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THE EXACT SURFACE IMPEDANCE APPROACH TO PROBLEMS OF DIFFRACTION IN OPEN WAVEGUIDES

A novel, exact approach to diffraction problems for open planar waveguides is described. This is based on total field analysis instead of on a model-matching technique. Within it, the rigorous impedance boundary condition is derived in integral form suitable for further treatment of the problem by Wiener--Hopf methods. As an example, the problem is considered of guided and leaky modes scattering at a junction between two planar, isotropic, symmetric waveguides. The Wiener-Hopf-Hilbert method is used and an exact, closed-form solution is obtained. Possible applications in integrated optics are indicated.

1. Introduction

Constant impedance approximate boundary conditions [1] are commonly exploited in electromagnetics, since they convert two or more media scattering problems into one medium problems, and hence substantially simplify analysis. As regards the canonical problems of diffraction by planar, semi-infinite impedance structures, exact, analytical methods of solution, such as the Wiener-Hopf method, are well known [1]. For more complicated geometries, approximate solutions also are available making use of the solutions of the canonical problems.

In dealing with infinite, open planar waveguides, constancy approximation for the surface impedance no longer holds, and only rigorous definition can be applied to each mode of the field independently. Consequently, the surface impedance (with spatial dispersion) at a fixed surface - expressed by the ratio of appropriate tangential components of electric and magnetic field - is dependent on the wave number, direction of propagation, polarization or even symmetry properties of the spectral field constituent in question. Even so, field propagation along the open planar waveguide and along the constant surface impedance plane have many features in common [2]. Thus, since the Wiener-Hopf methods do not seem to possess any true limitations as regards application of the rigorous boudary conditions, it might be worth while to try to use them in problems of diffracion in open planar waveguides [3]. The educement of the appropriate surface impedance approach proves to be possible and the object of the present paper is to demonstrate this procedure by way of the case of diffraction by the junction of two multimode, planar, isotropic, symmetric, and step index waveguides with the same thickness of core. The approach is not limited to this simple case and can be generalized for more practical cases of waveguides and geometries.

Unlike the mode-matching technique, commonly exploited in such problems [4], the surface impedance approach does not utilize the continuity relations of tangential field components in a plane of the wavegiude junction. Instead, the diffraction problem can be formulated in the homogeneous - in transverse and longitudinal directions - semi-infinite free-space region bounded by the plane placed on or over surfaces of the waveguide's cores. All necessary information about the structure beneath this plane is contained in the integral boundary conditions defined at this plane together with the so-called edge and radiation conditions.

Every single spectral constituent of the postulated longitudinal spectral representation of the scattered field does not in general satisfies the continuity relations at the core surface. Such a general representation allows to define the incident field in a form suitable for further treatment of the problem by the Wiener-Hopf techniques. Moreover, that can be done without resort to the assumption of the completness of the normal mode representation of the total field. Thus, the completness of this representation will possibly prove to be a direct consequence of the solution obtained.

The impedance boundary condition devised enables formulation of the diffraction problem rigorously in the semi-infinite space outside the core. It is assumed at the outset that only one, not necessarily fundamental, guided mode is incident on the junction, while the multimode guided or leaky incidence presents no greater difficulties. Although application of the standard Wiener--Hopf technique seems to be possible, the extension of it, namely the Wiener-Hopf-Hilbert method [5] is used. Apart from its simplicity and potential applicability in treating coupled Wiener-Hopf equations, this method avoids the difficulties of factorization procedure and transforms the equations into Hilbert equations, which may easily be solved. The integral form of the boundary conditions is well suited to this method and an exact, closed-form solution can be obtained, in which the procedure applied is similar to that for the problem of surface and leaky waves diffraction by a constant impedance half plane [6] . The exact analytical expressions for coefficients of any guided or leaky mode excited by the discontinuity are derived, and the radiation field is represented in the integral form suitable for asymptotic approximation.

The problem of scattering of guides modes by waveguide discontinuities has received considerable attention in recent years. The cases in which both film thickness and refractive index changes - or either of these - are present in the guiding structure have been analysed in many papers and a representative list of such is given in [7] . It is not the object of this paper to discuss the results derived from them from the point of view of the surface impedance approach at such an early stage of the latter. Nevertheless, it is to be noted that everywhere only the normal mode representation of the field has been assumed, and different approaches has been applied to overcome the difficulties connected with the continuous nature of constituents of the mode spectrum. Although, in addition to various approximate techniques, rigorous analysis has been applied also, none of them lead to an exact closed-form solution.

2. Impedance boundary conditions

Consider a planar, symmetric waveguide consisting of a core of thickness 2d with relative refractive index n and relative instrinsic impedance Z with respect to the surrounding homogeneous medium in a region of semi-infinite space. The core-space interface is hereinafter referred to as the interface. The junction between two such waveguides is shown at Figure 1.

