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A PRESENT-DAY PROBLEMS IN IDENTIFICATION OF DYNAMIC SYSTEMS - A REVIEW

1. Introduction

In a present time the literature devoted to identification problems is very extensive. There are several papers and monographs concerned with system identification problems. A multitude of papers reported at different scientific meetings have been connected with identification of different systems. The identification problems of automatic control systems have been discussed in a hundreds of papers. But in the recent years more and more works are devoted to other systems: economic, biological, mechanical etc. It seems that the apperance and spreading of powerful electronic computers is one of the reasons of these trends.

This paper reviews the problems connected with identification procedures which can be treated as a step in model building theory.

2. Basic approaches to the identification problem

A mathematical model is a representation, in a convenient form, of the essential aspects of an existing

(or to be constructed) system.

A model building process consists of the following steps:

1. Selection of a model structure based on physical behaviour of the system,
2. Fitting of parameters to available data (parameter estimation problem),
3. Verification and testing of the model.

In the vast majority of cases the model building process is based on " a priori" knowledge provided that the structure of the system is available and is expressed in terms of mathematical dependences. In this restricted case the identification procedure is reduced to the experimental determination of parameter values ("parametric identification" or "Parameter estimation").

The fact that identification procedures are based on measurements is conformable with warning given by Lord Kelvin to the physicists: "If you are talking about things you cannot measure, you don't know what are you talking about".

Fig.1. gives an illustration of the relation between measurements and the model building process.

It is a very important feature that identification procedures give us only mathematical approximation of the physical reality. Information about real system are obtained from input/output data.

Fig.2. gives an illustration of the relation between measurement and modelling processes but otherwise than Fig. 1.

The parameter values are not directly observable in the vast majority of cases. Therefore many identification methods are based on output error concept, where the output error is defined as the difference between output signal of the system and output signal of the model when the system and the model are excited by the same input signal.

Parameter estimation may be based on active experiments (harmonic input) or passive observation of the system during exploatation conditions (stochastic input).

The physical properties of a real dynamical system lead us to nonlinear continuous model and next by the other reduction to a linear model represented most frequently by the system of ordinary differential equations. It is clear that mathematical model can be represented by another forms e.g. algebraic equations, difference equations, partial differential or difference equations, differential-difference equations, integral equations and so on - according to areas of applications.

Fig.3. gives a number of relations between different types of characteristics used for description of

a linear model when the input is continuous deterministic or stochastic signal.

3. Mathematical models representations

We are concerned here with dynamic systems which can be described by ordinary differential equations.

A well known, general model has the following state-space system equation:

$$\dot{\underline{x}} = \underline{g}(\underline{x},\underline{u},\underline{a},\underline{n},t) \quad \underline{x}(0) = \underline{x}_0 \qquad (1)$$

where $\underline{x}(t)$ is n dimensional vector of state variables characterising the system dynamic behaviour; $\underline{u}(t)$ is m dimensional vector of input variables which will normally be assumed to be measured exactly; $\underline{a}(t)$ represents r dimensional set of unknown parameters to be determined; $\underline{n}(t)$ is l dimensional vector of unmeasurable input disturbances that affect the system. The model is commonly obtained from physical laws thus the vector function $\underline{g}(.)$ is, in general case, nonlinear and time depended.

In the control theory the system state-equation (1) is completed by an observation (or output) equation (observation device - see Fig.4):

$$\underline{y} = \underline{h}(\underline{x},\underline{u},\underline{m},t) \qquad (2)$$

where $\underline{y}(t)$ is p dimensional vector of observed output of the system and $\underline{m}(t)$ is k dimensional

vector of output disturbances.

It is usually assumed that $\underline{n}(t)$ and $\underline{m}(t)$ disturbances are independent white Gaussian noise processes.

The vector function $\underline{h}(.)$ is in general case nonlinear and time depended.

If the r dimensional parameter vector $\underline{a}(t)$ is time depended the system is nonstationary. The elements of this vector may often be assumed constant i.e. $\dot{\underline{a}} = \underline{0}$ or slowly variable. In this case the system is stationary.

