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Influence of the offset on the experimental yield surfaces of metals: a theoretical evaluation

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A THEORETICAL study of the offset influence on the shape of the subsequent yield surfaces it developed. The analysis is performed according to the slip-theory model of an hardening material with ideal Baushinger effect. A comparison of the theoretical predictions of the yield surface in the hardening range is made for different loading paths.

Praca zawiera teoretyczną analizę wpływu definicji umownej granicy plastyczności na interpretację wyników badań eksperymentalnych dotyczących wpływu odkształcenia plastycznego na kształt powierzchni plastyczności. Wykazano, że przy badaniach metali przy złożonych stanach naprężenia wyniki wyraźnie zależą od przyjętej definicji granicy plastyczności.

В работе содержится теоретический анализ влияния определения условного предела текучести на интерпретацию опытных данных, относящихся к зависимости формы поверхности текучести от пластических деформаций. Показано, что при исследовании металлов в условиях сложного напряженного состояния эта зависимость различна для различных определений предела текучести.

1. Some remarks on the experimental evaluation of the subsequent yield surfaces in tension-torsion tests

EXPERIMENTAL research in plasticity has experienced a revival in the last 15 years. In particular, the determination of the subsequent yield surfaces of metals has been tackled in several laboratories but with different levels of accuracy and without systematic coordination of research. With a few exceptions (e.g. [6, 10, 18]), most experimental researches are, to the best of our knowledge, based on tension-torsion tests on tubular specimens. After the first yield surface has been obtained, the specimen is subjected to a strain hardening process and then unloaded; then, several points of the new yield surface are evaluated.

It is very difficult to recognize the first yielding; as matter of fact small plastic strains are present in polycrystalline agregates even under very small stress.

Different conventional definitions of yielding have therefore been proposed, corresponding to different values of the plastic offset [12] (Fig. 1).

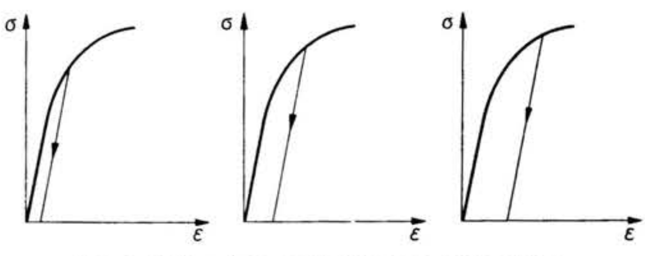

FIG. 1. Different conventional values of the offset.

Consequently, we have to establish a fundamental distinction between the *ideal test*, with an offset equal to zero, and the *actual test* with a measurable offset; the influence of the offset on the experimental yield surface was first pointed out by LIN and ITO [11].

In this paper we attempt a theoretical explanation of the offset sensitivity of the experimental yield surfaces. To this end, we make use of an earlier treatment of the plasticity of metals exhibiting strain-hardening and Bauschinger effect [15], which in turn was based on the theory of slip [1].

The essential feature of the proposed extension of the slip theory was the introduction of two different values of the limit shear stress τ_L^+ and τ_L^- for every pair of planes, in order to take into account the Bauschinger effect.

Consider a *Pabc* coordinate system in a small region around a point P: let the yield criterion for the *ab* planes be

$$\tau_{ab} - \tau_L = 0, \tag{1.1}$$

where τ_{ab} is the shear stress acting on the *ab* planes and τ_L the corresponding limit value (Fig. 2).

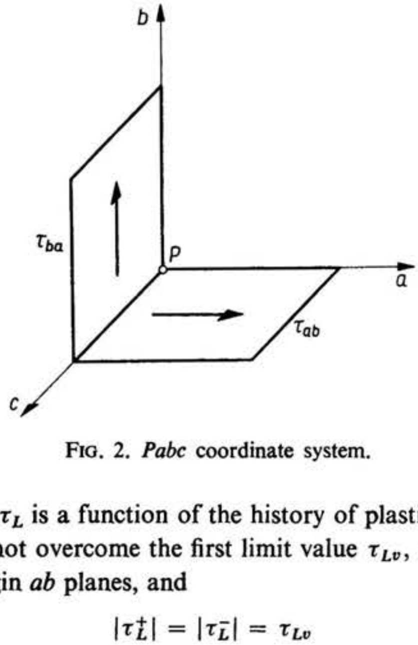

FIG. 2. *Pabc* coordinate system.