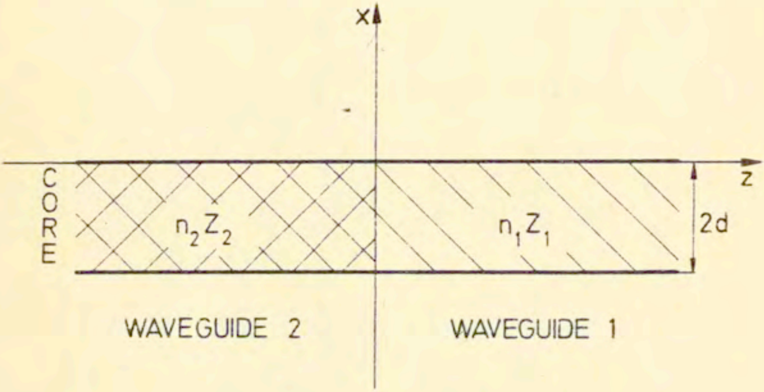

Fig.1 Discontinuity at the junction of two open waveguides.

The z and y, axes of the Cartesian coordinates x,y,z system, are set on the interface, where the structure and the field are assumed to be invariant with respect to the y coordinate and the guidance direction is indicated by the z coordinate. In view of the symmetry of the structure it sufficies to restrict further considerations to one part of the waveguide bisected - here for $x \geq -d$.

Let us first concentrate on the even TM electromanetic field - that is the symmetric about the plane $x = -d$, where the H_x, H_z and E_y components vanish. We formulate the y component of the magnetic field in terms of a longitudinal spectral representation for $x > 0$:

$$(1) \quad H_y(x,z) = \int_{\mathcal{C}} A(\beta) \exp\left(i(\alpha x + \beta z)\right) d\beta \,,$$

where

$$\alpha(\beta) = (k^2-\beta^2)^{1/2};\ \mathrm{Re}(k) > 0,\ \mathrm{Im}(k) \geq 0$$

and k, α, β denote the wave number outside the core and its components along x and z directions, respectively. An explicit dependendence of the spectral amplitude $A(\beta)$ on α and β is shown and a time dependence on $\exp -i\omega t$ is assumed and supressed hereinafter. To specify the double-valued function $\alpha(\beta)$ uniquely, we consider the complex β-plane to be a two-leafed surface with the branch cuts Γ_+ and Γ_-, and choose the top Riemann sheet from:

$$\alpha(0) = k.$$

The choice of the branch cuts Γ_+, Γ_- is, as shown in Fig. 2, well adapted to the latter asymptotic evaluation of the far field [6]. The integration contour $\mathfrak{C}$, placed on the top Riemann sheet follows the real axis except in the case of the indentations about the branch points at $\beta=-k$ above and $\beta=k$ below for the lossless surrouding medium. The other possible deformations of $\mathfrak{C}$ will be determined latter.

In order to take into account the continuity of the tangential field components H_y and E_z at the interface let us assume the following form of the field for $x \geq -d$:

$$(2)\qquad H_y(x,z) = \int_{\mathfrak{C}} B(\beta)(\alpha-k\eta(\beta))^{-1}\,\Psi(\beta,x)\exp(i\beta z)\,d\beta + \int_{\mathfrak{C}} C(\beta)(\alpha-k\eta(\beta))^{-1}\,\Phi(\beta,x)\exp(i\beta z)\,d\beta$$

where

$$(3)\qquad \Psi(\beta,x) = (\alpha-k\eta(\beta))\begin{cases}\exp(i\alpha x) & 0<x\\ \cos\varkappa(x+d)/\cos\varkappa d & -d\leq x<0\end{cases}$$

$$(4)\qquad \Phi(\beta,x) = (\alpha-k\eta(\beta))\begin{cases}-k\eta(\beta)\exp(i\alpha x) & 0<x\\ \cos\varkappa(x+d)/\cos\varkappa d & -d\leq x<0\end{cases}$$

$$(5)\qquad \eta(\beta) = -iZk^{-1}n^{-1}\varkappa\,\mathrm{tg}\,\varkappa d$$

and

$$\varkappa(\beta) = (k^2 n^2 - \beta^2)^{1/2}$$

is the x component of the wave number in the core.

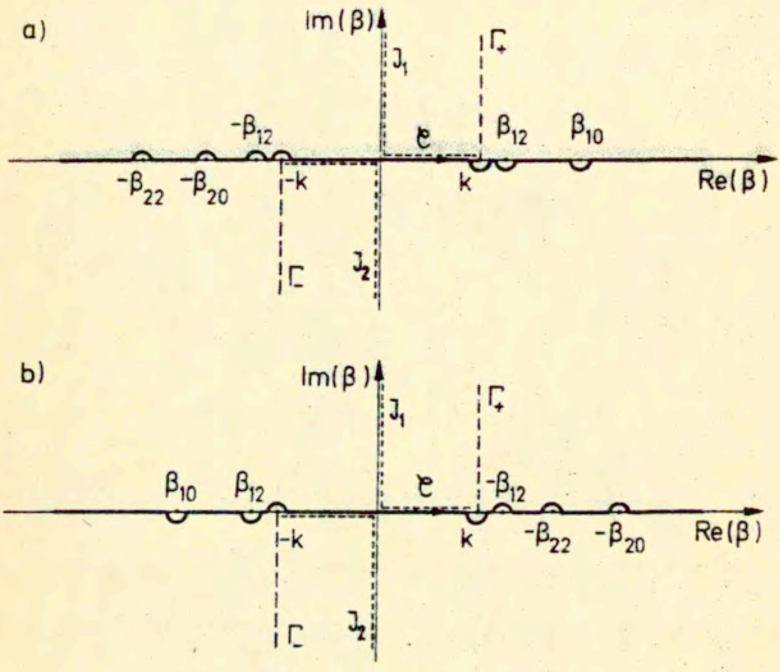

Fig. 2. The complex β-plane showing the integration contour C, the branch cuts Γ_+, Γ_-, the alternative branch cuts J_1, J_2 and the positions of poles at $\beta=\beta_{10}$, $\pm\beta_{12}$ of even guided modes for the waveguide 1 and at $\beta=-\beta_{20}$, β_{22} for the waveguide 2. Only two, forward modes are assumed to be guided by each waveguide:

(a) incidence of a single guided mode with the pole at $\beta=-\beta_{12}$,

(b) excitation of a single guided mode with the pole at $\beta=-\beta_{12}$

The first (second) integral in (2) expresses the part of the field for which the H_y (E_z) field component is continuous at the interface. Equivalence of the field representations (1) and (2) for $x > 0$ yelds:

$$(6) \quad A(\beta) = B(\beta) - C(\beta)k\eta(\beta)$$

It is not difficult to realize that anti-symmetric, with respect to α, parts $\{\Psi(\beta,x)\}_o$ and $\{\Phi(\beta,x)\}_o$ of the functions $\Psi(\beta,x)$ and $\Phi(\beta,x)$ describe the modes from the entire continuous and discrete spectrum of the ideal waveguide;

$$(7) \quad \{\Psi(\beta,x)\}_o = \frac{1}{k\eta(\beta)}\{\Phi(\beta,x)\}_o = \begin{cases} \alpha\cos\alpha x - ik\eta(\beta)\sin\alpha x, & 0<x \\ \alpha\cos\alpha(x+d)/\cos\alpha d, & -d<x<0 \end{cases}$$

where for the discrete spectrum the transverse resonance relation must be satisfied:

$$(8) \quad \alpha + k\eta(\beta) = 0 \quad \text{for } \beta=\beta_n,\ \alpha=\alpha_n=\alpha(\beta_n);\ n=0,2,4,\ldots$$

The term 'ideal' indicates the waveguide infinite along the guidance direction, undisturbed by any inhomogeneities or discontinuities. Every mode of the ideal waveguide yields the conventional impedance boundary condition at $x=0$.

$$(9) \quad [\partial/\partial x + ik\eta(\beta)]\{\Psi(\beta,x)\}_o = 0 \quad \text{for } x=0^+$$

Thus, $\eta(\beta)$ describes at the interface the relative surface impedance as seen by the field in the free-space ($x=0^+=\lim x$ where $x>0$ and $x \longrightarrow 0$).

In order to generalize the impedance boundary condition (9), let us impose on (2) the continuity - at the interface - condition of the transverse field components H_y and E_z. Then, we arrive at two integral equations for $x=0^+$:

$$(10) \quad \int_{\mathcal{C}} B(\beta)(\alpha+k\eta(\beta))\exp(i\beta z)\,d\beta = 0$$

$$(11) \quad \int_{\mathcal{C}} C(\beta)(\alpha+k\eta(\beta))\exp(i\beta z)\,d\beta = 0$$

Eqs (10) and (11) states the most general impedance boundary conditions for the fields expressed by the longitudinal spectral representation (2) .

Similarly, for taking into account in the total field presence of a discrete - not necessarily modal - part in a form:

$$a_i \exp i(\alpha_i x+\beta_i z)$$

we have, for $x > 0$

$$(1') \quad H_y(x,z) = \int_{\mathcal{C}} A(\beta)\exp(i(\alpha x+\beta z))\,d\beta + a_i \exp(i(\alpha_i x+\beta_i z))$$

and for $x \geq -d$

$$(2') \quad H_y(x,z) = \int_{\mathcal{C}} \left[B(\beta)\Psi(\beta,x) + C(\beta)\Phi(\beta,x)\right](\alpha - k\eta(\beta))^{-1}\exp(i\beta z)\,d\beta$$
$$+\left[b_i \Psi(\beta_i,x) + c_i \Phi(\beta_i,x)\right](\alpha_i - k\eta(\beta_i))^{-1}\exp(i\beta_i z)$$

where $\alpha_i^2 + \beta_i^2 = k^2$ and

$$(6') \quad a_i = b_i - k\eta(\beta_i)c_i$$

Imposing on (2') the continuity condition of H_y and E_z at the interface we finally obtain for $x = 0^+$

$$(10') \quad \int_{\mathcal{C}} B(\beta)(\alpha+k\eta(\beta))\exp(i\beta z)\,d\beta + b_i(\alpha_i+k\eta(\beta_i))\exp(i\beta_i z) = 0$$

$$(11') \quad \int_{\mathcal{C}} C(\beta)(\alpha+k\eta(\beta))\exp(i\beta z)\,d\beta + c_i(\alpha_i+k\eta(\beta_i))\exp(i\beta_i z) = 0$$

Now it is evident that (10) and (11) are in fact a substantial extension of (9). They can be as well applied to cases in which a single spectral constituent or even the entire spectral integral do not satisfy the interface continuity field conditions.

The application of the boundary conditions (10') and (11') to the solution procedure avoids the need to calculate the field within the core. Formulation of the problem is then carried out in terms of surface or leaky waves and spectral amplitudes of the plane wave expansion (1) rather, than in terms of guided, leaky or radiation modes of the waveguide. As it emerges in the next section, the integral form of (10-11) can be handled analytically be the Wiener-Hopf-Hibert method.