Unknown parameter vector $\underline{a}(t)$ may be of stochastic nature. Application of stochastic functions is provided by the observation of the behaviour of physical phenomena and trend , frequently occurence in recent years, to rely more and more heavily on probabilistic models. One reason of this trend is the desire of the part of investigators to have mathematical models which more accurately describe tha physical processes under study. In the end case we obtain the nonlinear, nonstationary and stochastic model which is very difficult for identification.

One of the most difficult problem in identification of nonlinear systems is the adequacy of used model to real nonlinear system. It is connected with divergence between the model structure given by the function $g(.)$ and the structure of a real nonlinear system. Further difficulties are connected with

the "local behaviour" and "global behaviour" of the system. Input signals of large amplitude would excite global behaviour which may be quite different from the local behaviour. The local behaviour of nonlinear system may have suddenly changes which mathematically corresponds to the loss of structural stability. There are other special phenomena like hysteresis, limit cycles and another behaviour which are not possible in linear system. Therefore identification procedures for nonlinear systems are in general more complex than those used for linear systens. So the linearized models are commonly used.

The problem of validity of linearization is to be considered in this case. This important question is answered by the Center Manifold Theorem [21] which says that the nonlinear behaviour is exhibited in terms of those states (more precisely, eigenvectors) which have eigenvalues with zero real parts associated with them.

Mechanical systems are frequently described by a mathematical model, called sometimes "equations of motion", in the form:

$$M\ddot{q} + C\dot{q} + Kq = Bf \qquad (3)$$

where $\ddot{q}(t)$, $\dot{q}(t)$, $q(t)$ are the column vectors of acceleration, velocity and displacement respectively, M is the mass matrix, C - damping matrix and K - stiffness matrix of the order n_m . The matrix B is the distribu-

tion matrix of the exciting forces $\underline{f}$.

The second-order equations (3) may be reduced to the first-order state-space form. Defining the $n=2n_m$ dimensional state-vector

$$\underline{x} = \begin{bmatrix} M^{1/2}\underline{q} \\ \hline M^{1/2}\dot{\underline{q}} \end{bmatrix} \tag{4}$$

the model given by eqs. (3) may be written in the form:

$$\dot{\underline{x}} = A\underline{x} + B\underline{f} \tag{5}$$

where

$$A = \left[\begin{array}{c|c} 0 & I \\ \hline -M^{-1/2}KM^{-1/2} & -M^{-1/2}CM^{-1/2} \end{array}\right] \tag{6}$$

$$D = \left[\begin{array}{c} 0 \\ \hline M^{-1/2}B \end{array}\right] \tag{7}$$

The "spring" matrix $\tilde{K}=M^{-1/2}KM^{-1/2}$ and the "damping" matrix $\tilde{C}=M^{-1/2}CM^{-1/2}$ remain symmatric if matrix K and matrix C were symmetric.

Note that the models (3) and (5) are built for deterministic case i.e. $\underline{n}(t)=\underline{m}(t)=\underline{0}$, $\forall\, t$.

The class of time-discrete models are very useful for computer applications. The state-space linear representation of eqs. (1), (2) may be written

in the form:

$$\underline{x}_{k+1} = A\underline{x}_k + D\underline{u}_k + \underline{n}_k \qquad (8)$$

$$\underline{y}_{k+1} = F\underline{x}_k + \underline{m}_k \qquad (9)$$

4. Controllability, observability, identifiability

One has to assume that the system under consideration is observable, controllable and identifiable.

The concept of controllability and its dual concept of observability were first introduced by R.E.Kalman [15] in 1960. Later the concept were discussed by many authors [10] ,[8] , [18] .

Let us consider the discrete deterministic models (8), (9) i.e. the case when disturbances $\underline{n}_k=\underline{m}_k=\underline{0}$, for all subscript k.

A system is called controllable if it is possible to find an input vector which brings the system from any initial state to any specified state in finite time. It is possible if the rank $[A^{n-1}D \,\vdots\, A^{n-2}D \,\vdots\, \ldots \,\vdots\, D] = n$. The couple (A,D) is called a controllable pair.

A system is called observable if from the measurements of the output vector it is possible to determine the state of the system. It is possible if the rank $[F^T \,\vdots\, A^TF^T \,\vdots\, \ldots \,\vdots\, A^{T(n-1)}F^T] = n$, where A^T, F^T are the transpose of the A,F. The couple (A,F)is called an observable pair.