The limit shear stress τ_L is a function of the history of plastic shear strain γ_{ab} between the *ab* planes. If τ_{ab} has not overcome the first limit value τ_{Lv}, no plastic shear strain has occurred between the virgin *ab* planes, and

$$|\tau_L^+| = |\tau_L^-| = \tau_{Lv} \tag{1.2}$$

If (1.2) holds for all the pairs of orthogonal planes through P, the considered small region of the body is in the virgin state.

If, on the contrary, the *ab* pair of planes have undergone some plastic shear strain, let τ_{ab}^* be the stress that produced the last increment of plastic strain, and τ_{ab} the current stress. Because of the Bauschinger effect, two different values of the limit stress, τ_L^+ and τ_L^-, must be considered.

If $\dot{\tau}_{ab}\dot{\tau}_{ab}^* > 0$

$$\tau_L^+ = \tau_{ab}, \tag{1.3}$$

while if $\dot{\tau}_{ab}\dot{\tau}^*_{ab} < 0$

$$\tau_L^- = -B_1\tau_L^+ + B_2\left(\tau_L^+ - 2\tau_{Lv}\frac{\dot{\tau}^*_{ab}}{|\dot{\tau}^*_{ab}|}\right), \tag{1.4}$$

where B_1 and B_2 are the coefficients of the local Bauschinger effect. For a virgin pair of planes, continuity between (1.2) and (1.4) gives:

$$B_1 + B_2 = 1. \tag{1.5}$$

The relationship (1.4) between the shear stress τ_L^+ and τ_L^- is unreliable when the Bauschinger effect or the value of the strain-hardening stress are very high, but is valid with sufficient accuracy for the usual behaviour of metals and for the loading paths commonly followed in experimental tests.

On the basis of these assumptions, an analysis of the evolution of the yield surface with strain-hardening has been developed [15] and subsequent yield surfaces in $\sigma-\tau$ and $\sigma_1-\sigma_2$ planes (corresponding to the tension-torsion and the tension-internal pressure tests) have been obtained [16, 17].

In the present paper, the gap between theoretical and experimental researches on these themes is examined.

2. Theoretical subsequent yield surfaces of metals in $\sigma-\tau$ plane

In a biaxial stress state — e.g., due to tension and torsion of a tubular specimen — the theoretical subsequent yield surfaces, following the theory summarized above, can be

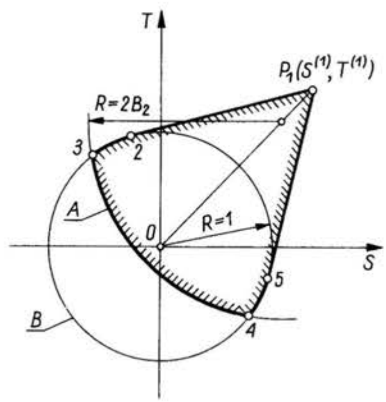

FIG. 3. Construction of the theoretical subsequent yield surfaces of metals exhibiting strain-hardening and Bauschinger effect in the plane S-T according to the slip theory: A — Bauschinger circle, B — unitary circle.

obtained by means of the geometrical construction of Fig. 3 [17], where S, T are dimensionless stresses defined by:

$$S = \frac{\sigma}{2\tau_{Lv}}, \quad T = \frac{\tau}{\tau_{Lv}}. \tag{2.1}$$

In turn, $\sigma-\tau$ are the stress components acting on the cross section of the tubular specimen. The theoretical yield surface, after loading from 0 to P_1, is P_1 –2–3–4–5– P_1, (Fig. 3). P_1–2 and P_1–5 are straight lines from P_1 tangent to the unitary circle; 2–3 and 4–5 are segments of the unitary circle of radius

$$R_B = 2B_2 \tag{2.2}$$

and centered at the point of the coordinates:

$$S_B = S^{(1)}(B_2-B_1), \quad T_B = T^{(1)}(B_2-B_1). \tag{2.3}$$

The segments 2–3 and 4–5, corresponding to the possibility of yielding on virgin pairs of planes, disappear if either the strain-hardening or the Bauschinger effect is very large.