Although we have discussed the even TM field, (10)-(11) are valid also for the odd TM field, which means that they are anti--symmetric about the $z = -d$ plane, provided that:

$$(5') \qquad \eta(\beta) = iZk^{-1}n^{-1}\varkappa\cot\varkappa b.$$

For the TE field ($E_x = E_z = H_y = 0$), in all expressions from this section the relative instrinsic impedance Z of the core medium must be replaced by the relevant relative intrinsic admitance $Y=Z^{-1}$. The function $\eta(\beta)$ describes then relative surface admittance of the interface.

Now, we are in a position to formulate the problem of guided modes diffraction by the junction between two planar, symmetric, step-index waveguides.

3. Statement of the problem

Consider a junction between two planar waveguides with the same thickness 2d of the core, whose properties relative to the surrounding homogeneous medium are described by their surface impedances $\eta_1(\beta)$, $\eta_2(\beta)$ respectively (Fig.1). All other quantities, related to the waveguides 1 or 2, will be labelled in the same manner. It is assumed that surface impedance discontinuity èxists at $z = 0$. Due to the absence of the coupling between TM and TE, or the even and odd parts of the electromagnetic field, we assume the definite polarization and symmetry of the total field - namely, the same as that of the incident guided mode in waveguide 1. This needs appropriate choice of the surface impedance function from (5) or (5').

Let us define the incident field $u_i(x,z)$ in the region $x > 0$ (for all z) as a surface wave:

$$(12) \qquad u_i(x,z) = a_{1i}\exp\left(i(\alpha_{1i}x - \beta_{1i}z)\right),$$

where a_{1i} is the amplitude constant. Longitudinal β_{1i} and transverse $\alpha_{1i}=\alpha(\beta_{1i})$ wave numbers are determined by the resonance relation for the waveguide 1:

$$(13) \quad \alpha_{1i} + k\eta_1(\beta_{1i}) = 0.$$

The subscripts 1,i indicate the i-th guided mode of the waveguide 1.

We decompose the total field u(x,z) into a sum of incident and scattered fields: (see also Appendix) :

$$(14) \quad u(x,z) = u_i(x,z) + u_s(x,z)$$

and assume the spectral representation for the scattered field

$$(1) \quad u_s(x,z) = \int_C A(\beta)\exp(i(\alpha x + \beta z))\,d\beta \qquad \text{for } x>0$$

where the spectral amplitude $A(\beta)$ is to be determined.

The total field satisfies:

(I) the Helmholtz equation for $x>0$

$$(15) \quad [\nabla^2 + k^2]u(x,z) = 0$$

where continuity of the total field and its first derivatives are assumed as x approaches zero;

(II) impedance boundary conditions at $x=0^+$:

for $z>0$

$$(16) \quad \int_C B_1(\beta)(\alpha+k\eta_1(\beta))\exp(i\beta z)\,d\beta = 0$$

$$(17) \quad \int_C C_1(\beta)(\alpha+k\eta_1(\beta))\exp(i\beta z)\,d\beta = 0$$

and for $z<0$

$$(18) \quad \int_C B_2(\beta)(\alpha+k\eta_2(\beta))\exp(i\beta z)\,d\beta + b_{2i}(\alpha_{1i} + k\eta_2(-\beta_{1i}))\exp(-i\beta_{1i}z) = 0$$

$$(19)\quad \int_{\mathcal{C}} C_2(\beta)(\alpha + k\eta_2(\beta)) \exp(i\beta z)d\beta + c_{2i}(\alpha_{1i} + k\eta_2(-\beta_{1i})) \exp(-i\beta_{1i}z) = 0$$

where for j=1,2

$$(20)\quad B_j(\beta) - C_j(\beta)k\eta_j(\beta) = A(\beta)$$

$$(21)\quad b_{ji} - c_{ji}k\eta_j(-\beta_{1i}) = a_{1i} .$$

(III) radiation condition for $k > 0$:

$$(22)\quad \int_{SDP} A(\beta)\exp(i(\alpha x+\beta z))d\beta \longrightarrow 0 \text{ when } r \longrightarrow \infty$$

where $r = (x^2+z^2)^{1/2}$ and SDP is the steepest descent path;
(IV) edge condition at x=z=0:

$$(23)\quad (\partial/\partial r)u(x,z) = O(r^{-1+\mu}),\ \mu > 0 \text{ when } r \longrightarrow 0.$$

The above conditions now specify the problem completely and uniquely.

The edge condition implies:

$$(24)\quad A(\beta) = O(\beta^{-1-\mu}),\ \mu > 0, \text{ when } \beta \longrightarrow \infty$$

while the definitions of the contour $\mathcal{C}$, branch cuts Γ_+, Γ_- and the top Riemann sheet in (1) assure compliance with conditions (I) and (III) provided that:

$$(25)\quad A(\beta) = O((\beta+k)^{-1+\lambda}),\ \lambda > 0 \text{ when } \beta \longrightarrow -k.$$

For further analysis, it may be as well to note that for the structure which can support surface or leaky waves (8) is satisfied on the top Riemann sheet of the β-plane. Solutions of (8) appear always in pairs $\beta=\pm\beta_{jn}$, j=1,2, that is immediately seen from the equivalent form of the transverse resonance relation:

$$(26)\quad [k\delta_j(\beta) - \beta][k\delta_j(\beta) + \beta] / [\alpha - k\eta_j(\beta)] = 0,\ j=1,2$$

where

$$\delta_j(\beta) = \left[1 - \eta_j^2(\beta)\right]^{1/2}$$

and equality $\eta_j(\beta) = 0$ implies $\delta_j(\beta) = 1$.

