LV

MEMORANDUM RESPECTING A NEW SYSTEM OF ROOTS OF UNITY

[*Phil. Mag.* vol. XII (1856), p. 446.]

I have lately been led to the conception of a new system, or rather *family of systems*, of *non-commutative roots of unity*, which are entirely distinct from the *ijk* of the quaternions, though having some general analogy thereto; and which admit, even more easily than the quaternion symbols do, of *geometrical interpretation*. In the system which seems at present to be the most interesting one, among those included in this new family, I assume three symbols, ι, κ, λ, such that

$$\iota^2 = 1, \quad \kappa^3 = 1, \quad \lambda^5 = 1, \quad \lambda = \iota\kappa; \tag{A}$$

where $\iota\kappa$ must be *distinguished* from $\kappa\iota$, since otherwise we should have $\lambda^6 = 1$, $\lambda = 1$. As a very simple *specimen* of the symbolical conclusions deduced from these fundamental assumptions, I may mention that if we make

$$\mu = \iota\kappa^2 = \lambda\iota\lambda,$$

we shall have also*

$$\mu^5 = 1, \quad \lambda = \mu\iota\mu;$$

so that μ is a new fifth root of unity, connected with the former fifth root λ by relations of perfect reciprocity. A long train of such symbolical deductions is found to follow: and every one of the results may be *interpreted*, as having reference to the passage from *face to face* (or from corner to corner) of the *icosahedron* (or of the dodecahedron): on which account, I am at present disposed to give the name of the 'Icosian Calculus,' to this new system of symbols, and of rules for their operation. Some additional remarks on this subject may soon be offered to the *Philosophical Magazine*, under the title, already sanctioned by the Editors, of 'Extensions of the Quaternions.'†

* In fact, by (A),

$$\iota\kappa = (\iota\kappa)^{-4} = (\kappa^{-1}\iota^{-1})^4 = (\kappa^2\iota)^4, \quad 1 = \iota . \iota\kappa . \kappa^2 = \iota(\kappa^2\iota)^4\kappa^2 = (\iota\kappa^2)^5;$$

also

$$\mu\iota\mu = \mu\kappa^2 = \iota\kappa^4 = \iota\kappa = \lambda.$$

† [Not to be confused with XIII. See postscript in LVII.]