The cross effect may or may not take place according to the intensity of the strain-hardening and of the Bauschinger effect.

3. Offset sensitivity of experimental yield surfaces

Among the many tests on the evolution of the yield surface in the strain-hardening range [2–10, 13], we have first of all to distinguish between the experimental surfaces obtained by testing always the same specimen and those obtained with the use of different specimens.

A sufficient identity of geometry and material properties as between a number of specimens is very difficult to achieve and even harder to measure. Therefore, many researchers prefer to test on one single specimen when evaluating each yield surface. Consequently, there arises, as will now be shown, the problem of the influence of the assumed offset on the yield surfaces.

For instance, let O–P_1 (Fig. 4) be the first strain-hardening loading: the specimen is then unloaded and reloaded, into the plastic range, following the path O–P_2. The yield

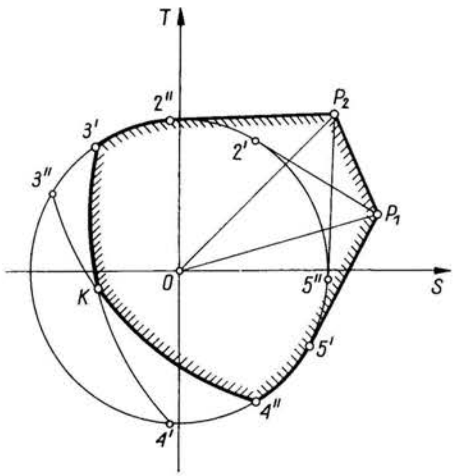

FIG. 4. The theoretical yield surface in the S, T-plane corresponding to two subsequent plastic strain-hardenings.

surface in this condition can be obtained, as shown in Fig. 4, by two successive applications of the construction in Fig. 3.

Consider now how the yield surface Y_{P_1} corresponding to loading to P_1 (Fig. 5) would be experimentally evaluated. After unloading to O, the specimen would be re-loaded

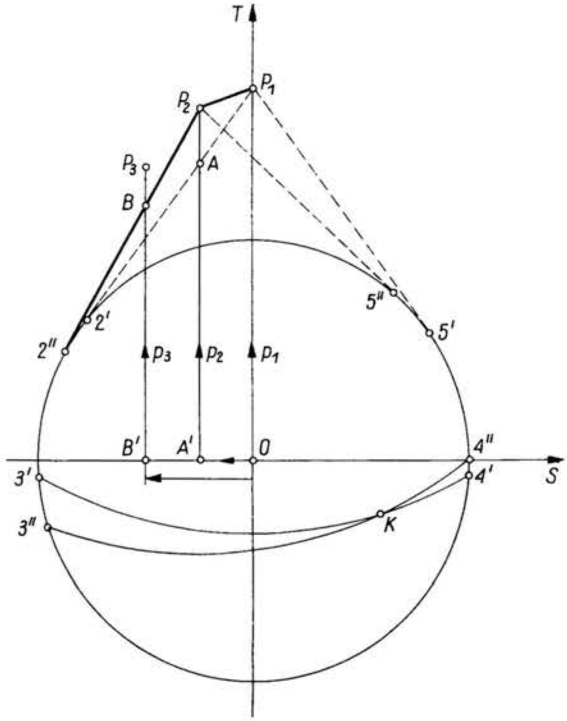

FIG. 5. Construction of the experimenter's theoretical yield surface in the S-T-plane.

following, for instance, the path P_2 $(OA'A, \ldots)$. Let A be the intersection point between this loading path and the yield surface Y_{P_1}: the experimenter does not become aware of such intersection point and, therefore, continues to twist the specimen until the point P_2, corresponding to non-zero offset, is reached. Therefore some further, unwanted but unavoidable strain-hardening has in this process been imposed on the specimen. The actual yield surface at this point $Y_{P_1P_2}$ is P_1–P_2–$2''$–$3'$–K–$4''$–$5'$–P_1 (Fig. 5).

