

216.

TABLES OF THE DEVELOPMENTS OF FUNCTIONS IN THE THEORY OF ELLIPTIC MOTION.

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THE notation made use of is as follows; viz., r, f, a, e, g denote

- r , the radius vector;
- f , the true anomaly;
- a , the mean distance;
- e , the eccentricity;
- g , the mean anomaly;

so that

$$\frac{r}{a} = \text{elqr } (e, g),$$

and

$$f = \text{elta } (e, g),$$

are known functions of e, g . Moreover, x denotes the periodic part of the quotient radius $\frac{r}{a}$, and y the equation of the centre, or periodic part of the true anomaly f ;
so that

$$\begin{aligned}\frac{r}{a} &= 1 + x, \\ f &= g + y,\end{aligned}$$

and (x, y) are therefore also known functions of e, g .

Formulæ for the development in multiple cosines or sines, up to the terms in e^7 , of the functions

$$(x^0, x^1, \dots x^7) \frac{\cos}{\sin} jy,$$

where j is an indeterminate symbol, are given by Leverrier in the *Annales de l'Observatoire de Paris*, tom. I. (1855), pp. 346—348. It occurred to me that it would be desirable to form tables such as are contained in the present memoir, of various functions of the forms

$$x^m \frac{\cos}{\sin} jf, \quad \left(\frac{r}{a}\right)^{\pm m} \frac{\cos}{\sin} jf,$$

obtainable from Leverrier's formulæ by assigning particular integer values to the symbol j . The calculations were performed under my superintendence by Messrs Creedy and Davis; and I have to acknowledge my obligation for a grant to defray the expenses, made to me out of the Donation Fund at the disposal of the Council of the Royal Society.

A cosine series is in general represented in the form

$$\Sigma [\cos]^i \cos ig,$$

where i extends from $-\infty$ to $+\infty$, and the coefficients $[\cos]^i$ satisfy the condition

$$[\cos]^{-i} = [\cos]^i;$$

and in like manner a sine series is represented in the form

$$\Sigma [\sin]^i \sin ig,$$

where i extends from $-\infty$ to $+\infty$, and the coefficients $[\sin]^i$ satisfy the condition

$$[\sin]^{-i} = -[\sin]^i, \text{ and in particular}$$

$$[\sin]^0 = 0.$$

Thus $[\cos]^i$ is a general symbol denoting the coefficient of $\cos ig$, in a cosine series considered as involving as well negative as positive multiples of the argument g , or, what is the same thing, $[\cos]^0$ is the constant term and $[\cos]^i$ is the half coefficient of $\cos ig$, when the series is considered as containing only positive multiples of the argument. And in like manner $[\sin]^i$ is a general symbol for the coefficient of $\sin ig$, in a sine series considered as involving as well negative as positive multiples of the argument g , or, what is the same thing, $[\sin]^i$ is the half coefficient of $\sin ig$, when the series is considered as containing only positive multiples of the argument.

In the case of a pair of corresponding functions $x^m \cos jf$ and $x^m \sin jf$, or $\left(\frac{r}{a}\right)^{\pm m} \cos jf$ and $\left(\frac{r}{a}\right)^{\pm m} \sin jf$, one of them expanded in the form

$$\sum [\cos]^i \cos ig,$$

and the other in the form

$$\sum [\sin]^i \sin ig,$$

the sums and differences of the corresponding coefficients $[\cos]^i$, $[\sin]^i$, are represented by the notation

$$[\cos \pm \sin]^i,$$

and it is clear that we have

$$[\cos + \sin]^{-i} = [\cos - \sin]^i, [\cos \pm \sin]^0 = [\cos]^0.$$

These coefficients $[\cos \pm \sin]^i$ are for many purposes equally useful with the coefficients $[\cos]^i$, $[\sin]^i$, and they are here tabulated accordingly.

The functions tabulated are as follows:—

$$(x^1, x^2, \dots, x^7), [\cos],$$

$$(x^0, x^1, x^2, \dots, x^7) \frac{\cos}{\sin} jf, j=1 \text{ to } j=7, [\cos], [\sin], [\cos \pm \sin],$$

$$\left(\left(\frac{r}{a}\right)^{+4}, \dots, \left(\frac{r}{a}\right)^{+1}, \log \frac{r}{a}, \left(\frac{r}{a}\right)^{-1}, \dots, \left(\frac{r}{a}\right)^{-5} \right) [\cos],$$

$$\left(\left(\frac{r}{a}\right)^{+4}, \dots, \left(\frac{r}{a}\right)^{+1}, \left(\frac{r}{a}\right)^{-1}, \dots, \left(\frac{r}{a}\right)^{-5} \right) \frac{\cos}{\sin} jf, j=1 \text{ to } j=5, [\cos], [\sin], [\cos \pm \sin],$$

all the developments being carried up to e^7 , the limit of the formulæ from which they are deduced.