Both assumptions are necessary in system identification because uncontrollable and/or unobservable parts of a system cannot be identified from observation of the input vector $\underline{u}_k$ and output vector $\underline{y}_k$.

The complete controllability and observability imply further that the triple (A,D,F) forms a minimal realization of order n .

In parameter identification case the unknown parameter vector should be identifiable. The concept of identifiability has been discussed by different authors [13] , [27] , [28] . We can say that a system is identifiable if from measurements of the state variables it is possible to determine the matrix A or if rank $\left[x(0) \vdots Ax(0) \vdots \ldots \vdots A^{(n-1)}x(0)\right] = n$ [9] .

5. Reduction of degrees of freedom

The reduction of degrees of freedom applied to the linear system given in the form (3) may be of interest to some identification technique.

The normal mode techniques applied for oscillation systems may be usefull method for this purpose.

Let us consider the linear model (3) with n_m degrees of freedom. The response $\underline{q}$ may be expressed by

$$\underline{q} = Z\underline{\eta} \qquad (10)$$

where $\underline{\eta}$ is generalized coordinates vector and the matrix Z is so-called modal matrix with n_n column

vectors of the normal modes of the order n_m ; n_n is here the number of normal modes in an interesting frequency range where $n_n < n_m$. Introducing eq. (10) into eq. (3) and premultiplying eq. (3) by Z^T yields

$$\tilde{M}\ddot{q} + \tilde{C}\dot{q} + \tilde{K}q = \tilde{B}\,f \qquad (11)$$

where $\tilde{B}=Z^TB$, $\tilde{M}=ZMZ^T$, $\tilde{C}=ZCZ^T$ and $\tilde{K}=ZKZ^T$.

From a physical point of view these matrix transformations denotes a transfer from a force to an energy equation and a reduction of the physical degrees of freedom of the order n_m to n_n where $n_n < n_m$. By analogy with eq.(3) the matrices $\tilde{M}$, $\tilde{C}$, $\tilde{K}$ are called generalized mass, damping and stiffness matrices, respectively. Note that matrices $\tilde{M}$ and $\tilde{K}$ are diagonal.

6. Output error approach in identification

In the vast majority cases some "a priori" knowledge about structure of identified system is available and in this restricted case the parameter identification (parameter estimation) problem is under the study. The parameter values are not directly observable. Consequently the optimal set of the parameter values has to be defined using a criterion with respect to the output error. The output error methods are probably the most intuitively obvious approach to the problem of system parameter estimation.

Let us consider the state-space nonlinear system

(1) and observation equation (2).

The output error is defined as:

$$\underline{e}(t,\underline{a}) = \hat{\underline{y}}(t) - \underline{y}(t,\underline{a}) \qquad (12)$$

where $\hat{\underline{y}}(t)$ is the observed output vector and $\underline{y}(t,\underline{a})$ is the output vector of the model, when the system and the model are excited by the same input vector $\underline{u}(t)$.

The choice of the form of quality function (cost function) $Q(e)$ differentiates the various estimation methodologies which have been developed over the last years.

The most common quality function is based on an integral of the L_2 norm in e ("distance" between system output and model output). In the purely deterministic case (system without disturbances i.e. $\underline{n}(t)=\underline{m}(t)=\underline{0}, \forall t$) the criterion is given in the form:

$$Q = \int_{t_o}^{t_1} \|\underline{e}\|_R^2 \, dt = \int_{t_o}^{t_1} \underline{e}^T R \underline{e} \, dt \qquad (13)$$

where R is a positive definite matrix that weights the individual components of the vector $\underline{e}(t)$. In the simplest scalar case this reduces to the integral of the squared error.

In the discrete-time equivalent of the integral form (13) there is

$$Q = \sum_{i=1}^{N} \|\underline{e}_i\|_R^2 \qquad (14)$$

where the subscript i indicates the value of the vector $\underline{e}(t)$ at the i-th sampling instant and N denotes the number of samples available over the observational interval (t_0, t_1).

The set of optimal parameters $\underline{a}^*$ is commonly obtained by minimising a scalar quality function $Q(\underline{a})$

$$\min_{\underline{a}} Q(\underline{a}) = Q(\underline{a}^*) \tag{15}$$

Note that the parameter vector $\underline{a}^*$ satisfies the model in relation to the real system only in the sense of the choosen quality function.