Upon unloading and re-loading following the new path P_3, plastic strains begin to appear again when the loading path intersects the yield surface $Y_{P_1P_2}$ in B: but the experimenter continues loading until the points P_3, outside of $Y_{P_1P_2}$, is reached. The previous arguments can be repeated: the new yield surface $Y_{P_1P_2P_3}$ appears.

In conclusion, when using repeatedly the same specimen, the experimenter obtains a sequence of points P_1, P_2 ,P_3, ..., P_n, each lying on a different yield surface, Y_{P_1}, $Y_{P_1P_2}$, $Y_{P_1P_2}, \ldots,_{P_n}$, corresponding to successive increments.

Therefore, the area inside P_1–P_2, ..., –P_m cannot be considered as an elastic region bounded by a yield surface; in other words P_1–P_2, ..., P_n cannot be regarded as a true yield locus. However, by applying the previous theoretical considerations it is possible to draw an appropriate figure connecting the points $P_1 P_2, \ldots, P_n$: we define this as the theoretical experimenter's yield surface.

An example of construction of an experimenter's yield surface is shown in Fig. 6.

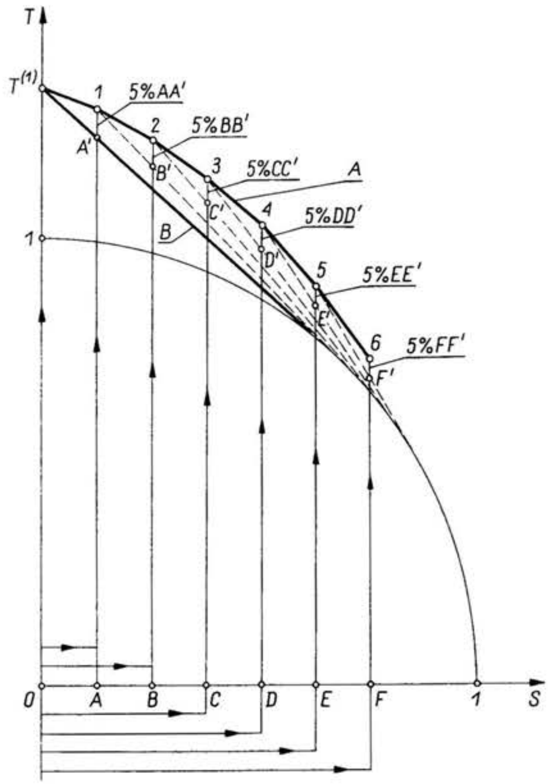

FIG. 6. Construction of the experimenter's theoretical yield surface. A – Experimenter's theoretical yield curve. B – Ideal yield curve.

A difficult task is the evaluation of the correct values of the stress offset ratio:

$$\psi = \frac{AP_2}{A'A}, \tag{3.1}$$

(cf. Fig. 5), actually used in the tests. Note first that during a test, while the strain offset is usually kept constant the corresponding stress ratio ψ varies. But above all it must be emphasized that the evaluation of ψ cannot ever be accurate: otherwise we could evaluate by the experimental diagrams $\sigma-\varepsilon$ or $\tau-\gamma$ the point of first yielding. We cannot assert that the assumed value of $\psi = 5\%$ represents an average value among the values that we may find in tests, but we can say that the value of ψ, to be deduced from the experiences reported in [9], is of some units %.

The value $\psi = 5\%$, above all, can represent an acceptable value for investigation of the qualitative changes produced by the testing on the theoretical yield surfaces.

It is also of interest to consider the effects of different loading paths on the distorsion of the experimenter's surfaces.