The calculations are based upon Leverrier's formulæ for

$$(x^0, x^1, \dots, x^7) \frac{\cos}{\sin} jy,$$

which are made use of under the following forms, which I call the Datum Tables

$$(x^0, \dots, x^7) \frac{\cos}{\sin} jy.$$

Cosine Table. $\frac{1}{2}$ of	[] ⁰				[] ¹			
	$\frac{e^0}{2}$	$\frac{e^2}{16}$	$\frac{e^4}{768}$	$\frac{e^6}{92160}$	$\frac{e}{4}$	$\frac{e^3}{96}$	$\frac{e^5}{7680}$	$\frac{e^7}{1290240}$
$x^0 \cos jy$	+ 1	- 8j ²	+ 96j ⁴	- 1280j ⁶	o	- 30j ²	+ 800j ⁴	- 16800j ⁶
		- 54j ²	+ 3920j ⁴	- 3440j ²		- 120j ²	+ 30590j ⁴	- 29470j ²
$x \cos jy$	+ 4	- 48j ²	o j ⁴	- 360j ²	- 1	+ 12j ²	- 160j ⁴	+ 2240j ⁶
					+ 9	- 370j ²	- 980j ⁴	
					- 50	+ 9695j ²		+ 245
$x^2 \cos jy$	+ 4	- 96j ²	+ 1920j ⁴	- 1320j ²	- 12	- 244j ²	+ 16800j ⁴	
					+ 80	- 6300j ²		- 420
$x^3 \cos jy$		+ 144	- 2880j ²		- 18	+ 480j ²	- 10080j ⁴	
					+ 360	- 22050j ²		- 3150
$x^4 \cos jy$		+ 144	- 5760j ²			- 960	- 5040j ²	
						+ 10080		
$x^5 \cos jy$			+ 14400			- 1200	+ 50400j ²	
						+ 37800		
$x^6 \cos jy$			+ 14400				- 151200	
$x^7 \cos jy$			o				- 176400	

Datum Table $(x_0 \dots x^7) \cos jy \dots$

Cosine Table. $\frac{1}{2}$ of	$[]^2$			$[]^2$		
	$\frac{e^2}{16}$	$\frac{e^4}{768}$	$\frac{e^6}{92160}$	$\frac{e^3}{96}$	$\frac{e^5}{7680}$	$\frac{e^7}{1290240}$
$x^0 \cos jy$	+ 4j ²	- 64j ⁴	+ 960j ⁶	+ 30j ²	- 1200j ⁴	+ 30240j ⁶
		- 256j ²	+ 11820j ⁴		- 3240j ²	+ 243810j ⁴
			+ 4710j ²			+ 173502j ²
$x \cos jy$	- 2	+ 192j ²	- 4800j ⁴	- 12j ²	+ 240j ⁴	- 4032j ⁶
		+ 64	- 8190j ²	- 9	+ 2745j ²	- 122220j ⁴
			- 1440		+ 675	- 225099j ²
						- 35721
$x^2 \cos jy$	+ 2	0j ²	- 480j ⁴	+ 12	- 840j ²	+ 10080j ⁴
		- 96	+ 1590j ²		- 1080	+ 102060j ²
			+ 2400			+ 61236
$x^3 \cos jy$		0	+ 7920j ²	- 6	- 240j ²	+ 10080j ⁴
			+ 1440		+ 540	+ 85050j ²
						- 17010
$x^4 \cos jy$		+ 96	- 1440j ²		+ 480	- 95760j ²
			- 8640			- 90720
$x^5 \cos jy$			+ 3600		- 600	- 10080j ²
						+ 113400
$x^6 \cos jy$			+ 10800			+ 30240
$x^7 \cos jy$			0			- 105840

... Datum Table ($x^0 \dots x^7$) $\cos jy \dots$

Cosine Table. $\frac{1}{2}$ of	[] ⁴		[] ⁵		[] ⁶		[] ⁷	
	$\frac{e^4}{768}$	$\frac{e^6}{92160}$	$\frac{e^5}{7680}$	$\frac{e^7}{1290240}$	$\frac{e^6}{92160}$	$\frac{e^7}{1290240}$	$\frac{e^7}{1290240}$	
$x^0 \cos jy$	+ 16j ⁴	- 384j ⁶	+ 400j ⁴	- 16800j ⁶	+ 64j ⁶	+ 3360j ⁶		
	+ 283j ²	- 22440j ⁴	+ 3360j ²	- 467530j ⁴	+ 8660j ⁴	+ 193130j ⁴		
		- 51528j ²		- 972790j ²	+ 48538j ²	+ 828758j ²		
$x \cos jy$	- 168j ²	+ 7680j ⁴	- 80j ⁴	+ 2240j ⁶	- 2880j ⁴	- 448j ⁶		
	- 64	+ 46260j ²	- 2375j ²	+ 205660j ⁴	- 37890j ²	- 82460j ⁴		
		+ 9216	- 625	+ 905135j ²	- 7776	- 689731j ²		
				+ 153125		- 117649		
$x^2 \cos jy$	+ 48j ²	- 960j ⁴	+ 1080j ²	- 50400j ⁴	+ 480j ⁴	+ 23520j ⁴		
	+ 96	- 22380j ²	+ 1000	- 540540j ²	+ 21450j ²	+ 444780j ²		
		- 15360		- 262500	+ 12960	+ 201684		
$x^3 \cos jy$	- 72	+ 1440j ²	- 240j ²	+ 3360j ⁴	- 7920j ²	- 3360j ⁴		
		+ 11520	- 900	+ 147630j ²	- 12960	- 210630j ²		
				+ 236250		- 216090		
$x^4 \cos jy$	+ 24	+ 2880j ²	+ 480	+ 35280j ²	+ 1440j ²	+ 65520j ²		
		+ 0		- 84000	+ 8640	+ 164640		
$x^5 \cos jy$		- 7200	- 120	- 30240j ²	- 3600	- 10080j ²		
				- 63000		- 88200		
$x^6 \cos jy$		+ 4320		+ 90720	+ 720	+ 30240		
$x^7 \cos jy$		0		- 35280	0	- 5040		

... Datum Table $(x^0 \dots x^7) \cos jy$.

