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Applications of a ray reflection model in the problem of highly rarefied gas flow past bodies

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RAREFIED hypersonic gas flow past a convex body is studied within the framework of the single collision approximation. Gas-surface interaction is described by the ray reflection model. In the case of a sphere, detailed results are given for gasdynamic fields and fluxes on the surface for different interaction parameters.

Zbadano hipersoniczny opływ gazu rozrzedzonego ciała wypukłego w ramach założeń aproksymacji pojedynczych zderzeń. Powierzchnia oddziaływania gazu jest opisana modelem odbicia promieni. W przypadku powierzchni kulistej podano szczegółowe rezultaty dla pól i strumieni gazodynamiki na powierzchni dla różnych parametrów oddziaływania.

Изучается гиперзвуковое обтекание выпуклого тела сильно разреженным газом. Задача рассматривается в рамках приближения однократными столкновениями, причем взаимодействие газа с поверхностью описано лучевой моделью отражения. Для случая сферической поверхности подробно излагаются решения для газодинамических полей и потоков на поверхности при различных значениях параметров взаимодействия.

WE CONSIDER axisymmetric steady hypersonic ($M_\infty = \infty$) highly rarefied ($Kn \gg 1$) gas flow past a strictly convex body. Gas-surface interaction is described by the ray model of the scattering function

$$V(\bar{u}_1, \bar{u}) = \delta(\bar{u} - \bar{u}_m(\bar{u}_1)), \tag{1}$$

$\bar{u}_m$ being a given function of the incidence velocity $\bar{u}_1$. Interaction between atoms is described by the normalized differential scattering cross-section

$$T_\omega(\vartheta) = (1 + \beta\cos\vartheta)/(4\pi), \quad 0 \leqslant \beta \leqslant 1, \tag{2}$$

and the total cross-section

$$\sigma(v_0) = \sigma_0 v_0^{-\gamma}, \quad 0 \leqslant \gamma < 4, \tag{3}$$

v_0 being the impact velocity, ϑ the scattering angle. Parameter β defines the scattering anisotropy, γ — interaction potential $U(r)$ hardness. For small β, the function (2) corresponds to a potential barrier of inclination

$$(dU/dr)_{r=r_{\max}} = -4/\beta.$$

In the present paper, exact expressions are obtained for the first terms of asymptotic expansions of aerodynamic quantities in inverse Knudsen number powers. Such a problem was solved in [1] for hard atoms ($\beta = 0$, $\gamma = 0$). In the case of a sphere with reflection along the normal ($\bar{u}_m = u_m \bar{n}$) mass, momentum and energy fluxes on the surface were calculated. Here, the solution is generalized in three aspects:

1. Atom pliancy ($\beta \neq 0$) and its radius dependence on the impact velocity ($\gamma \neq 0$) are taken into account;

2. In addition to the one-parametric ray model

$$u_m(\bar{u}_1) = u_m, \quad \theta_m(\bar{u}_1) = 0, \tag{4}$$

the two-parametric model (see [2])

$$u_m(\bar{u}_1) = u_0\left[1-\frac{4\cos\theta_0\cos\theta_1}{1+4\cos^2\theta_0}\right]^{1/2}, \quad \theta_m(\bar{u}_1) = \operatorname{arctg}\frac{\sin\theta_1}{2\cos\theta_0-\cos\theta_1} \tag{5}$$

is used, u_0 being a maximum value of the reflection velocity reached for $\theta_1 = \pi/2$ and $\theta_0 \in (0, 60°)$ — an angle for which the reflection changes from underspecular into overspecular.

Parameters u_m, θ_m are the average magnitude and direction of scattered atoms.