4. Solution

From the boundary conditions (16) - (19), after enclosing at infinity the contour $\mathfrak{C}$ by semicircles in the upper (for $z > 0$) and lower (for $z < 0$) half-planes of β-plane, by virtue of (24) we obtain:

(27) $$B_1(\beta)(\alpha + k\eta_1(\beta)) = U_1(\beta)$$

(28) $$C_1(\beta)(\alpha + k\eta_1(\beta)) = U_2(\beta)$$

(29) $$B_2(\beta)(\alpha + k\eta_2(\beta)) = L_1(\beta) + b_{2i}(2\pi i)^{-1}(\alpha_{1i} + k\eta_2(-\beta_{1i}))(\beta + \beta_{1i})^{-1}$$

(30) $$C_2(\beta)(\alpha + k\eta_2(\beta)) = L_2(\beta) + c_{2i}(2\pi i)^{-1}(\alpha_{1i} + k\eta_2(-\beta_{1i}))(\beta + \beta_{1i})^{-1}$$

where the unknown functions $U_j(\beta)$ and $L_j(\beta)$ are holomorphic above and below $\mathfrak{C}$, respectively and the pole at $\beta = -\beta_{1i}$ must be placed under $\mathfrak{C}$ (Fig. 2). Let us take the poles of $\eta_1(\beta)$ and $\eta_2(\beta)$ to be placed below and above $\mathfrak{C}$ respectively. Then a function

$$(2\pi i)^{-1}(\alpha_{1i} - k\eta_2(-\beta_{1i}))(k\eta_2(\beta) - k\eta_2(-\beta_{1i}))(\beta + \beta_{1i})^{-1}$$

is holomorphic under $\mathfrak{C}$. Therefore, in view of (20) and (21), from (27) - (30) we finally obtain:

(31) $$A(\beta)(\alpha + k\eta_1(\beta)) = U(\beta)$$

(32) $$A(\beta)(\alpha + k\eta_2(\beta)) = L(\beta) + a_{1i}(2\pi i)^{-1}(\alpha_{1i} + k\eta_2(-\beta_{1i}))(\beta + \beta_{1i})^{-1}$$

where $U(\beta)$ ($L(\beta)$) are holomorphic above (below) $\mathfrak{C}$. The spectral amplitude $A(\beta)$, as a solution of (31) and (32) determines the scattered field only outside the core but it will prove to be all we need to determine the total field everywhere (see Appendix).

In order to obtain the Wiener-Hopf equation in the vicinity of $\mathfrak{C}$ we eliminate $A(\beta)$ from (31) and (32). Hence, we obtain:

$$(33) \quad L(\beta) = \frac{\alpha + k\eta_2(\beta)}{\alpha + k\eta_1(\beta)} U(\beta) - \frac{a_{1i}}{2\pi i} \frac{\alpha_{1i} + k\eta_2(-\beta_{1i})}{\beta + \beta_{1i}}$$

The key step of the Wiener-Hopf-Hilbert method consists in analytic continuation of all components of (33) into the lower part of β-plane – that is, below the contour $\mathfrak{C}$ and in re-expressing (33) into a new Hilbert problem at the contour Γ_- (see Fig. 2) [5]. To make this procedure to be possible, the left and right hand parts of (33) must everywhere be holomorphic under $\mathfrak{C}$, excluding the branch cut Γ_-, which in general is not applicable as regards (33) . To overcome this difficulty, let us assume the following form of $U(\beta)$:

$$(34) \quad U(\beta) = \frac{V(\beta)\,[\beta + k\delta_1(\beta)]}{[\beta + \beta_{1i}][\beta + k\delta_2(\beta)]}$$

where $V(\beta)$ is holomorphic in the entire β-plane but Γ_-. Furthermore, the following restriction imposed on $U(\beta)$ ensures regularity of the right hand part of (33) at $\beta = -\beta_{1i}$:

$$(35) \quad U(-\beta_{1i}) = a_{1i}(2\pi i)^{-1}[\beta_{1i}/\alpha_{1i} + k\eta_1'(-\beta_{1i})]$$

where primes are taken to mean derivatives with respect to β. Eq. (33) may now be rewritten as

$$(36) \quad L(\beta) = \frac{[\alpha + k\eta_2(\beta)][\beta + k\delta_1(\beta)]\; V(\beta)}{[\alpha + k\eta_1(\beta)][\beta + k\delta_2(\beta)][\beta + \beta_{1i}]} + \frac{a_{1i}[\alpha_{1i} + k\eta_2(-\beta_{1i})]}{2\pi i[\beta + \beta_{1i}]}$$

where the left and right hand parts of (36) are now holomorphic below $\mathfrak{C}$ but Γ_-. Taking limits of both parts from the left and right hand side of Γ_-, after elementary calculations we arrive at the Hilbert equation:

$$(37) \qquad H(t)\, V_-(t) = V_+(t)\,, \qquad t \in \Gamma_-,$$

where

$$(38) \qquad H(\beta) = \frac{[\alpha + k\eta_1]\,[\alpha - k\eta_2]}{[\alpha - k\eta_1]\,[\alpha + k\eta_2]}$$