Sine Table. $\frac{1}{2}$ of	[] ⁰				[] ¹			
	$\frac{e^0}{2}$	$\frac{e^2}{16}$	$\frac{e^4}{768}$	$\frac{e^6}{92160}$	$\frac{e}{4}$	$\frac{e^3}{96}$	$\frac{e^5}{7680}$	$\frac{e^7}{1290240}$
$x^0 \sin j y$					$+ 2j$	$- 24j^3$	$+ 320j^5$	$- 4480j^7$
					$- 6j$	$+ 260j^3$	$- 1400j^5$	
						$+ 100j$	$- 17290j^3$	
							$+ 7490j$	
$x \sin j y$					$+ 21j$	$- 400j^3$	$+ 5040j^5$	
						$- 180j$	$+ 35j^3$	
							$+ 6125j$	
$x^2 \sin j y$					$+ 12j$	$- 320j^3$	$+ 6720j^5$	
						$+ 160j$	$- 420j^3$	
							$+ 3780j$	
$x^3 \sin j y$						$+ 840j$	$- 25200j^3$	
							$- 10080j$	
$x^4 \sin j y$						$+ 480j$	$- 20160j^3$	
							$+ 25200j$	
$x^5 \sin j y$								$+ 88200j$
$x^6 \sin j y$								$+ 50400j$
$x^7 \sin j y$								o

Datum Table $(x^8 \dots x^7) \sin j y \dots$

Sine Table. $\frac{1}{2}$ of	[] ²			[] ³		
	$\frac{e^2}{16}$	$\frac{e^4}{768}$	$\frac{e^6}{92160}$	$\frac{e^3}{96}$	$\frac{e^5}{7680}$	$\frac{e^7}{1290240}$
$x^0 \sin jy$	+ 5j	- 240j ³	+ 6000j ⁵	+ 8j ³	- 160j ⁵	+ 2688j ⁷
		- 88j	+ 8895j ³	+ 26j	- 3070j ³	+ 125160j ⁵
			+ 2040j		- 1290j	+ 257166j ³
						+ 59850j
$x \sin jy$	- 4j	+ 64j ³	- 960j ⁵	- 27j	+ 1400j ³	- 36624j ⁵
		+ 184j	- 8340j ³		+ 2300j	- 207165j ³
			- 5700j			- 145971j
$x^2 \sin jy$		- 72j	- 1200j ³	+ 12j	- 160j ³	+ 1344j ⁵
			+ 4260j		- 1600j	+ 39900j ³
						+ 130788j
$x^3 \sin jy$		- 96j	+ 2880j ³		- 180j	+ 55440j ⁵
			+ 6120j			+ 32760j
$x^4 \sin jy$			- 8280j		+ 720j	- 20160j ³
						- 156240j
$x^5 \sin jy$			- 7200j			+ 37800j
$x^6 \sin jy$			o			+ 90720j
$x^7 \sin jy$			o			o

... Datum Table $(x^0 \dots x^7) \sin j - \dots$

Sine Table. $\frac{1}{2}$ of	[] ⁴		[] ⁵		[] ⁶		[] ⁷	
	$\frac{e^4}{768}$	$\frac{e^6}{92160}$	$\frac{e^5}{7680}$	$\frac{e^7}{1290240}$	$\frac{e^6}{92160}$	$\frac{e^7}{1290240}$		
$x^0 \sin j y$	+ 120j ³	- 4800j ⁵	+ 32j ⁵	- 896j ⁷	+ 1200j ⁵	+ 128j ⁷		
	+ 206j	- 49200j ³	+ 1790j ³	- 124600j ⁵	+ 29835j ³	+ 35560j ⁵		
		- 21648j	+ 2194j	- 939722j ³	+ 29352j	+ 563486j ³		
				- 416990j		+ 472730j		
$x \sin j y$	- 32j ³	+ 768j ⁵	- 760j ³	+ 34608j ⁵	- 192j ⁵	- 9744j ⁵		
	- 236j	+ 27840j ³	- 2640j	+ 599935j ³	- 15780j ³	- 339745j ³		
		+ 35736j		+ 657601j	- 36324j	- 595231j		
$x^2 \sin j y$	+ 156j	- 7680j ³	+ 160j ³	- 4032j ⁵	+ 5520j ³	+ 1344j ⁵		
		- 30120j	+ 2080j	- 237300j ³	+ 31620j	+ 152460j ³		
				- 610764j		+ 552468j		
$x^3 \sin j y$	- 48j	o j ³	- 1020j	+ 35280j ³	- 960j ³	- 45360j ³		
		+ 8640j		+ 303240j	- 19320j	- 384720j		
$x^4 \sin j y$		+ 7200j	+ 240j	+ 6720j ³	+ 7560j	+ 6720j ³		
				+ 11760j		+ 193200j		
$x^5 \sin j y$		- 5760j		- 113400j	- 1440j	- 63000j		
$x^6 \sin j y$		o		+ 50400j	o	+ 10080j		
$x^7 \sin j y$		o		o	o	o		

... Datum Table ($x^0 \dots x^7$) $\sin j y$.