If the system structure is equivalent to the model structure it is sufficient to take the partial derivatives with respect to the unknown parameters equal to zero

$$\frac{\partial Q}{\partial \underline{a}}\bigg|_{\underline{a}=\underline{a}^*} = \underline{0} \tag{16}$$

and from this condition to obtain $\underline{a}^*$. Condition (16) assures minimum of the quality function $Q(\underline{a})$ for $\underline{a}^*$ if the matrix

$$\frac{\partial^2 Q}{\partial \underline{a}\, \partial \underline{a}^T}\bigg|_{\underline{a}=\underline{a}^*} \tag{17}$$

is positive definite.

If the disturbances $\underline{n}(t)$ and/or $\underline{m}(t)$ exist then the quality function is a random variable and it has

been considered as expected value

$$J(\underline{a}) = E[Q(\underline{a})] \qquad (18)$$

Note that commonly no solution of eq.(18) in the sense of eq.(16)for all input/output data sets. Minimising the quality function $J(\underline{a})$ is possible only if some "a priori" knowledge is available with respect to probability densities. We never obtain, in this case, the exact optimal parameter vector $\underline{a}^*$ but only an estimate $\hat{\underline{a}}$. It is required to express the quality of these estimates. It means that the estimates satisfy some common type of statistical convergence.

There are well known in statistical literature some desirable properties defined for the estimators $\hat{\underline{a}}$ of the parameters $\underline{a}^*$:

- unbiasedness: $E[\underline{a}] = a^*$,
- consistency: $\lim_{k\to\infty} P[|\underline{a} - \underline{a}^*| > \varepsilon] = 0$, where k - number of observations, P[.] - probability,
- efficiency: $\det[\mathrm{cov}(\hat{\underline{a}}^i) - \mathrm{cov}(\hat{\underline{a}})] \geq 0$, where $\hat{\underline{a}}$ is the "best" estimator of $\underline{a}^*$,
- sufficiency: the above condition is true and for all estimators $\hat{\underline{a}}^i$, i=1,2, ... the conditional probability-density function $p(\hat{\underline{a}}^i|\hat{\underline{a}})$ is not dependent on $\underline{a}^*$ [9] .

Another important property is a possibility (or not) of introducing "a priori" knowledge which

may be available from physical insight or from prior measurements.

Increasing number of information should succesively increase tha quality of estimates. But it is important to have in mind that the quality of estimates is formulated for infinite number of sampled data. On the other hand we never have infinite number of samples and infinite precision of measurements. Then the estimates can never reach the true values.

7. Identification methods

A classification of methods used in system identification is very complicated and some people think that the field of identification appear to look more like a bag of tricks than a unified subject. One of the simple way is classification, based upon four main factors:

- the nature of the system under study,
- the model used to characterise the system,
- the type of input/output data,
- the type of estimation method utilized.

The last aspect is the most importantly in the present context.

The nature of system and/or model may be classified by some well known properties, for example: continuous-time/discrete-time, linear/nonlinear, deterministic/stochastic, single input (output)/ multivariable. The cases off-line/on-line and

time domain/frequency domain analysis may be considered too.

The mathematical approach used in identification procedures are either of the deterministic or stochastic type , as it was mentioned above. In the first case the disturbances (noise) is negligible or is not acting on the system. But commonly it is assumed that the disturbances are acting on the system to be identified. In general case the disturbances are unknown and it is assumed that they have some statistical properties e.g. they have a certain type of distribution when the actual values of the disturban are not known (they may be unobservable).

Regarding the stochastic approach to system identification we are tempted to look for the "best" estimation procedure. In the statistical literature a number of different estimation procedures have been developed. It is unfortunate that the choice between these criteria has more or less subjective aspects and that the mathematical approach is strongly dependent on these criteria.

The main problem in parameter estimation is to finde the estimate $\hat{\underline{a}}$ of an unknown parameter vector $\underline{a}$ based on observation of input/output data. Based on available "a priori" knowledge one can make a distinction between different kinds of estimates. There are three basic approaches:

- least-squares estimation (additional knowledge is not necessary).
- maximum likelihood estimation (the covariance matrix and the join probability density function of the disturbance has to be known),
- Bayes' estimation ("a priori" information on the probability disturbances is included in the formulation of the problem).