and the limiting values of functions on the left and right sides of Γ_- are distinguished by the (+) and (-) subscripts, respectively. It is assumed that $\alpha_+(t)=\alpha(t)$ and $\alpha(t)\pm k\eta_j(t)\neq 0$ for t placed on Γ_- and j = 1,2. The eqs. (37), (38) state the typical homogeneous Hilbert problem on an open contour Γ_- [9] where the positive direction is taken from $-k-i\infty$ to $-k$. The solution of (37) has the general form:

$$(39) \qquad V(\beta) = w_m(\beta)\sum_n c_n(\beta+k)^n \exp(Q(\beta))$$

where the function $Q(\beta)$, holomorphic everywhere except on Γ_-, is defined by the Cauchy integral:

$$(40) \qquad Q(\beta) = \frac{1}{2\pi i}\int_{\Gamma} \ln[H(t)]\,(t-\beta)^{-1}dt,$$

with m as unspecified natural number, n as integers, c_n as constant coefficients and $w_m(\beta)$ as undetermined polynomial of m-th order. Taking into account the boundeness of $Q(\beta)$ at $\beta = -k$ and at infinity, from (24), (25) we infer that m=n=0 . Thus, calculating from (35) the sole non-zero coefficient c_0 and returning to (31), we finally obtain:

$$(41) \quad A(\beta) = -\frac{a_{1i}\,\beta_{1i}[k\delta_2(-\beta_{1i})-\beta_{1i}]\,[\alpha - k\eta_1(\beta)]}{2\pi i\,\alpha_{1i}[\beta+k\delta_2(\beta)][\beta - k\delta_1(\beta)]\,[\beta+\beta_{1i}]}$$

$$\cdot \exp\left[Q(\beta) - Q(-\beta_{1i})\right].$$

The spectral amplitude $A(\beta)$ possesses simple poles at $\beta=-\beta_{1i};-\beta_{2n}$ belowe $\mathfrak{C}$ and at $\beta=\beta_{1n}$ above $\mathfrak{C}$, where n are even (odd) natural numbers enumerating even (odd) spectral modes in each waveguide.

The positions of poles in relation to $\mathfrak{C}$ determine the appropriate indentations of $\mathfrak{C}$ as, for the presence of surface wave poles, is indicated in Fig.2. The poles of the function $\eta_1(\beta)$ and $\eta_2(\beta)$ do not lead to singularities in $A(\beta)$ and so do not affect the shape of $\mathfrak{C}$.

Although (41) together with (1) and (5) or (5') already solve the problem, for far field analysis the original contour $\mathfrak{C}$ may be deformed into the steepest descent path SDP, dependent on an angle of observation defined by:

(42) $$x=r\sin\theta \quad , \quad z=r\cos\theta \quad , \quad 0\leqslant\theta\leqslant\pi$$

Then, the total field decomposes into an exact diffracted field:

(43) $$u_d(x,z)=\int_{SDP} A(\beta)\exp(i(\alpha x+\beta z))\,d\beta \quad ,$$

and a geometrical optical field as a sum of the incident field and all residue contributions from the poles of $A(\beta)$ situated between $\mathfrak{C}$ and SDP:

(44) $$u_g(r,\theta)=a_{1i}h(\pi-\psi_{1i}-\theta)\exp\left[-ikr\cos(\varphi_{1i}+\theta)\right]$$
$$+\sum_n p_{1n}h(\psi_{1n}-\theta)\exp\left[ikr\cos(\varphi_{1n}-\theta)\right]$$
$$+\sum_n p_{2n}h(\theta-\pi+\psi_{2n})\quad\exp\left[-ikr(\varphi_{2n}+\theta)\right] ,$$

where for $j=1,2$, and for $-\pi/2<\mathrm{Re}(\varphi_{jn})<3\pi/2$, we have:

(45) $$\varphi_{jn}=\cos^{-1}(\beta_{jn}/k) , \qquad \sin\varphi_{jn}=\alpha_{jn}/k ,$$

(46) $$\psi_{jn}=\mathrm{Re}(\varphi_{jn})+\mathrm{sgn}\left[\mathrm{Im}(\varphi_{jn})\right]\cos^{-1}(1/\cosh(\mathrm{Im}(\varphi_{jn}))) .$$

The Heaviside unit function $h(\cdot)$ describes the domain of definition of the incident, reflected and transmitted surface or leaky waves with the complex angles of propagation $\pi-\varphi_{1i}$, φ_{1n}, $\pi-\varphi_{2n}$, with the shadow boundary angles $\pi-\psi_{1i}$, ψ_{1n}, $\pi-\psi_{2n}$ and with the constant amplitudes a_{1i}, p_{1n}, p_{2n}, respectively. Calculation of the residues at $\beta=\beta_{1n}$, $-\beta_{2n}$ yields:

$$(47)\quad p_{1n} = -\frac{2a_{1i}\beta_{1i}\left[k\delta_2(-\beta_{1i})-\beta_{1i}\right]\exp\left[Q(\beta_{1n})-Q(-\beta_{1i})\right]}{\left[k\delta_2(\beta_{1n})+\beta_{1n}\right]\left[1-k\delta'_1(\beta_{1n})\right]\left[\beta_{1n}+\beta_{1i}\right]}$$

$$(48)\quad p_{2n} = -\frac{a_{1i}\beta_{1i}\left[k\delta_2(-\beta_{1i})-\beta_{1i}\right]\left[\alpha_{2n}-k\eta_1(-\beta_{2n})\right]}{\alpha_{1i}\left[1+k\delta'_2(-\beta_{2n})\right]\left[k\delta_1(-\beta_{2n})+\beta_{2n}\right]\left[\beta_{1i}-\beta_{2n}\right]}$$

$$\cdot\exp\left[Q(-\beta_{2n})-Q(-\beta_{1i})\right].$$

As might be expected, for the limiting case of two identical waveguides the total field reduces to the incident field.