3. The quantities calculated are not only fluxes on the surface but also gas-dynamic fields in front of the sphere.

Owing to $M_\infty = \infty$, the incident distribution function is $f_\infty(\bar{u}) = \delta(\bar{u}-\bar{u}_\infty)$, $\bar{u}_\infty = \{0, 0, -1\}$. The part of the space $\bar{r}$ filled by the rays passing from points $\bar{r}'_s$ of the front part of the body surface in directions $\bar{u}_m$ will be designated by Λ. In the free molecule limit we have

$$f_0(\bar{r}, \bar{u}) = \delta(\bar{u}-\bar{u}_\infty)+\frac{\cos\theta_1}{|J|}\delta(\bar{u}-\bar{u}_m), \tag{6}$$

$\theta_1 = \sphericalangle(\bar{n}, -\bar{u}_\infty)$, and $|J|$ connected with the ray divergence has been found in [1]. In the rest of the space, the second term is absent; in the wake, both are absent.

In the near-free-molecule regime at any distance $r < 0(Kn)$, the asymptotic expansion (see [3])

$$f = f_0 - \frac{1}{Kn} f_1 + \ldots \tag{7}$$

is valid, $Kn = (n_\infty \sigma_0 \mathscr{L})^{-1}$, n_∞ being the numerical density of oncoming flow, $\mathscr{L}$ a characteristic measure of the body. An exact expression of the coefficient at Kn^{-1} is

$$f_1(\bar{r}, \bar{u}) = \int\limits_{\Lambda_1(\bar{r},\bar{u})} \left\{ f_0\left(\bar{r}-\frac{\bar{u}}{u}\lambda, \bar{u}\right) Q_0\left(\bar{r}-\frac{\bar{u}}{u}\lambda, \bar{u}\right) - \frac{\Phi_0\left(\bar{r}-\frac{\bar{u}}{u}\lambda, \bar{u}\right)}{\left|1-\frac{\lambda}{u}\frac{du}{d\lambda}\right|} \right\} \frac{d\lambda}{u}, \tag{8}$$

$\left|1-\frac{\lambda}{u}\frac{du}{d\lambda}\right|$ being a divergence factor (see [1]). The integration domain Λ_1 depends on the form of the body. The collision and creation functions can be written as

$$Q_0(\bar{r}, \bar{u}) = |\bar{u}-\bar{u}_\infty|^{1-\gamma} + \frac{\cos\theta_1}{|J|}|\bar{u}-\bar{u}_m|^{1-\gamma}, \tag{9}$$

$$\Phi_0(\bar{r}, \bar{u}) = \frac{2\cos\theta_1}{|J|}|\bar{u}_\infty-\bar{u}_m|^{1-\gamma} T(\bar{u}_\infty, \bar{u}_m, \bar{u}), \tag{10}$$

where

$$T = \frac{2}{|\bar{u}_\infty - \bar{u}_m|^2}\left[T_\omega(\vartheta) + T_\omega(\pi - \vartheta)\right]\delta\left\{\left|\bar{u} - \frac{\bar{u}_\infty + \bar{u}_m}{2}\right| - \frac{|\bar{u}_\infty - \bar{u}_m|}{2}\right\}, \tag{11}$$

$$\vartheta = \sphericalangle\left(\bar{u} - \frac{\bar{u}_\infty + \bar{u}_m}{2}, \bar{u}_m - \bar{u}_\infty\right). \tag{12}$$

It is clear from (11) that in the case (2), f_1 does not in fact depend on β.

Designating

$$g_1(\bar{r}) = \int_{u_n < 0} f_1(\bar{r}, \bar{u}) G(\bar{r}, \bar{u}) d\bar{u}, \tag{13}$$

with proper $G(\bar{r}, \bar{u})$, we can find coefficients at Kn^{-1} corresponding to (7) expansions of gasdynamic quantities.