It may be remarked that with a view to the convenience of calculation, the Datum Tables give the half values of the several functions; thus, e.g., we have

$$\begin{aligned} \frac{1}{2} x^0 \cos jy = & \left\{ +1 \frac{e^0}{2} - 8j^2 \frac{e^2}{16} + (96j^4 - 54j^2) \frac{e^4}{768} + \&c. \right\} + \left\{ 0 \frac{e}{4} - 30j^3 \frac{e^3}{96} + \&c. \right\} 2 \cos g \\ & + \left\{ 4j^2 \frac{e^2}{16} + \&c. \right\} 2 \cos 2g + \&c. \end{aligned}$$

and so for the other functions in the two tables.

The calculation, for a given value of j , of the functions (x^0, x^1, \dots, x^7) $\frac{\cos}{\sin} jy$ is performed as follows; the numerical value of j is in the first instance substituted throughout the two Datum tables, so as to obtain the numerical values of the separate terms $\pm Aj^m$; when this is done, the terms in each compartment are added together with the proper signs, and the results written down in a single table as follows, viz. if $j = 1$, and if we write down only the portion of the table which corresponds to $x^0 \frac{\cos}{\sin} y$, (the portions corresponding to $x^1 \frac{\cos}{\sin} y, x^2 \frac{\cos}{\sin} y \&c. \dots x^7 \frac{\cos}{\sin} y$, being of a precisely similar form, and coming into the table in order one under the other) this is

	[] ⁰				[] ¹			
	$\frac{e^0}{2}$	$\frac{e^2}{16}$	$\frac{e^4}{768}$	$\frac{e^6}{92160}$	$\frac{e}{4}$	$\frac{e^3}{96}$	$\frac{e^5}{7680}$	$\frac{e^7}{1290240}$
$\frac{1}{2} x^0 \frac{\cos}{\sin} y$								
[cos]	+ 1	- 8	+ 42	- 800	0	- 30	+ 680	- 15680
[sin]	0	0	0	0	2	- 30	+ 680	- 15680
[cos + sin]	+ 1	- 8	+ 42	- 800	+ 2	- 60	+ 1360	- 31360
[cos - sin]	Do.	„	„	„	- 2	0	0	0

	[] ²			[] ³		
	$\frac{e^2}{16}$	$\frac{e^4}{768}$	$\frac{e^6}{92160}$	$\frac{e^3}{96}$	$\frac{e^5}{7680}$	$\frac{e^7}{1290240}$
$\frac{1}{2} x^0 \frac{\cos}{\sin} y$						
[cos]	+ 4	- 320	+ 17490	+ 30	- 4440	+ 447552
[sin]	+ 5	- 328	+ 16935	+ 34	- 4520	+ 444864
[cos + sin]	+ 9	- 648	+ 34425	+ 64	- 8960	+ 892416
[cos - sin]	- 1	+ 8	+ 555	- 4	+ 80	+ 2688

$\frac{1}{2} x^0 \cos y$	$[]^4$		$[]^5$		$[]^6$		$[]^7$	
	$\frac{e^4}{768}$	$\frac{e^6}{92160}$		$\frac{e^5}{7680}$	$\frac{e^7}{1290240}$		$\frac{e^6}{92160}$	
$[\cos]$	+ 299	- 74352		+ 3760	- 1457120		+ 57262	+ 1025248
$[\sin]$	+ 326	- 75648		+ 4016	- 1482208		+ 60387	+ 1071904
$[\cos + \sin]$	+ 625	- 150000		+ 7776	- 2939328		+ 117649	+ 2097152
$[\cos - \sin]$	- 27	+ 1296		- 256	+ 25088		- 3125	- 46656

where the lines $[\cos + \sin]$, $[\cos - \sin]$ are obtained by the addition and subtraction of the lines $[\cos]$, $[\sin]$.

The lines $[\cos + \sin]$, $[\cos - \sin]$, are then rearranged by reading them backwards and forwards from the terms which are in the $[\cos - \sin]$ line, under $[]^j$; viz., in the present case, where $j=1$, from the terms $-2, 0, 0, 0$ under $[]^1$, as follows:—

$x^0 \cos f$	$[]^0$				$[]^1$			
	$\frac{e}{4}$	$\frac{e^3}{96}$	$\frac{e^5}{7680}$	$\frac{e^7}{1290240}$	$\frac{e^0}{2}$	$\frac{e^2}{16}$	$\frac{e^4}{768}$	$\frac{e^6}{92160}$
$\frac{1}{2} [\cos + \sin]$	- 2	o	o	o	+ 1	- 8	+ 42	- 800
$\frac{1}{2} [\cos - \sin]$	Do.	„	„	„	o	- 1	+ 8	+ 555
$[\cos]$	- 4	o	o	o	+ 1	- 9	+ 50	- 245
$[\sin]$	o	o	o	o	$\div 1$	- 7	+ 34	- 1355

$x^0 \cos f$	$[]^2$				$[]^3$			$[]^4$	
	$\frac{e}{4}$	$\frac{e^3}{96}$	$\frac{e^5}{7680}$	$\frac{e^7}{1290240}$	$\frac{e^2}{16}$	$\frac{e^4}{768}$	$\frac{e^6}{92160}$	$\frac{e^3}{96}$	$\frac{e^5}{7680}$
$\frac{1}{2} [\cos + \sin]$	+ 2	- 60	+ 1360	- 31360	+ 9	- 648	+ 34425	+ 64	- 8960
$\frac{1}{2} [\cos - \sin]$	o	- 4	+ 80	+ 2688	o	- 27	+ 1296	o	- 256
$[\cos]$	+ 2	- 64	+ 1440	- 28672	+ 9	- 675	+ 35721	+ 64	- 9216
$[\sin]$	+ 2	- 56	+ 1280	- 24048	+ 9	- 621	+ 33129	+ 64	- 8704