Identification techniques based on the above estimation methods are expanded on many different methods. For example the least-squares approach are used in the following modifications [26] :

- ordinary least squares,
- weighted least squares,
- Markov estimate,
- stochastic approximation,
- Kalman-Bucy filtering,
- instrumental variable method,
- generalized least squares,
- extended least squares,
- square-root filtering.

The maximum likelihood estimation is based on the likelihood function L defined as a joint probability density function L . In this approach the estimates $\hat{\underline{a}}$ of $\underline{a}$ are those for which the likelihood function L is maximized.

The maximum likelihood estimation has a long

history in the statistical literature. It has been applied in different industrial application, to time-series analysis and so on. This method is based in identification approach on the definition of an error funstion, but the formulation is restricted by the additional assumption that the stochastic disturbances have specified probability distribution function. In most applications this assumption is restricted to the case of Gaussian, normal distribution, since this implies that the amplitude distribution can be described completely by their firstt two moments i.e. the mean vector and the covariance matrix.

The maximum likelihood estimation can be extenkes ded to the Bayesian approach where the Bayes' rule lin- a priori and a posteriori probability density functions. The Bayes' rule may be written in the form:

$$p(\underline{y}|\underline{a})p(\underline{a}) = p(\underline{y},\underline{a}) = p(\underline{a}|\underline{y})p(\underline{y}) \qquad (19)$$

where $p(\underline{y})$, $p(\underline{a})$ are the probability density functions of the output vector $\underline{y}$ and parameter vector $\underline{a}$ respectively ; $p(\underline{y},\underline{a})$ - joint probability density function; $p(\underline{y}\ \underline{a})$, $p(\underline{a}\ \underline{y})$ - conditional distribution functions „a priori" and „a posteriori", respectively. The identification techniques based on the above estimation may be expanded on many different methods.

Frequently used and well known identification methods are based on:

- regression technique,
- gradient technique.

The regression technique is based on the least-squares estimation and is applicable to both linear and nonlinear multi-variable systems. The input/output measurements obtained during normal process operation are used, thus on-line identification is possible.

The minimization of the error $e=\hat{y}-y$ yields the so called "normal equation" for the estimation a:

$$\hat{\underline{a}} = \left[U^T U\right]^{-1} U^T \underline{Y} \qquad (20)$$

where U^T is the transpose of observation matrix U. The inversion of the matrix $\left[U^T U\right]^{-1}$ involve some computational difficulties. These difficulties may be eliminated in the sequential approach in regression technique [12].

The weighted least squares and Markov estimate correspond to a specific form of the weighting matrix R in the criterion (13).

The Markov estimate

$$\hat{\underline{a}} = \left[U^T N^{-1} U\right]^{-1} U^T N^{-1} \underline{Y} \qquad (21)$$

is a result of minimizing the criterion $Q=e^T N^{-1} e$, where N is the covariance matrix of the disturbances $\underline{n}(t)$ [18]. This estimation is better than the estimation (20).

The gradient techniques represent a direct computational method where essential property, decreasing of the quality function at each iteration, is commonly secured. The nature of this technique is introduced by expanding the nonlinear terms in a Taylors series and using these series in the linear or quadratic forms.

The gradient techniques appear in many modifications in the technical literature and has a close relation to other procedures, for example adjustment methods. The major part of adjustment methods are based on simultaneous parameter adjustments and can be distinguished as direct-search methods and gradient methods.

The direct-search methods can be devided into two subclasses:

- tabulating methods, where the values of the quality function are calculated in several points in the parameter space e.g. at the junctions of a regular grid or randomly distributed (Monte Carlo method),
- exploratory methods, where the algorithms follow implicity the gradient path by step wise looking for a descending direction (Simplex method).

The gradient methods can be:

- continuous-adjustment (quite popular steepest-descent method),
- intermitted-adjustment, where the parameter are adjusted one by one or all parameters are adjusted simultaneously.

The most common adjustment gradient algorithm has the form:

$$\dot{\underline{a}} = -\mathcal{K} \frac{\partial Q}{\partial \underline{a}} \qquad (22)$$

in the continuous case or

$$\underline{a}^{i+1} = \underline{a}^{i} - \mathcal{K} \frac{\partial Q}{\partial \underline{a}}\Big|_{\underline{a}=\underline{a}^{i}} \qquad (23)$$

in the discrete form. Here $\mathcal{K}$ is a positive definite matrix by which the velocity of identification procedure can be controlled.