At surface points $\bar{r}_s$, for $G = |u_n|\{1, \bar{u} - \bar{u}_m(\bar{r}_s, \bar{u}), u^2 - u_m^2(\bar{r}_s, \bar{u})\}$, we have the particle flux and the momentum and energy exchange coefficients $g_1(\bar{r}_s) = \{\bar{\nu}_1(\bar{r}_s), \bar{p}_1(\bar{r}_s), q(\bar{r}_s)\}$. At any point $\bar{r}$, for $G = \{1, \bar{u}\, \frac{1}{2}(\bar{u} - \bar{U})^2\}$, we have the mean density, velocity and energy

$$g_1(\bar{r}) = \{n_1, (n\bar{U})_1, (nE)_1\}.$$

In accordance with (8), we can write

$$g_1 = g_0 \zeta - g_*. \tag{14}$$

The dislodging factor ζ is calculated as a single integral over λ owing to (6). The creation factor g_* is calculated as a triple integral over λ and a solid angle owing to the δ-function in (11). On the symmetry axis, this integral reduces to a double one.

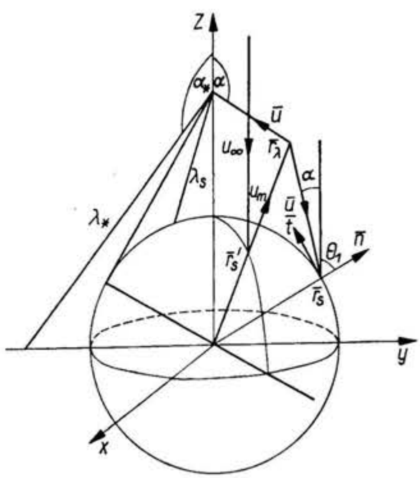

FIG. 1.

In the case of a sphere with reflection along the normal (Fig. 1), $\bar{u}_m = u_m \bar{n}$, $J = r^2 u_m$.

From $\bar{r}_\lambda = \bar{r} - \frac{\bar{u}}{u}\lambda$ we have

$$z_\lambda = z + \lambda\cos\alpha, \quad \alpha = \sphericalangle(z, -\bar{u}). \tag{15}$$

For sphere surface points $\bar{r} = \bar{r}_s$, the integration domain Λ_1 in (8) is determined by

$$\begin{aligned} &0 \leqslant \lambda < \infty \quad \text{if} \quad \cos\theta_1 > 0, \cos\alpha > 0, \\ &0 \leqslant \lambda \leqslant -\frac{\cos\theta_1}{\cos\alpha} \quad \text{if} \quad \cos\theta_1 > 0, \cos\alpha < 0, \\ &-\frac{\cos\theta_1}{\cos\alpha} \leqslant \lambda < \infty \quad \text{if} \quad \cos\theta_1 < 0, \cos\alpha > 0. \end{aligned} \tag{16}$$

For symmetry axis points in front of the sphere, the domain Λ_1 is determined by

$$\begin{aligned} &0 \leqslant \lambda < \infty \quad \text{if} \quad 0 \leqslant \alpha \leqslant \pi/2, \\ &0 \leqslant \lambda \leqslant \lambda_* = -\frac{z}{\cos\alpha} \quad \text{if} \quad \frac{\pi}{2} < \alpha \leqslant \alpha_* = \pi - \operatorname{arctg}\frac{1}{\sqrt{z^2-1}}, \\ &0 \leqslant \lambda \leqslant \lambda_s = -z\cos\alpha - \sqrt{1 - z\sin^2\alpha} \quad \text{if} \quad \alpha_* < \alpha \leqslant \pi. \end{aligned} \tag{17}$$

The functions $\nu_1(\theta_1), \bar{p}_1(\theta_1) = -\tau_1(\theta_1)\bar{z} - p_1(\theta_1)\bar{n}, q_1(\theta_1)$ on the sphere surface and $n_1(z)$, $U_1(z)$, $E_1(z)$ on the axis were calculated for three values of $u_m = 0.1; 0.5; 1$ for