5. Remarks and conclusions

The solution of the diffraction problem gives the exact analytical expressions (47) and (48) for the constant amplitudes of every guided or leaky mode supported by the guides structure. The radiation modes excited by the junction discontinuity build up the exact diffracted field as described by the integral (43), which can be evaluated asymptotically by the well known steepest descent method [10] . The total power loss at the junction can be computed exactly from a knowledge of constant amplitudes of discrete modes or more directly from the asymptotic evaluation of the integral (43), which gives in addition the directivity of the radiation field. Therefore, although the formulae (41) - (48) describe only the total field outside the core, all information of practical interest can be extracted directly from them.

Due to the linearity of the problem, for the incident field composed of several guided modes with arbitrary symmetry and polarization, solution is arrived at by summation of all solutions (41) for individual excitations, where the appropriate definitions (5) or (5') of surface impedances must be chosen.

It may seem surprising how the serious difficulties related to compliance with the continuity conditions of the tangent field components at the junction may be avoided in the solution procedure in question. In fact, those restrictions are replaced by the edge (24) and the integral convergenve (25) conditions. The simplicity of these conditions appears to be the main advantage of the surface impedance approach presented as regards the waveguide discontinuity problems.

It is pertinent to note that the directions of phase increase and local energy transfer of the plane waves outside the core are assumed to have equal signs. This need not hold in the core, so that the backward [8] as well as forward modes are admissible by the general form of the solution. In effect, for the certain forward or backward mode, additional restrictions must be imposed on the transverse resonance relation (8) and the definition (45) of the complex angle of propagation. For instance, it must be stipulated that $Re(\beta_{1n}) > 0$, $-\pi/2 \leqslant Re(\varphi_{1n}) < \pi/2$ for the n-th forward mode, whereas for the n-th backward mode $Re(\beta_{1n}) < 0$, $\frac{\pi}{2} < Re(\varphi_{1n}) \leqslant \frac{3\pi}{2}$ should be taken, which results in change of the position of the pole at $\beta = \beta_{1n}$ symmetrically about the origin in the β - plane. At the same time, the positions of the pole in relation to the contour $\mathcal{C}$ are the same for both cases. Figs. 2 (a) and 2 (b) show possible positions of the poles relevant to the two forward guided modes of each waveguide, while the replacement of Fig.2 (a) with Fig. 2 (b) gives the same for all guided modes with bacward characteristics. Backward waves may occur, when for example, the core region is filled by dispersive medium - e.g. plasma - and such the structure can be similarly analysed by means of the solution derived.

The solution can be applied also the reverse problem, where constant amplitudes are sought for discrete modes incident from the left and right to the junction, for which the only single reflected mode with amplitude a_{1i} is guided by the waveguide 1. The unknown amplitudes are determined by (47), (48), and the appropriate positions of the poles of incident forward guided modes are shown in Fig. 2 (b). Fig. 2 (a) is relevant to the reverse problem for guided modes with backward characteristics.

The impedance approach presented can be applied to all open, planar, semi-infinite guiding structures, assuming appropriate form of the surface impedance functions. Thus, generalization of the solution to the junction between two asymmetric slab waveguides with different heights of the slabs is possible. The cases of granded-index, periodic or anisotropic [3] planar waveguides seem also to be tractable. Diffraction problems at the and of the waveguide or in bifurcated waveguides can be considered similarly.

The exact solution procedure, or an approximate one based on it for more complicated geometries make it possible to analyse rigorously such integrated optic components as couplers, converters or splitters. In each case, solution has a simple analytic form, which gives a direct and intimate description of the diffraction processes investigated.

Finally, let us mention the problem of the modal representation of the total field. Such a representation is not adequate to the solution procedure where we have been seeking for the explicit form of the scattered field. Owing to the postulated form of the incident field, the scattered field does not satisfy separately the continuity relation of tangent field components at the interface. Therefore, it can not be decomposed into a complete set of the modes of the ideal waveguide. Nevertheless, this is the case for the total field (see Appendix). Hence, the solution obtained gives the direct evidence of the completness property of the modal representation for the diffraction problems of open waveguides - at least for the simplest case analysed here.

It is worth noting that in the longitudinal representation the spectral amplitude $A(\beta)$ of the scattered field has been found only for the field outside the core. However, the modal representation of the total field is entirely determined by this amplitude (see Appendix). Thus, the surface impedance approach solves the presented diffraction problem in all regions of the joint open waveguides.