This method is characterized by computational simplicity and slow convergence near the optimum. Another methods posesses just opposite properties and moreover it needs a closer to optimum initial values of the calculated parameters. Restriction of initial conditions and assumed initial control of the second order gradient methods are compensated by the quadratic convergance.

There are many modifications of the gradient techniques well described in literature (see e.g.[9] ,[25]).

The stochastic approximation method is well known and has a large number of different modifications. In contrast to the gradient techniques the stochastic approximation takes the random disturbing variable into consideration . This method has a close relation to the optimal filtering theory of Kalman, which in the linear case defines exactly the weighting factors associated with the corrections.

8. Nonparametric identification

In the case of nonparametric identification the frequency, step and impulse response estimation are basic techniques. These methods are usually formulated as equivalence of continuous-time differential equations in the form of the transfer function in the frequency domain or the impulse response (step response) in the time domain. All these methods can be applied to linear models. They may also be applicable to the linearized form of nonlinear models if the input levels are kept low.

Among the three types of input signals the step input is the simplest to apply. It may be realized by a sudden applaying (or removal) of input signal as is almost always possible without special instrumentation. This technique is an off-line technique , applicable to stationary process only. The impulse (delta function) input technique is very similar to the step technique.

The frequency response technique is based on the Nyquist and Bode works where the frequency response implies that sine-wave inputs are applied to the system over the range of interest. In this case, well known as transfer function analysis, some practical difficulties may be connected with generatin of sine-wave inputs of various frequencies.

The nonparametric identification also holds for stochastic input signals. In this case the following pair of relations are used:

convolution form in time domain τ :

$$R_{xy}(\tau) = h(\tau) * R_{xx}(\tau) \qquad (23)$$

multiplication form in frequency domain f:

$$S_{xy}(jf) = H(jf) \cdot S_{xx}(f) \qquad (24)$$

where $R_{xx}(\tau)$ and $R_{xy}(\tau)$ are respectively the autocorrelation and crosscorrelation functions of stationary stochastic input x(t) and output y(t); $S_{xx}(f)$ and $S_{xy}(jf)$ are the auto-spectrum and the cross-spectrum respectively; $h(\tau)$ and $H(jf)$ are the impulse response function and the transfer function respectively.

Moreover if x(t) is white noise, then the cross-correlation function $R_{xy}(\tau)$ is itself an estimate of the impulse response function.

Fig.3. gives some relations between nonparametric identification techniques and different types of descriptions obtained during these tests.

The frequency response technique is inconvenient because of long test time is involved in sequentially identyfying each relevant point on a frequency response curve. There are alternative methods of the technique, where there are possible to identify simultaneously a number of required points on the frequency response curve.

The class of techniques built around digital

spectral analysis and numerical Fourier transform are the most widely used. It is difficult to ascertain exactly who first proposed the use of numerical Fourier transform. One of them was Schuster [29] who applied this technique with the aim of determining periodicities but this application was not without certain statistical difficulties. It is well known that the estimate of the spectrum is inconsistent i.e. it has a high variance which is not decreasing by increasing the series N of sampled values. There are some ways of overcoming this inconsistency e.g. the averaging of several independent periodograms of the same process nowadays called the "direct" method [2] .

The computational difficulty lies in the large number of operations. The discrete Fourier transform (DFT) of an N point sequence requires N^2 multiplications and time of calculation is enormous for large N. The Fast Fourier Transform (FFT) introduced by Cooley and Tukey [6] is based on the recogniction of the fact that periodicities in the DFT may be exploited what permits to reduce the number of multiplications from N^2 to $N \log_2 N$.

Modern digital spectral analysis, on-line or off-line, are normally achieved by the DFT. The development of compact, high speed minicomputers and microcomputer array processors caused possibility calculate the digital spectral estimates during the time analysis.

Digital spectral estimates suffer from a statistical and systematic errors. These are connected with windowing error, variance errors, aliasing. The detailed discussion of these problems is given in literature (see for example [29]).