$x^0 \cos f$	$[]^5$		$[]^6$		$[]^7$		$[]^8$	
	$\frac{e^4}{768}$	$\frac{e^6}{92160}$		$\frac{e^5}{7680}$	$\frac{e^7}{1290240}$		$\frac{e^6}{92160}$	$\frac{e^7}{1290240}$
$\frac{1}{2} [\cos + \sin]$	+ 625	- 150000	+ 7776	- 2939328	+ 117649	+ 2097152		
$\frac{1}{2} [\cos - \sin]$	o	- 3125	o	- 46656	o	o		
$[\cos]$	+ 625	- 153125	+ 7776	- 2985984	+ 117649	+ 2097152		
$[\sin]$	+ 625	- 146875	+ 7776	- 2892672	+ 117649	+ 2097152		

where the lines $[\cos]$, $[\sin]$ are obtained by the addition and subtraction of the lines $\frac{1}{2} [\cos + \sin]$, $\frac{1}{2} [\cos - \sin]$; and it then only remains to reduce the fractions, at the same time multiplying by 2 the lines $\frac{1}{2} [\cos + \sin]$, $\frac{1}{2} [\cos - \sin]$, to obtain instead the lines $[\cos + \sin]$, $[\cos - \sin]$. We have thus,

$x^0 \cos f$	$[]^0$				$[]^1$			
	e	e^3	e^5	e^7	e^0	e^2	e^4	e^6
$[\cos + \sin]$	- 1	o	o	o	+ 1	- 1	$+\frac{7}{64}$	$-\frac{5}{288}$
$[\cos - \sin]$	Do.	„	„	„	o	$-\frac{1}{8}$	$+\frac{1}{48}$	$+\frac{37}{3072}$
$[\cos]$	- 1	o	o	o	$+\frac{1}{2}$	$-\frac{9}{16}$	$+\frac{25}{384}$	$-\frac{49}{18432}$
$[\sin]$	o	o	o	o	$+\frac{1}{2}$	$-\frac{7}{16}$	$+\frac{17}{384}$	$-\frac{271}{18432}$

$x^0 \cos f$	$[]^2$				$[]^3$			$[]^4$		
	e	e^3	e^5	e^7	e^2	e^4	e^6	e^3	e^5	e^7
$[\cos + \sin]$	+ 1	$-\frac{5}{4}$	$+\frac{17}{48}$	$-\frac{7}{144}$	$+\frac{9}{8}$	$-\frac{27}{16}$	$+\frac{765}{1024}$	$+\frac{4}{3}$	$-\frac{7}{3}$	$+\frac{83}{60}$
$[\cos - \sin]$	o	$-\frac{1}{12}$	$+\frac{1}{48}$	$+\frac{1}{240}$	o	$-\frac{9}{128}$	$+\frac{9}{320}$	o	$-\frac{1}{15}$	$+\frac{7}{180}$
$[\cos]$	$+\frac{1}{2}$	$-\frac{2}{3}$	$+\frac{3}{16}$	$-\frac{1}{45}$	$+\frac{9}{16}$	$-\frac{225}{256}$	$+\frac{3969}{10240}$	$+\frac{2}{3}$	$-\frac{6}{5}$	$+\frac{32}{45}$
$[\sin]$	$+\frac{1}{2}$	$-\frac{7}{12}$	$+\frac{1}{6}$	$-\frac{19}{720}$	$+\frac{9}{16}$	$-\frac{207}{256}$	$+\frac{3681}{10240}$	$+\frac{2}{3}$	$-\frac{17}{15}$	$+\frac{121}{180}$

$x^0 \frac{\cos f}{\sin f}$	$[]^5$		$[]^6$		$[]^7$		$[]^8$	
	e^4	e^6	e^5	e^7	e^6	e^7		
[cos + sin]	+ $\frac{625}{384}$	- $\frac{625}{192}$	+ $\frac{81}{40}$	- $\frac{5103}{1120}$	+ $\frac{117649}{46080}$		+ $\frac{1024}{315}$	
[cos - sin]	o	- $\frac{625}{9216}$	o	- $\frac{81}{1120}$	o		o	
[cos]	+ $\frac{625}{768}$	- $\frac{30625}{18432}$	+ $\frac{81}{80}$	- $\frac{81}{35}$	+ $\frac{117649}{92160}$		+ $\frac{512}{315}$	
[sin]	+ $\frac{625}{768}$	- $\frac{29375}{18432}$	+ $\frac{81}{80}$	- $\frac{2511}{1120}$	+ $\frac{117649}{92160}$		+ $\frac{512}{315}$	

which is, in fact, part of the table $(x^0 \dots x^7) \frac{\cos}{\sin} f$. It is to be remarked, that when in any column the second term is zero, the third and fourth terms are each equal to one-half of the first term, and that I have in the tables represented them by the letters H, H.