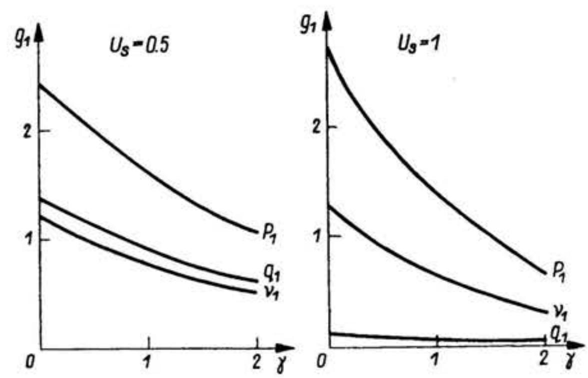

FIG. 2. Sphere, $Q_1 = 0$.

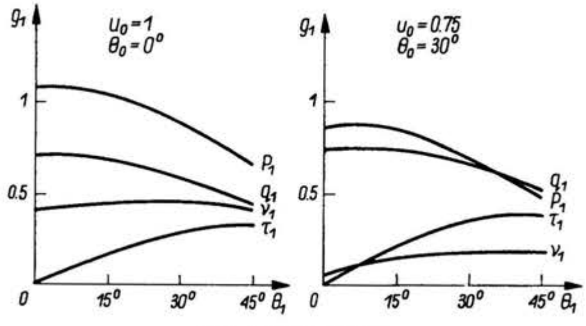

FIG. 3. Segment 45°.

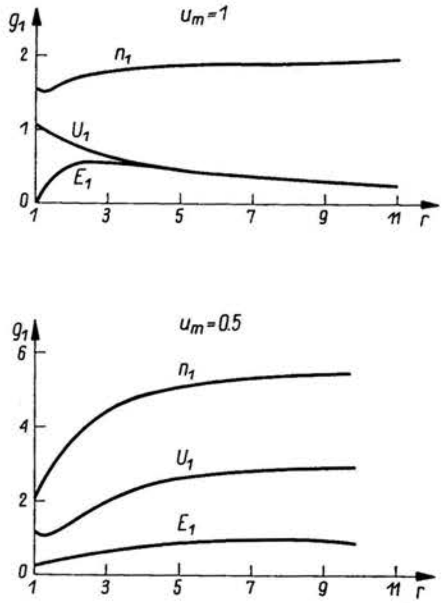

Fig. 4. $\gamma = 0, \theta_1 = 0°$.

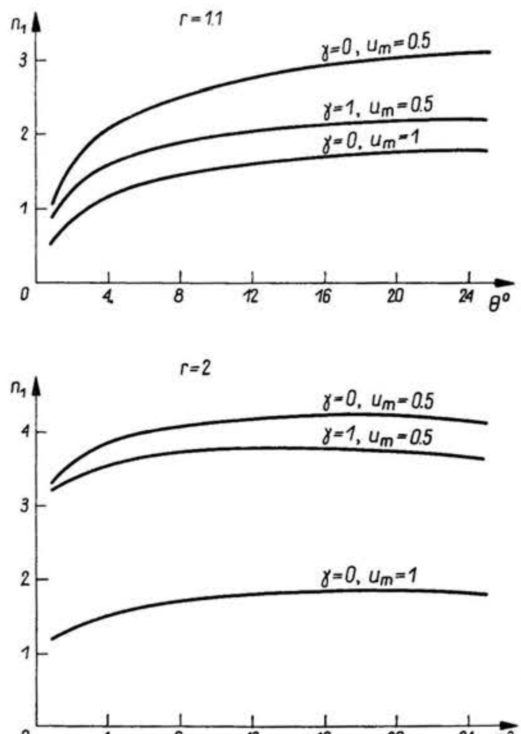

FIG. 5.

$\gamma = 0$ and $\gamma = 1$. In the case of (5), the flow past a spherical segment was considered, and mass, momentum and energy fluxes on the body surface were calculated. Some of the results are shown in Figs. 2–5.

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