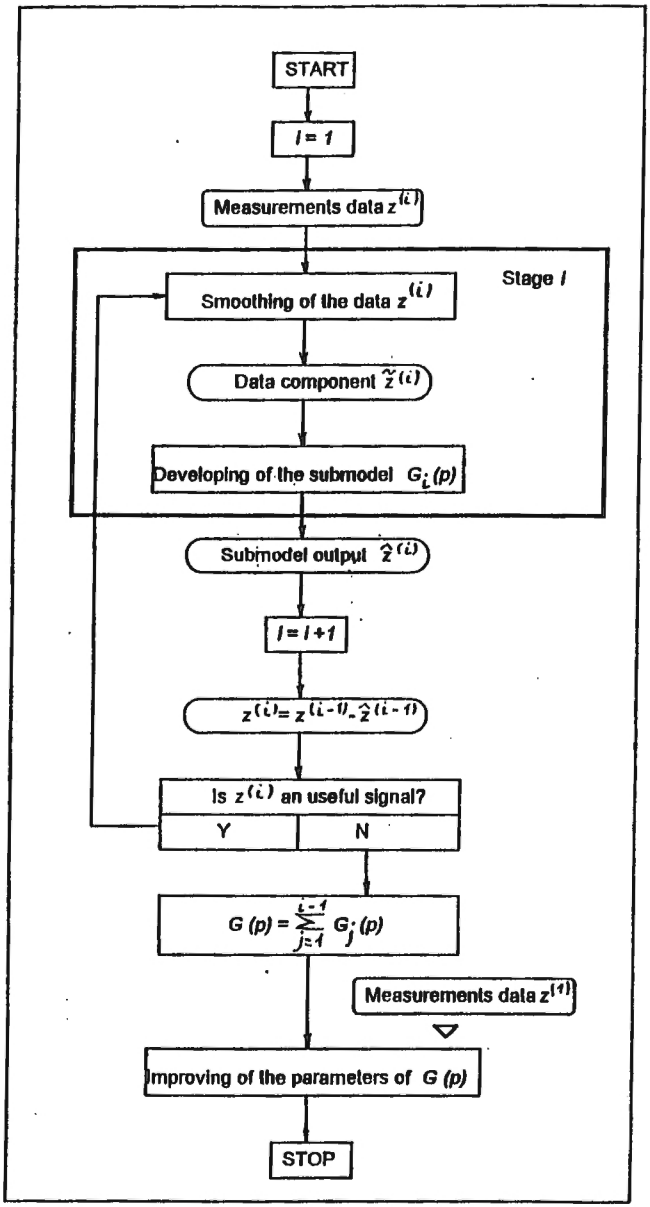

START
l = 1
Measurements data $z^{(i)}$
Stage l
Smoothing of the data $z^{(i)}$
Data component $\tilde{z}^{(i)}$
Developing of the submodel $G_i(p)$
Submodel output $\hat{z}^{(i)}$
l = l + 1
$z^{(i)} = z^{(i-1)} - \hat{z}^{(i-1)}$
Is $z^{(i)}$ an useful signal?
Y
N
$G(p) = \sum_{j=1}^{i-1} G_j(p)$
Measurements data $z^{(1)}$
Improving of the parameters of G (p)
STOP

Figure 3.2.