9. Concluding remarks

The field of identification is, as it was described above, rather complex. This complexity is due to the applications, goals, conditions, methods, class of systems etc. The consequence of this complexity is difficulties of prediction of the future development of identification theory understood as a step in mathematical modelling.

Most identification methods are adapted to computational technique. The great part of them is based on output error concept. There is a problem connected with convergence of gradient algorithms. This problem can be solved by using the Lyapunov function approach [23] , [32] , [7] or basing on hyperstability theory [17] .

The computational difficulties increase with the complexity of the considered systems. Some difficulties are connected with the possibility that the state variable may be measurable or not. It is clear that the estimation of parameters in a case of of known state variables is always less difficult than the join state and para-

meter estimation representing a nonlinear problem. Identification of parameters of nonlinear systems must relate to a specific nonlinear formulation of approximation , since an infinite number of nonlinear function does exist and a parameters which fits one function to measurements is not the best for another nonlinear function. Then the identification of nonlinear systems is still an open question.

It is most likely that some future investigation in the field of identification will be connected with modifications and extensions of the methods well known in the present time.

The significance of accuracy analysis of different identification methods is indisputable. This accuracy is connected, among other things, with computational operations. The improvements in this area may be connected with development of computational methods and techniques.

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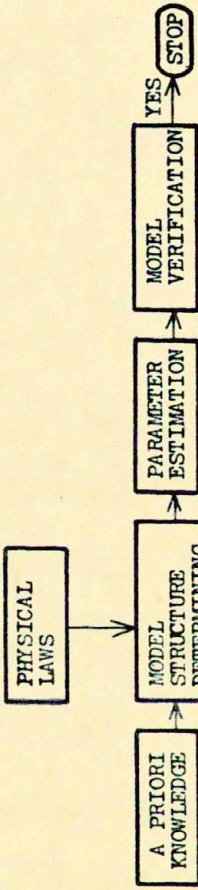

Fig.1

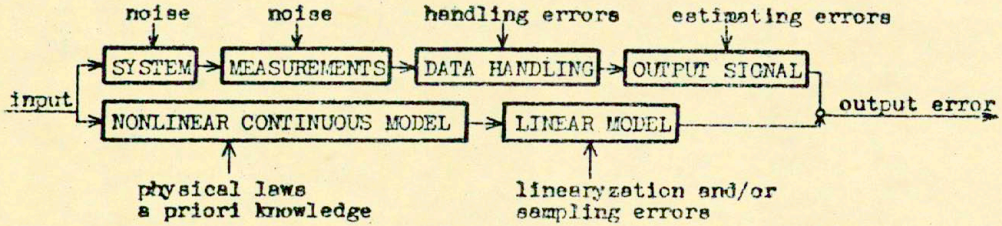

Fig. 2

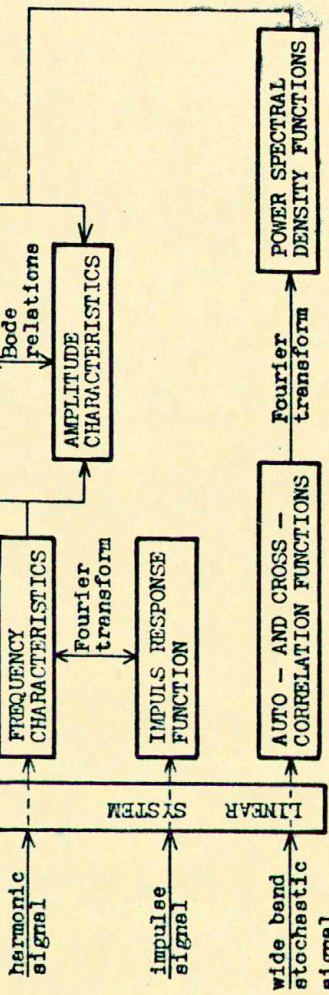

Fig.3

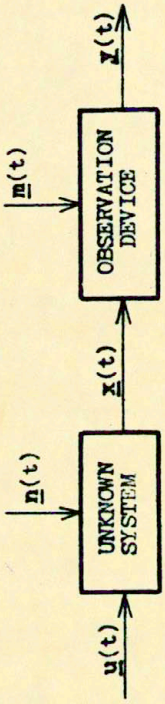

Fig. 4