The foregoing method depends, it is easy to see, on the following proposition; viz., writing

$$x^m \cos jf = \sum [\cos]_f^i \cos ig, \quad x^m \sin jf = \sum [\sin]_f^i \sin ig,$$

$$x^m \cos jy = \sum [\cos]_y^i \cos ig, \quad x^m \sin jy = \sum [\sin]_y^i \sin ig,$$

where, as before, $f = g + y$, then we have

$$\begin{aligned} [\cos + \sin]_f^i &= [\cos + \sin]_y^{i-j}, \\ [\cos - \sin]_f^i &= [\cos - \sin]_y^{i+j}, \end{aligned}$$

the latter of which follows immediately from the former. To prove this, it is to be remarked that in general

$$\begin{aligned} \cos \alpha \sum [\cos]^i \cos ig &= \sum [\cos]^i \cos (ig + \alpha), \\ \sin \alpha \sum [\sin]^i \sin ig &= - \sum [\sin]^i \cos (ig + \alpha), \\ \sin \alpha \sum [\cos]^i \cos ig &= \sum [\cos]^i \sin (ig + \alpha), \\ \cos \alpha \sum [\sin]^i \sin ig &= \sum [\sin]^i \sin (ig + \alpha); \end{aligned}$$

whence, putting $\alpha = jg$,

$$\begin{aligned} \cos jf &= \cos (jy + jg) = \sum [\cos + \sin]_y^i \cos (i + j)g \\ &= \sum \frac{1}{2} \{ [\cos + \sin]_y^{i-j} + [\cos + \sin]_y^{-i-j} \} \cos ig, \\ \sin jf &= \sin (jy + jg) = \sum [\cos + \sin]_y^i \sin (i + j)g \\ &= \sum \frac{1}{2} \{ [\cos + \sin]_y^{i-j} - [\cos + \sin]_y^{-i-j} \} \sin ig; \end{aligned}$$

and therefore

$$[\cos]_f^i = \frac{1}{2} \{ [\cos + \sin]_y^{i-j} + [\cos - \sin]_y^{-i-j} \},$$

$$[\sin]_f^i = \frac{1}{2} \{ [\cos + \sin]_y^{i-j} - [\cos - \sin]_y^{-i-j} \};$$

and, adding and subtracting, we have the theorem. The coefficients contained in the Datum Tables $(x^0 \dots x^r) \frac{\cos}{\sin} jy$ were taken to be $\frac{1}{2} [\cos]_y^i$, $\frac{1}{2} [\sin]_y^i$, in order that the calculations might in the first instance give $\frac{1}{2} [\cos + \sin]_f^i$, $\frac{1}{2} [\cos - \sin]_f^i$, and thence by an addition and subtraction, without a division by 2, give $[\cos]_f^i$, $[\sin]_f^i$.

The resulting values admit of a very simple verification; in fact, writing down the two equations

$$x^m \cos jf = \sum [\cos]^i \cos ig, \quad x^m \sin jf = \sum [\sin]^i \sin ig,$$

or, what is the same thing, the summation now extending only from $i=1$ to $i=\infty$,

$$\frac{1}{2} x^m \cos jf = \frac{1}{2} [\cos]^0 + \sum' [\cos]^i \cos ig, \quad \frac{1}{2} x^m \sin jf = \sum' [\sin]^i \sin ig;$$

if in the first equation, and in the second equation differentiated with respect to g , we put $g=0$, then, observing that $g=0$ gives $f=0$, $x=-e$, $\frac{df}{dg} = \frac{(1+e)^2}{(1-e^2)^{\frac{3}{2}}}$, we have

$$\frac{1}{2} [\cos]^0 + \sum' [\cos]^i = \frac{1}{2} j (-e)^m,$$

$$\sum' i [\sin]^i = \frac{1}{2} j (-e)^m (1 + 2e + \frac{5}{2} e^2 + 3e^3 + \frac{27}{8} e^4 + \frac{15}{4} e^5 + \frac{65}{16} e^6 + \frac{35}{8} e^7 + \dots).$$

The values of $(x^0 \dots x^r) \frac{\cos}{\sin} jf$ being known, those of $\left(\frac{r}{a}\right)^{\pm m} \frac{\cos}{\sin} jf$ can, of course, be determined by merely substituting for $\frac{r}{a}$ its value $(1+x)$, expanding and reducing; and a convenient mode of arranging the calculations presents itself so obviously, that no explanation is required. The verification is similar to that for $x^m \frac{\cos}{\sin} jf$. The only difference is, that for $g=0$ we have the value of r , viz. $1-e$, in the place of the value $-e$ of x . A similar verification is applicable to the Datum Tables.

To make the arrangement of the Tables clear, I have given at the end of them some examples to show how they are to be read off.

Table ($x^0, x^1, x^2, x^3, x^4, x^5, x^6, x^7$).