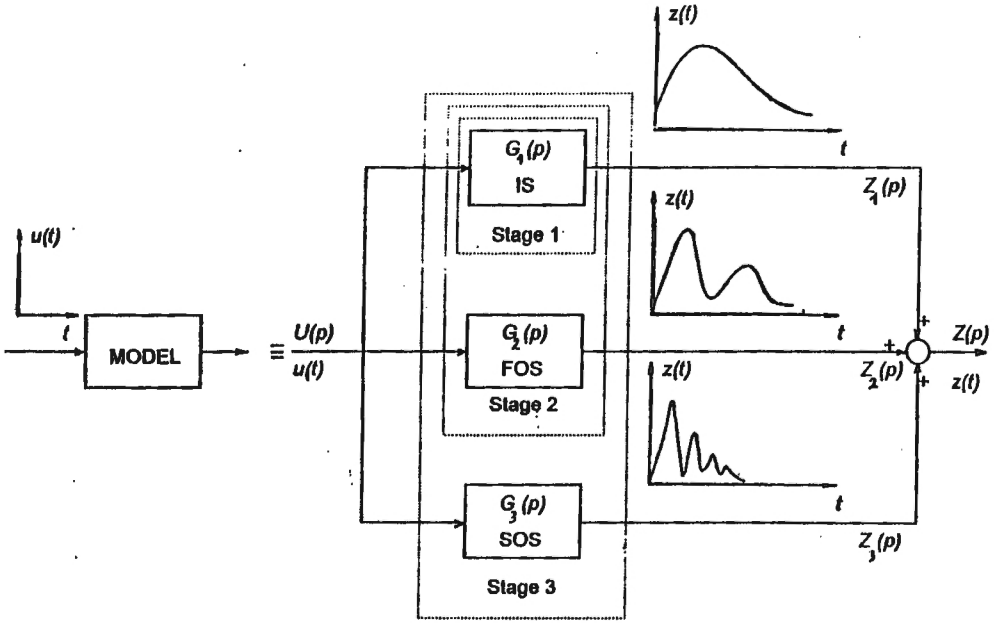

u(t)
t
MODEL
≡
U(p)
u(t)
G1(p)
IS
Stage 1
G2(p)
FOS
Stage 2
G3(p)
SOS
Stage 3
z(t)
t
Z1(p)
Z2(p)
Z3(p)
Z(p)
z(t)

Figure 3.3.