Figure 7 shows the portion of the experimenter's yield surface contained in the positive quadrant of the T–S plane, with $\psi = 5\%$ and the tension-torsion loading path having increasing torque. Analogous constructions are presented in Fig. 8 and Fig. 9, respectively

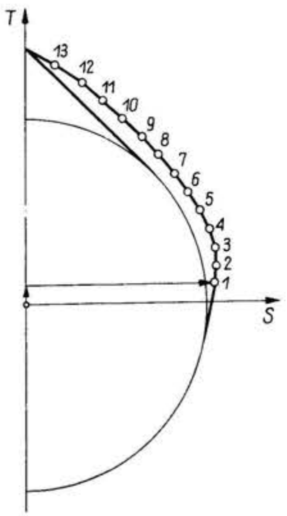

FIG. 7. The experimenter's theoretical yield surface corresponding to the tension-torsion loading with increasing torque.

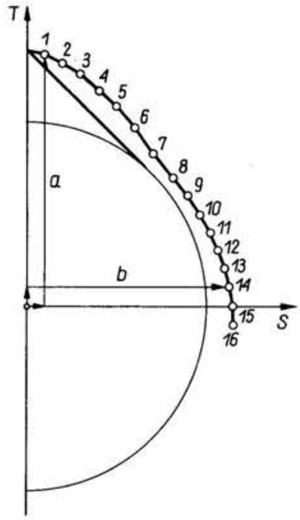

FIG. 8. The experimenter's theoretical yield surface corresponding to the torsion-tension loading with decreasing tension.

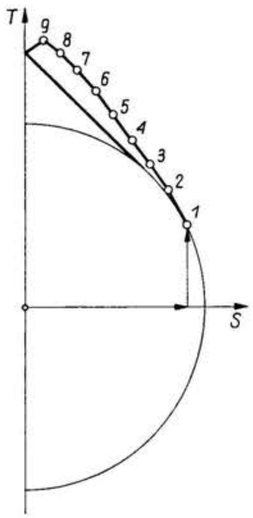

FIG. 9. The experimenter's theoretical yield surface corresponding to a sequence of tension-torsion and torsion-tension paths. A — Points 1–7, loading path of type a; B — Points 8–16, loading path of type b.

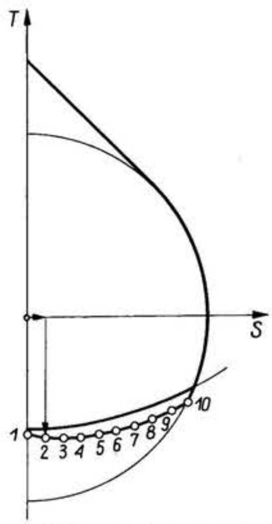

FIG. 10. Back boundaries sensitive to the Bauschinger effect.

for torsion-tension paths with decreasing tension and for a sequence of tension-torsion and torsion-tension paths.

This latter sequence is very similar to the loading paths followed in many tests: it is possible to notice some concavity in the apparent (experimenter's) yield surface.

Finally, in Fig. 10 (in the fourth quadrant of the T–S plane, there is a picture of the portion of the experimenter's yield surface sensitive to the Bauschinger effect.

4. Concluding remarks

Inspection of the theoretical experimenter's yield surfaces of Figs. 6–10, obtained by considering different loading paths, shows that the experimental yield surfaces may have various shapes and be significantly different from the theoretical ones of the ideal tests. Note also that the previous constructions concerned only a portion of the stress plane; a complete loading path, moving through all the four quadrants of the T–S plane, would produce even larger distorsions of the theoretical yield loci.

It is necessary, however, to remember that while in this comparative analysis a constant ratio ψ has been used during an actual test, the coefficient ψ changes because of the variable slope of the curves $\sigma-\varepsilon$ and $\tau-\gamma$, as already pointed out.

Nevertheless we can affirm that the theoretical yield surfaces corresponding to an ideal test and the experimental curves obtained by testing the same specimen are not comparable because of the offset effect.

On the contrary, what we have defined as the theoretical experimenter's yield surfaces are certainly comparable with the experimental results. Unfortunately, in the literature not enough information is given to enable a thorough comparison between theory and the published tests. However, there is a general qualitative agreement between our theoretical experimenters' yield surfaces and those deduced by tests. On the other hand it is worth-while to notice that the Naghdi's results [5], obtained by testing a different specimen for each point, agree well with the theoretical first yield surfaces given by our modified slip theory [14].

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