USING PRIMAL-DUAL INTERIOR POINT METHOD TO DETERMINE LEAST-WEIGHT TRUSS LAYOUTS

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1. Abstract

We are concerned with finding the least-weight truss layouts and consider two variations of such problems: a basic one which is cast as a potentially very large scale linear programming problem and a more elaborate one which involves stability constraints and therefore leads to a semidefinite programming problem. We address computational optimization techniques which allow to solve both problems in an efficient way. We demonstrate that a specialized primal-dual interior point methods handle large instances of such problems and, in acceptable time, deliver accurate solutions to practical layout optimization problems.

2. The problems

Let a_i, $i = 1, ..., n$ denote the cross-sectional areas of the member bars and let $m(\approx Nd$, where N is the dimension of the design domain and d is the number of nodes in the design domain) be the number of the non-fixed degrees of freedom, $f_\ell \in \mathcal{R}^m, \ell \in \{1, \dots, n_L\}$ be a set of external forces applied to the structure, and $q_\ell \in \mathcal{R}^n$ be the associated tensile and compressive forces of the bars. Then, the classical multiple-load case least-weight truss layout optimization problem can be cast as the following linear program

$$
\begin{aligned}
\underset{a,q_\ell}{\text{minimize}} \quad & l^T a \\
\text{subject to} \quad & Bq_\ell = f_\ell, && \ell = 1, \cdots, n_L \\
& -\sigma^- a \leq q_\ell \leq \sigma^+ a, && \ell = 1, \cdots, n_L \\
& a \geq 0,
\end{aligned}
\tag{1}
$$

where $l \in \mathcal{R}^n$ is a vector of bar lengths, and $\sigma^- > 0$ and $\sigma^+ > 0$ are the material's yield stresses in compression and tension, respectively. For each load case $\ell = 1, \cdots, n_L$, the equation $Bq_\ell = f_\ell$ states the nodal equilibrium with $B \in \mathcal{R}^{m\times n}$ the reduced geometry matrix.

Next, we consider the truss problem with global stability constraints based on linear buckling. This leads to the following semidefinite programming formulation

$$
\begin{aligned}
\underset{a,q}{\text{minimize}} \quad & l^T a \\
\text{subject to} \quad & Bq = f \\
& -\sigma^- a \leq q \leq \sigma^+ a \\
& K(a) + \tau G(q) \succeq 0 \\
& a \geq 0,
\end{aligned}
\tag{2}
$$

where the stiffness matrix K and the geometry stiffness matrix G are given by

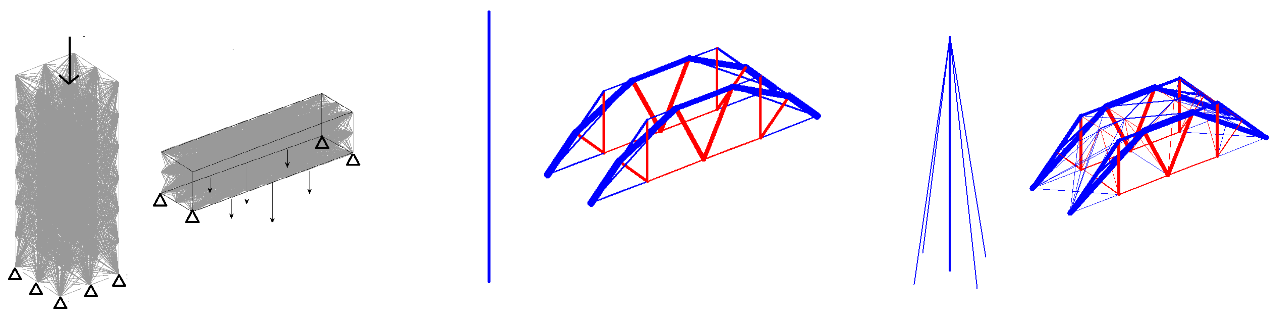
$$K(a) = \sum_{j=1}^{n} a_j K_j, \text{ with } K_j = \frac{E_j}{l_j}\gamma_j\gamma_j^T \quad \text{and} \quad G(q) = \sum_{j=1}^{n} q_j G_j, \text{ with } G_j = \frac{1}{l_j}(\delta_j\delta_j^T + \eta_i\eta_j^T).$$

E denotes the Young's modulus and directions $(\delta_j, \gamma_j, \eta_j)$ are mutually orthogonal ($\eta = 0$ for 2D problems).

3. The method and example applications

The challenge when solving both these problems originates from very large number of bars and consequently very large dimensions (number of variables and constraints) in the resulting optimization problems. Therefore to tackle the problems we apply interior point methods [2] which are known to excel on large scale problems. In both cases, we incorporate the *member adding technique* [1] which translates the column generation principle [3] into engineering applications. Namely, we start with a structure constituting a minimum connectivity and gradually append those missing bars which violate dual feasibility constraints. The member adding process terminates when there are no such bars. The procedure allows to keep the size of underlying optimization problems small enough to reach optimal solutions of otherwise untractable design instances. Additionally, we specialize the linear algebra solver inside the interior point algorithm to take into account particular graph structure of the reduced geometry matrix. The linear system is reduced to a significantly smaller one and solved with a preconditioned conjugate gradient method. A specially designed preconditioner exploits the sparsity pattern and particular numerical properties of the reduced geometry matrix. The preconditioner improves the clustering of eigenvalues in the linear system and delivers fast convergence of the conjugate gradients method.

The application of the method to the multiple-load case truss layout optimization problems (1) is described in detail in [4] and its application is discussed in [5]. An extension to problems with global stability constraints based on linear buckling (2) is now being developed.



Domains and loads No stability constraints With stability constraints

We illustrate our findings on two small 3D examples presented in a figure above: a rectangular three-dimensional body clamped at its lower surface with a single load applied on top of it and a bridge with loads distributed at two edges on the bottom of it. Formulations (1) without stability constraints produce a solution of a single bar without any bracing for the clamped body and two independent planar trusses for the bridge. When stability constraints (2) are added, an extra bracing for the single bar of the clamped body appears and extra connections between planar trusses of the bridge are present.

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References

[1] Matthew Gilbert and Andrew Tyas. Layout optimization of large-scale pin-jointed frames. *Engineering Computations*, 20(8):1044–1064, 2003.
[2] J. Gondzio. Interior point methods 25 years later. *European Journal of Operational Research*, 218(3):587–601, 2012.
[3] J. Gondzio, P. González-Brevis, and P. Munari. New developments in the primal-dual column generation technique. *European Journal of Operational Research*, 224(1):41–51, 2013.
[4] A. Weldeyesus and J. Gondzio. A specialized primal-dual interior point method for truss layout optimization problems. Technical Report ERGO 17-012, School of Mathematics, University of Edinburgh, November 2017.
[5] J. Ye, P. Shepherd, L. He, M. Gilbert, A. Tyas B. Davison, J. Gondzio, A. G. Weldeyesus, and H. Fairclough. Computational layout design optimization of frame structures. In *Proceedings of the IASS Annual Symposium 2017*.