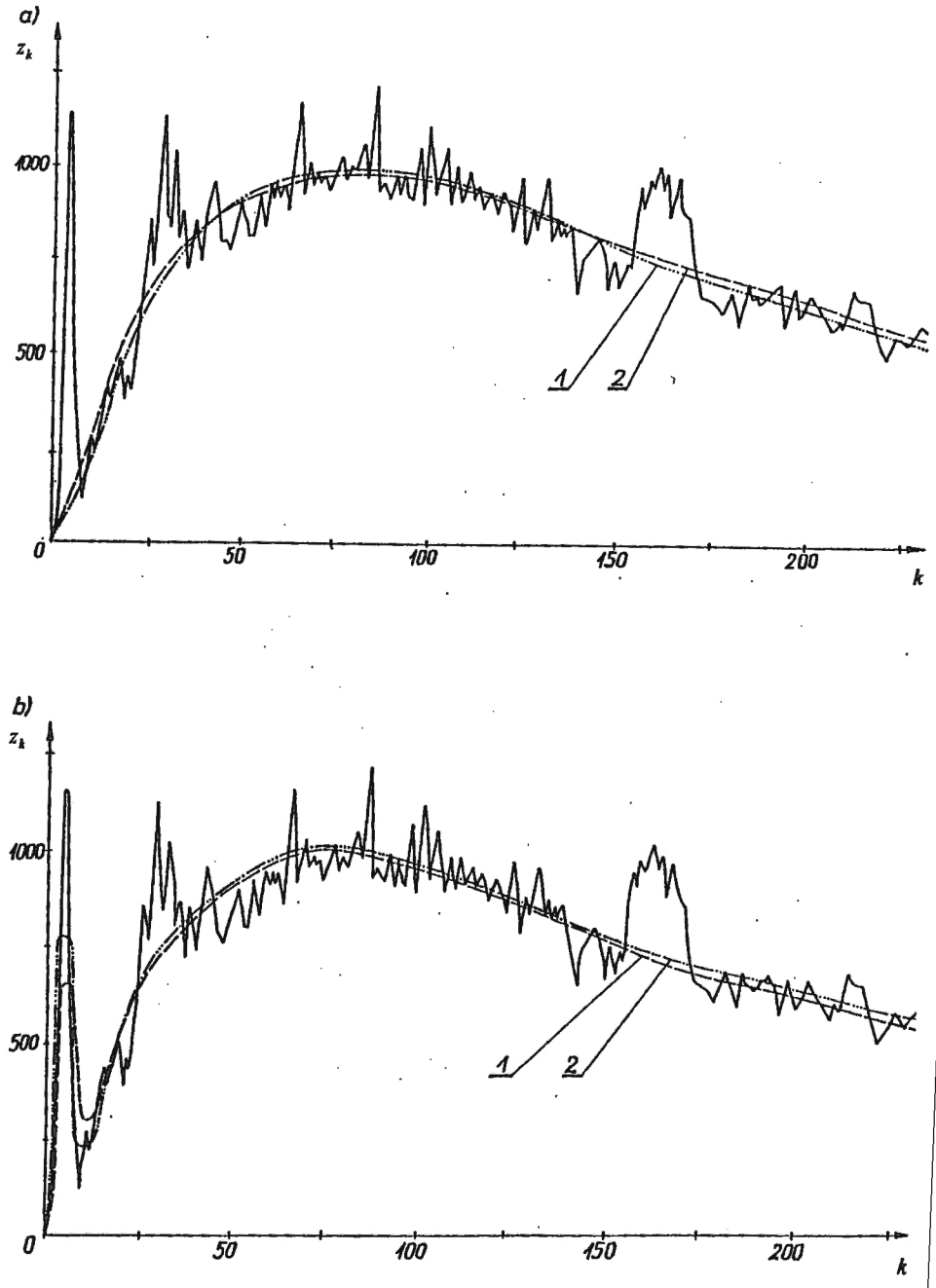

a)
z_k
1000
500
0
50
100
150
200
k
1
2
b)
z_k
1000
500
0
50
100
150
200
k
1
2

Figure 3.4. a), b).

c)

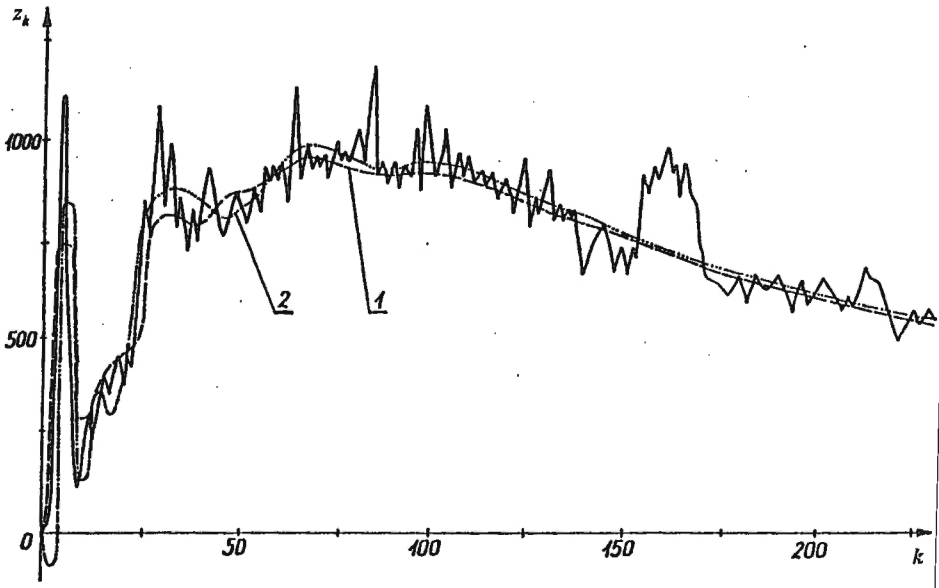
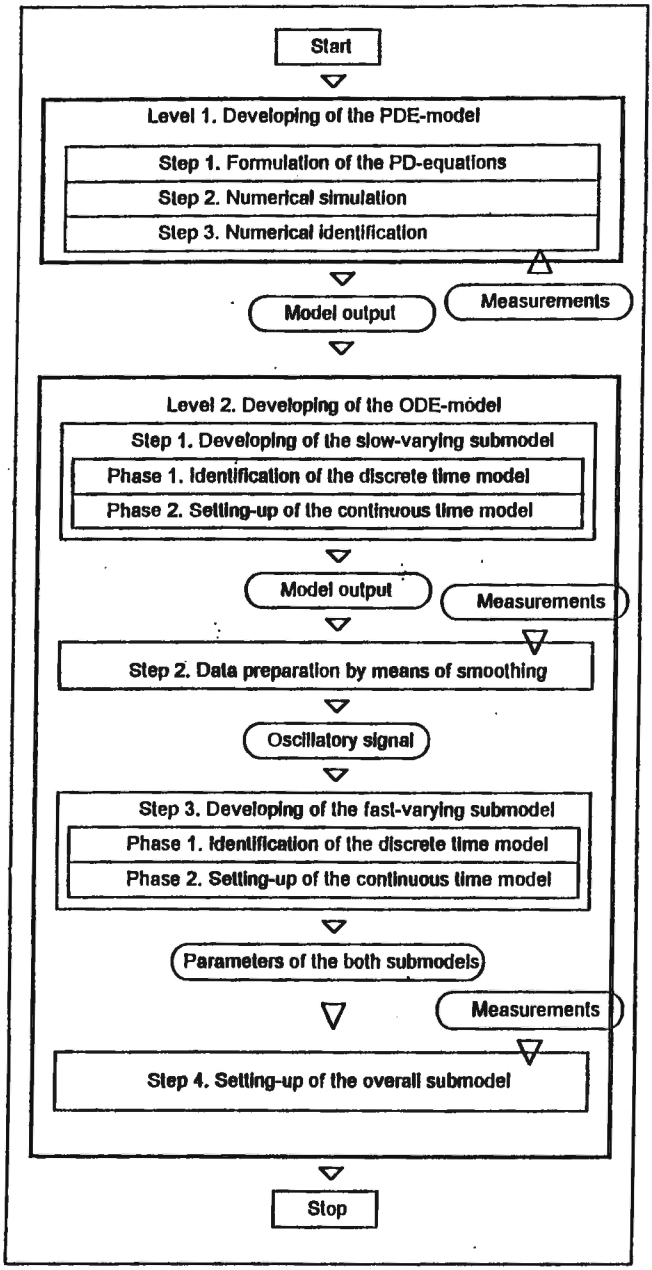