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Appendix

Modal representation of the total field

The total field in the first ($z > 0$, $j=1$) and in the second ($z < 0$, $j=2$) waveguides has been sought in the following longitudinal spectral representation:

(A1) $u(x,z) =$

$$\left[b_{ji}\Psi_j(-\beta_{1i},x)+c_{ji}\Phi_j(-\beta_{1i},x)\right](\alpha_{1i}-k\eta_j(-\beta_{1i}))^{-1}\exp(-i\beta_{1i}z)$$
$$+\int_{C}\left[B_j(\beta)\Psi_j(\beta,x)+C_j(\beta)\Phi_j(\beta,x)\right](\alpha-k\eta_j(\beta))^{-1}\exp(i\beta z)\,d\beta$$

where $\Psi_j(\beta,x)$ and $\Phi_j(\beta,x)$ were expressed by the substitution of $\eta(\beta)=\eta_j(\beta)$ in (3) and (4). By virtue of (20) and (21) the total field representation can also take the equivalent form:

(A2) $u(x,z) =$

$$\left\{\begin{matrix} a_{1i}\exp(i\alpha_{1i}x) \\ \left[a_{1i}+c_{ji}(\alpha_{1i}+k\eta_j(-\beta_{1i}))\right]\cos\varkappa_{ji}(x+d)/\cos\varkappa_j d \end{matrix}\right\}\exp(-i\beta_{1i}z)$$

$$+\int_{C}\begin{cases}A(\beta)\exp(i\alpha x)\\ [A(\beta)+C_j(\beta)(\alpha+k\eta_j(\beta)]\cos\varkappa_j(x+d)/\cos\varkappa_j d\end{cases}\exp(i\beta z)d\beta$$

where $\varkappa_{ji}=\varkappa_j(-\beta_{1i})$. The upper line is hereinafter reserved for the free-space region $(x>0)$ and the lower - for the core $(-d\leq x<0)$. In (A2) the spectral amplitude $A(\beta)$ is determined completely by (41), whereas only general analytic properties of $C_j(\beta)$ are available from (28) and (30).

In order to obtain the modal representation of the total field we transform (A2) into a transverse spectral representation, that is, the integral along the real axis in the complex β-plane we replace by an integral along the real axis in a complex α-plane. It can be effected by deforming - along semicircles at infinity - the original contour C of integration into the paths around the alternative branch cuts J_1 ($\mathrm{Im}(\alpha)=0$, $\mathrm{Im}(\beta)>0$) for the waveguide 1 and J_2 ($\mathrm{Im}(\alpha)=0$, $\mathrm{Im}(\beta)<0$) for the waveguide 2 (see Fig. 2). Owing to the behavour of $A(\beta)$ at infinity, to a presence of the exponent factor of the integrand in(A2) for $x>0$ and to the symmetry of this integrand with respect to $\varkappa$ for $-d\leq x<0$, the semicircles contribute nothing to the integral. What is more, $A(\beta)$ satisfies on J_2 the same Hilbert problem as on Γ_-, which results in:

$$\text{(A3)}\qquad \left[\frac{A(t)}{\alpha-k\eta_j(t)}\right]_- = \left[\frac{A(t)}{\alpha-k\eta_j(t)}\right]_+, \qquad t\in J_j$$

Hence, in view of (28)and (30), by Cauchy's theorem we finally obtain:

(A4) $u(x,z) =$

$$a_{1i}h(z)\begin{cases}\exp(i\alpha_{1i}x)\\ \cos\varkappa_{ji}(x+d)/\cos\varkappa_{ji}d\end{cases}\exp(-i\beta_{1i}z)$$

$$+\sum_n p_{jn}\begin{cases}\exp(i\alpha_{jn}x)\\ \cos\varkappa_{jn}(x+d)/\cos\varkappa_{jn}d\end{cases}\exp((-1)^{j-1}i\beta_{jn}z)$$

$$+ \int_0^\infty 2\,\frac{\alpha}{\beta}\left\{\frac{A(\beta,\alpha)}{\alpha - k\eta_j(\beta)}\right\}_e \left\{\begin{matrix}\alpha\cos\alpha x - ik\eta_j(\beta)\sin\alpha x \\ \alpha\cos\varkappa_j(x+d)/\cos\varkappa_j d\end{matrix}\right\}\exp(i\beta z)\,d\alpha$$

where

$$\varkappa_{jn} = \varkappa_j(\beta_{jn})$$

$$\alpha_{jn} + k\eta_j(\beta_{jn}) = 0$$

and

$$\left\{\frac{A(\beta,\alpha)}{\alpha - k\eta_j(\beta)}\right\}_e = \frac{1}{2}\left\{\frac{A(\beta,\alpha)}{\alpha - k\eta_j(\beta)} - \frac{A(\beta,-\alpha)}{\alpha + k\eta_j(\beta)}\right\}$$

states a symmetric part of $A(\beta)(\alpha - k\eta_j(\beta))^{-1}$ with respect to α. Here, an explicit dependence of the spectral amplitude $A(\beta,\alpha)$ on α is shown.

The transverse spectral representation (A4) is expressed by the decomposition of the total field into a complete set of normal modes of the ideal waveguide, described by the parameters equal to those of the first or second semi-infinite waveguide, respectively. In the succesive terms in (A4) we recognize the surface incident mode, reflected or transmitted guided modes and radiation modes for the first (j=1) and second (j=2) waveguide. It is of interest to underline, that the domain of definition of the incident wave is now restricted to the waveguide 1 only and - apart from the constant amplitude a_{1i} of the incident field - the total field is determined completely by the spectral amplitude $A(\beta)$.