		$[]^0$	$[]^1$	$[]^2$
		$e^0 \quad e^2 \quad e^4 \quad e^6$	$e \quad e^3 \quad e^5 \quad e^7$	$e^2 \quad e^4 \quad e^6$
x^0		+ I o o o	o o o o	o o o
x^1		+ $\frac{I}{2}$ o o	- $\frac{I}{2}$ + $\frac{3}{16}$ - $\frac{5}{384}$ + $\frac{7}{18432}$	- $\frac{I}{4}$ + $\frac{I}{6}$ - $\frac{I}{32}$
x^2		+ $\frac{I}{2}$ o o	- $\frac{I}{4}$ + $\frac{I}{48}$ - $\frac{I}{1536}$	+ $\frac{I}{4}$ - $\frac{I}{4}$ + $\frac{5}{96}$
x^3		+ $\frac{3}{8}$ o	- $\frac{3}{8}$ + $\frac{3}{32}$ - $\frac{5}{1024}$	o + $\frac{I}{32}$
x^4	[cos]	+ $\frac{3}{8}$ o	- $\frac{I}{4}$ + $\frac{I}{64}$	+ $\frac{I}{4}$ - $\frac{3}{16}$
x^5		+ $\frac{5}{16}$	- $\frac{5}{16}$ + $\frac{15}{256}$	+ $\frac{5}{64}$
x^6		+ $\frac{5}{16}$	- $\frac{15}{64}$	+ $\frac{15}{64}$
x^7			- $\frac{35}{128}$	

Table ($x^0, x^1, x^2, x^3, x^4, x^5, x^6, x^7$).

		$\overbrace{[\quad]^3}^{e^3 \quad e^5 \quad e^7}$	$\overbrace{[\quad]^4}^{e^4 \quad e^6}$	$\overbrace{[\quad]^5}^{e^5 \quad e^6}$	$\overbrace{[\quad]^6}^{e^6}$	$\overbrace{[\quad]^7}^{e^7}$
x^0		○ ○ ○	○ ○	○ ○	○	○
x^1		$-\frac{3}{16} + \frac{45}{256} - \frac{567}{10240}$	$-\frac{1}{6} + \frac{1}{5}$	$-\frac{125}{768} + \frac{4375}{18432}$	$-\frac{27}{160}$	$-\frac{16807}{91260}$
x^2		$+\frac{1}{4} - \frac{9}{32} + \frac{243}{2560}$	$+\frac{1}{4} - \frac{1}{3}$	$+\frac{25}{96} - \frac{625}{1536}$	$+\frac{9}{32}$	$+\frac{2401}{7680}$
x^3		$-\frac{1}{8} + \frac{9}{64} - \frac{27}{1024}$	$-\frac{3}{16} + \frac{1}{4}$	$-\frac{15}{64} + \frac{375}{1024}$	$-\frac{9}{32}$	$-\frac{343}{1024}$
$[\cos]$						
x^4		$+\frac{1}{8} - \frac{9}{64}$	$+\frac{1}{16}$ ○	$+\frac{1}{8} - \frac{25}{192}$	$+\frac{3}{16}$	$+\frac{49}{192}$
x^5		$-\frac{5}{32} + \frac{45}{256}$	$-\frac{5}{32}$	$-\frac{1}{32} - \frac{25}{256}$	$-\frac{5}{64}$	$-\frac{35}{256}$
x^6		$+\frac{3}{64}$	$+\frac{3}{32}$	$+\frac{9}{64}$	$+\frac{1}{64}$	$+\frac{3}{64}$
x^7		$-\frac{21}{128}$		$-\frac{7}{128}$		$-\frac{1}{128}$

Table $(x^0, x^1, x^2, x^3) \frac{\cos}{\sin} f \dots$

$\frac{\cos}{\sin} f$	$[]^0$				$[]^1$			
	e	e^3	e^5	e^7	e^0	e^2	e^4	e^6
x^0	[cos + sin]	- I	o	o	o	+ I	- I	$+\frac{7}{64}$
	[cos - sin]	„	„	„	„	o	$-\frac{I}{8}$	$+\frac{I}{48}$
	[cos]	- I	o	o	o	H	$-\frac{9}{16}$	$+\frac{25}{384}$
	[sin]	o	o	o	o	H	$-\frac{7}{16}$	$+\frac{17}{384}$
x^1	[cos + sin]	$-\frac{I}{2}$	o	o	o	$+\frac{I}{2}$	$-\frac{I}{8}$	$-\frac{I}{128}$
	[cos - sin]	„	„	„	„	$+\frac{I}{4}$	$+\frac{I}{48}$	$+\frac{19}{1536}$
	[cos]	$-\frac{I}{2}$	o	o	o	$+\frac{3}{8}$	$-\frac{5}{96}$	$+\frac{7}{3072}$
	[sin]	o	o	o	o	$+\frac{I}{8}$	$-\frac{7}{96}$	$-\frac{31}{3072}$
x^2	[cos + sin]	$-\frac{I}{2}$	o	o	o	$+\frac{I}{2}$	$-\frac{I}{4}$	$+\frac{5}{384}$
	[cos - sin]	„	„	„	„	$+\frac{I}{4}$	$-\frac{I}{16}$	$+\frac{5}{512}$
	[cos]	$-\frac{I}{2}$	o	o	o	$+\frac{3}{8}$	$-\frac{5}{32}$	$+\frac{35}{3072}$
	[sin]	o	o	o	o	$+\frac{I}{8}$	$-\frac{3}{32}$	$+\frac{5}{3072}$
x^3	[cos + sin]	$-\frac{3}{8}$	o	o	o	$+\frac{3}{8}$	$-\frac{I}{16}$	
	[cos - sin]	„	„	„	„	$+\frac{I}{4}$	$+\frac{I}{128}$	
	[cos]	$-\frac{3}{8}$	o	o	o	$+\frac{5}{16}$	$-\frac{7}{256}$	
	[sin]	o	o	o	o	$+\frac{I}{16}$	$-\frac{9}{256}$	

N.B.—In all the Tables H, H, in the third and fourth lines of a set of four lines, denote one half of the number above them in the first line. The terms [cos - sin]⁰, denoted by „, „, ... are the same as the terms [cos + sin]⁰ immediately above them.