Figure 3.4. c).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Level 1. Developing of the PDE-model
Step 1. Formulation of the PD-equations
Step 2. Numerical simulation
Step 3. Numerical identification
Model output
Measurements
Level 2. Developing of the ODE-model
Step 1. Developing of the slow-varying submodel
Phase 1. Identification of the discrete time model
Phase 2. Setting-up of the continuous time model
Model output
Measurements
Step 2. Data preparation by means of smoothing
Oscillatory signal
Step 3. Developing of the fast-varying submodel
Phase 1. Identification of the discrete time model
Phase 2. Setting-up of the continuous time model
Parameters of the both submodels
Measurements
Step 4. Setting-up of the overall submodel
Stop

Figure 4.1.

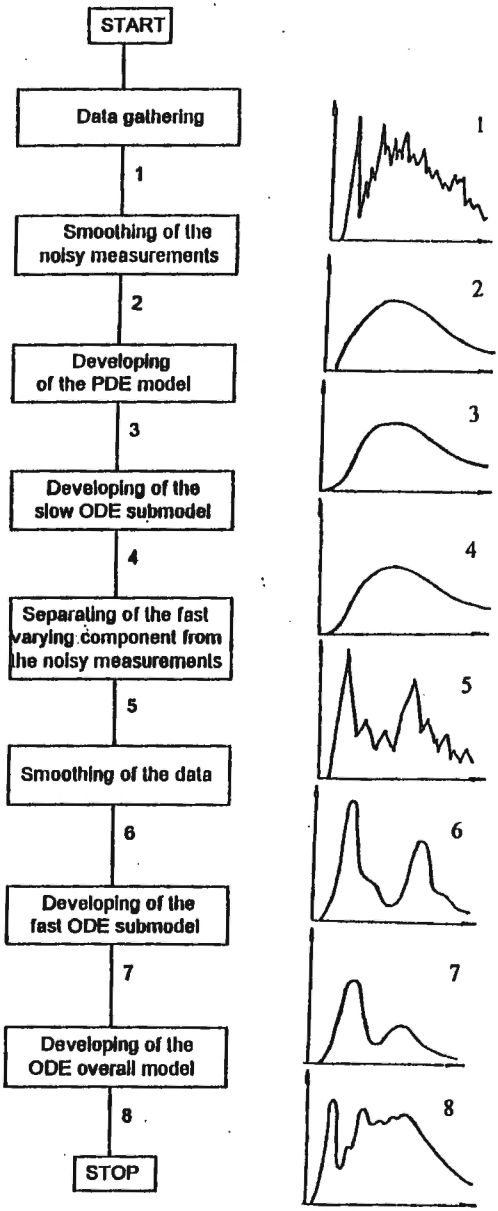
START
Data gathering
1
Smoothing of the noisy measurements
2
Developing of the PDE model
3
Developing of the slow ODE submodel
4
Separating of the fast varying component from the noisy measurements
5
Smoothing of the data
6
Developing of the fast ODE submodel
7
Developing of the ODE overall model
8
STOP
1
2
3
4
5
6
7
8

Figure 4.2.

