MICHELL CANTILEVER ON CIRCULAR SUPPORT FOR UNEQUAL PERMISSIBLE STRESSES IN TENSION AND COMPRESSION

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1. Introduction

In his pioneering work, Michell [1] derived optimality criteria of least-volume trusses and showed that the optimal truss is immersed in an appropriate field of virtual displacements. Michell started his investigation from Maxwell theorem [2] and deduced that the layout of the optimal truss can be determined by solving the simplified problem with equal permissible stresses in tension and compression $\sigma_T = \sigma_C$. This conclusion, however, is valid only for Maxwell class of problems, when all external loads together with reaction forces, if any, are known and fixed. This restriction of truss theory proposed originally by Michell was noticed by Prager [3] and Hemp [5,6], who independently derived more general optimality criteria for trusses with different properties of material in tension and compression and supported in any way (including the case of unknown reaction forces). This topic was studied later by Rozvany [7], who has shown that for unequal permissible stresses Michell's optimality criteria are valid only for a highly restricted class of support conditions. Rozvany indicated that Michell's solution of his very first example, involving a point load and a circular support seems to be incorrect for $\sigma_T \neq \sigma_C$ and would need to be revised. The aim of the present paper is to provide an exact analytical solution to this particular problem.

2. Some preliminary analytical and numerical results

The considered problem consists in finding the lightest truss (more precisely Michell structure) transmitting a point load to a circular support, see Fig. 1. Here we assume that: i) the force is applied outside the circle ($r > r_0$) and is oriented circumferentially, ii) the design domain is the exterior of the circle, iii) the maximum allowable tensile and compressive stresses in a structure are σ_T and σ_C, respectively.

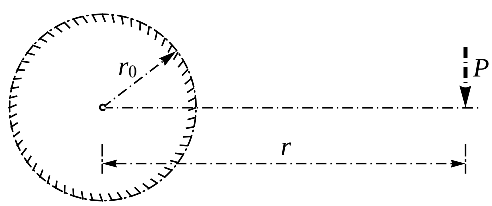

Figure 1. The problem setting.

The solution derived by Michell [1] is shown in Fig. 2a, and leads to the volume

$$V = P\,r\left(\frac{1}{\sigma_T} + \frac{1}{\sigma_C}\right)\log\frac{r}{r_0} \tag{1}$$

The layout consists of logarithmic spirals with a constant angle 45° between spirals and rays (or circles). It cannot satisfy kinematic boundary conditions derived by Hemp [6] for $\sigma_T \neq \sigma_C$. For example, if $\sigma_T = 3\sigma_C$ then the bars should intersect the supporting circle with angles 30° and 60° for tensile and compressive bars, respectively. This condition is satisfied in the improved solution proposed by Dewhurst and Srithongchai [8], see Fig. 2b. Here the layout consists again of logarithmic spirals but with different inclination angles of bars in tension and compression. It is interesting that the volume derived in [9] is identical with formula (1).

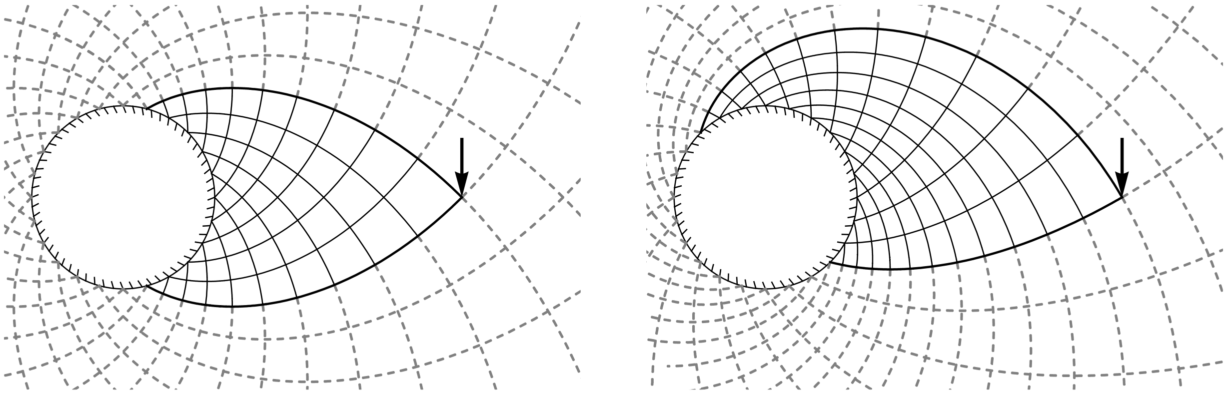

Figure 2. Suboptimal layouts of Michell structure for $\sigma_T/\sigma_C = 3$; solutions provided by a) Michell [1] and b) Dewhurst and Srithongchai [8].

Numerical tests performed with using adaptive ground structure method (see [9]) clearly indicate that both solutions of Fig. 2 are not optimal. A selected numerical solution for $r = 4r_0$, $\sigma_T = 2\sigma_0$, and $\sigma_C = (2/3)\sigma_0$ is presented in Fig. 3. It can be observed that the optimal layout is more complex because the inclination angles of bars are not constant and depend of the radius r. Moreover, the volume of the structure presented in Fig. 3 is smaller than the volume given by the formula (1). For assumed data the volume defined in (1) is equal to $8\log 4\, V_0 \approx 11.090354\, V_0$, where $V_0 = Pr_0/\sigma_0$, while the volume of numerical solution is equal to $10.8405\, V_0$, which is about 2% smaller. It means that formula (1) is wrong because the discretized truss approximation cannot be better than the exact analytical solution including infinite number of bars.

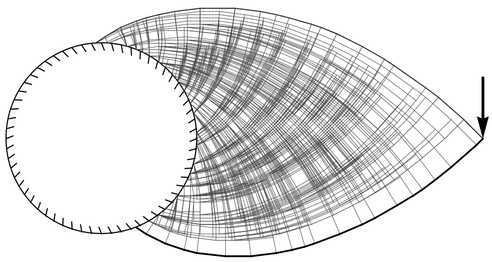

Figure 3. Numerical solution of Michell cantilever for $\sigma_T/\sigma_C = 3$; $r = 4r_0$.

3. Final remarks

The analytical solution of the considered problem is still not known but authors believe they can find and present it during the conference and in a full-length paper.

References

[1] A.G.M. Michell. The limit of economy of material in frame structures. *Philosophical Magazine*, 8(47):589, 1904.
[2] C. Maxwell. On Reciprocal Figures, Frames, and Diagrams of Forces. *Edinb Roy Soc Proc* 7:160, 1870.
[3] W. Prager. A problem of optimal design. *Proc. of the Union of Theoretical and Applied Mechanics*, Warsaw, 1958.
[5] W.S. Hemp. *Theory of the structural design. Report No. 115*. The College of Aeronautics, Cranfield, August 1958.
[6] W.S. Hemp. *Optimum structures*. Clarendon Press, Oxford, 1973.
[7] G.I.N. Rozvany. Some shortcomings in Michell's truss theory. *Structural Optimization*, 12:244, 1996.
[8] P. Dewhurst and S. Srithongchai. An Investigaton of Minimum-Weight Dual-Material Symmetrically Loaded Wheels and Torsion Arms. *ASME. J. Appl. Mech.*, 72(2):196, 2005.
[9] T. Sokół. On the Numerical Approximation of Michell Trusses and the Improved Ground Structure Method. In: Schumacher A., Vietor T., Fiebig S., Bletzinger KU., Maute K. (eds) *Advances in Structural and Multidisciplinary Optimization. WCSMO 2017*. pp. 1411-1417, Springer, 2018.