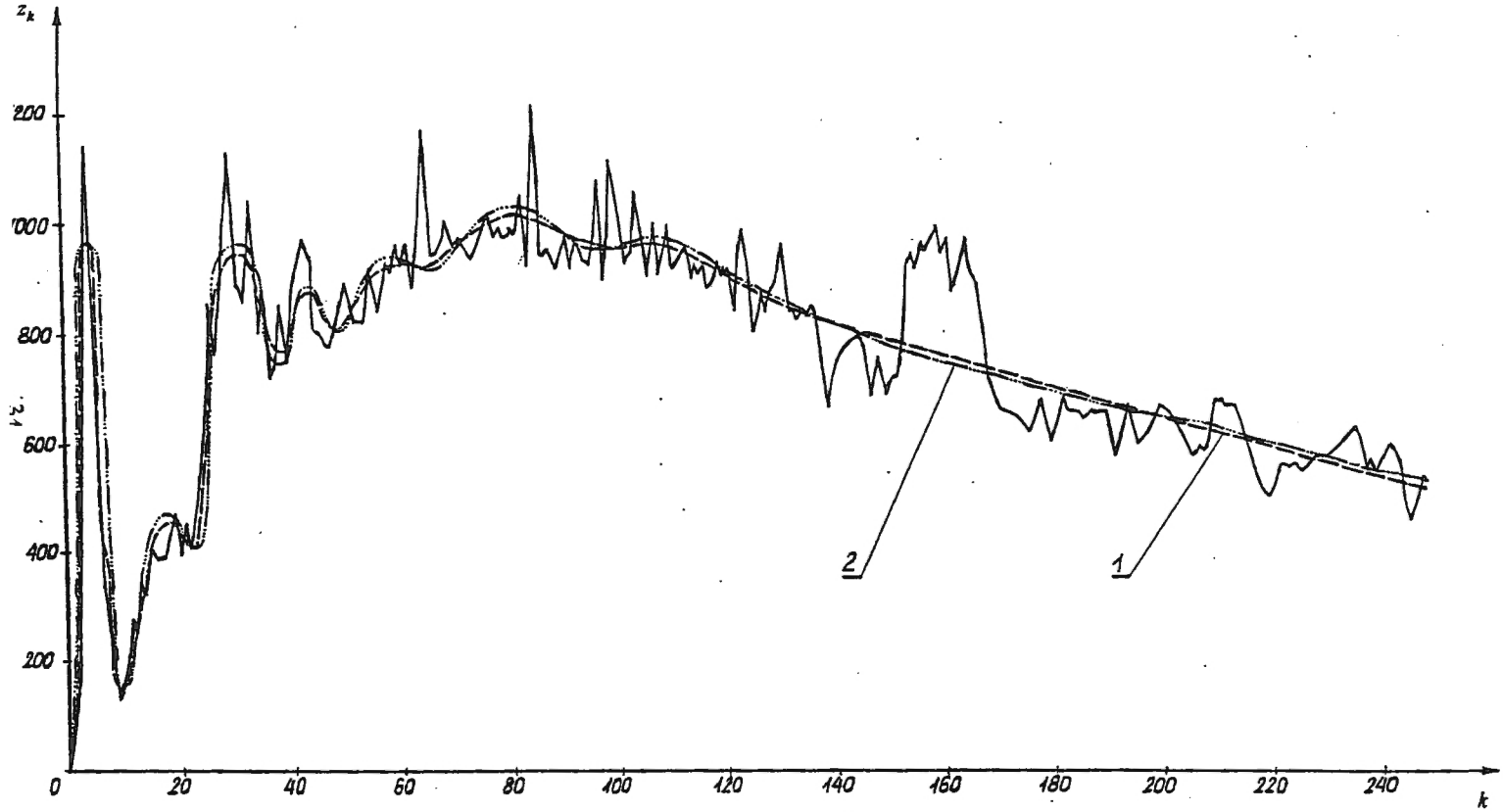
z_k
200
1000
800
600
400
200
0
20
40
60
80
100
120
140
160
180
200
220
240
k
2
1

Figure 4.3.

Tab. 3.1.

Model	R	T_1	T_2	k_1	ω_1	k_2	ω_2	$S^{1/2}$
LR								
1	2	188.68	41.15					119.38
2	4	188.68	41.15	7.39	19.51			100.96
③	6	188.68	41.15	7.39	19.51	24.45	13.79	97.51
NR								
①	2	217.39	33.67					116.38
②	4	222.22	31.35	4.75	17.24			96.57
3	6	185.19	45.87	23.81	33.51	5.06	8.11	83.26

Tab. 4.1.

Parameters	$R=6$	$R=5$	$R=11$
T_1	263.16	32.26	263.16
T_2	14.88		21.19
k_1	11.27	31.95	19.88
ω_1	237.99	27.83	33.39
k_2	1.68	3.77	11.48
ω_2	191.56	7.39	22.51
T_3			1.32
k_3			14.68
ω_3			13.45
k_4			3.29
ω_4			4.58