... Table $(x^0, x^1, x^2, x^3) \frac{\cos}{\sin} f \dots$

	$\frac{\cos}{\sin} f$	[] ²				[] ³			[] ⁴			
		e	e^3	e^5	e^7	e^2	e^4	e^6	e^3	e^5	e^7	
x^0	[cos + sin]	+ 1	- $\frac{5}{4}$	+ $\frac{17}{48}$	- $\frac{7}{144}$	+ $\frac{9}{8}$	- $\frac{27}{16}$	+ $\frac{768}{1024}$	+ $\frac{4}{3}$	- $\frac{7}{3}$	+ $\frac{83}{60}$	
	[cos - sin]	o	- $\frac{1}{12}$	+ $\frac{1}{48}$	+ $\frac{1}{240}$	o	- $\frac{9}{128}$	+ $\frac{9}{320}$	o	- $\frac{1}{15}$	+ $\frac{7}{180}$	
	[cos]	H	- $\frac{2}{3}$	+ $\frac{3}{16}$	- $\frac{1}{45}$	H	- $\frac{225}{256}$	+ $\frac{3969}{10240}$	H	- $\frac{6}{5}$	+ $\frac{32}{45}$	
	[sin]	H	- $\frac{7}{12}$	+ $\frac{1}{6}$	- $\frac{19}{720}$	H	- $\frac{207}{256}$	+ $\frac{3681}{10240}$	H	- $\frac{17}{15}$	+ $\frac{121}{180}$	
x^1	[cos + sin]	- $\frac{1}{2}$	+ $\frac{7}{8}$	- $\frac{29}{96}$	+ $\frac{5}{144}$	- $\frac{3}{4}$	+ $\frac{21}{16}$	- $\frac{327}{512}$	- 1	+ $\frac{23}{12}$	- $\frac{289}{240}$	
	[cos - sin]	o	+ $\frac{1}{8}$	- $\frac{1}{96}$	+ $\frac{1}{240}$	o	+ $\frac{3}{32}$	- $\frac{33}{1280}$	o	+ $\frac{1}{12}$	- $\frac{29}{720}$	
	[cos]	H	+ $\frac{1}{2}$	- $\frac{5}{32}$	+ $\frac{7}{360}$	H	+ $\frac{45}{64}$	- $\frac{1701}{5120}$	H	+ 1	- $\frac{28}{45}$	
	[sin]	H	+ $\frac{3}{8}$	- $\frac{7}{48}$	+ $\frac{11}{720}$	H	+ $\frac{39}{64}$	- $\frac{1569}{5120}$	H	+ $\frac{11}{12}$	- $\frac{419}{720}$	
x^2	[cos + sin]	o	- $\frac{1}{12}$	+ $\frac{1}{32}$	+ $\frac{1}{4}$	- $\frac{7}{16}$	+ $\frac{73}{512}$	+ $\frac{1}{2}$	- $\frac{23}{24}$	+ $\frac{257}{480}$		
	[cos - sin]	o	- $\frac{1}{24}$	+ $\frac{1}{480}$	o	- $\frac{1}{32}$	- $\frac{5}{256}$	o	- $\frac{1}{24}$	- $\frac{1}{480}$		
	[cos]	o	- $\frac{1}{16}$	+ $\frac{1}{60}$	H	- $\frac{15}{64}$	+ $\frac{63}{1024}$	H	- $\frac{1}{2}$	+ $\frac{4}{15}$		
	[sin]	o	- $\frac{1}{48}$	+ $\frac{7}{480}$	H	- $\frac{13}{64}$	+ $\frac{83}{1024}$	H	- $\frac{11}{24}$	+ $\frac{43}{160}$		
x^3	[cos + sin]	- $\frac{3}{8}$	+ $\frac{7}{16}$	- $\frac{7}{64}$	- $\frac{1}{4}$	+ $\frac{51}{128}$	- $\frac{1}{8}$	+ $\frac{1}{32}$	+ $\frac{33}{128}$			
	[cos - sin]	- $\frac{1}{8}$	+ $\frac{1}{8}$	- $\frac{1}{64}$	- $\frac{1}{16}$	+ $\frac{3}{32}$	o	- $\frac{1}{32}$	+ $\frac{29}{384}$			
	[cos]	- $\frac{1}{4}$	+ $\frac{9}{32}$	- $\frac{1}{16}$	- $\frac{5}{32}$	+ $\frac{63}{256}$	H	o	+ $\frac{1}{6}$			
	[sin]	- $\frac{1}{8}$	+ $\frac{5}{32}$	- $\frac{3}{64}$	- $\frac{3}{32}$	+ $\frac{39}{256}$	H	+ $\frac{1}{32}$	+ $\frac{35}{384}$			

... Table $(x^0, x^1, x^2, x^3) \frac{\cos}{\sin} f.$

	$\frac{\cos}{\sin} f$	$\overbrace{[]^5}_{e^4 \quad e^6}$	$\overbrace{[]^6}_{e^5 \quad e^7}$	$\overbrace{[]^7}_{e^6 \quad e^7}$	$\overbrace{[]^8}_{e^7}$
x^0	[cos + sin]	$+\frac{625}{384} - \frac{625}{192}$	$+\frac{81}{40} - \frac{729}{160}$	$+\frac{117649}{46080}$	$+\frac{1024}{315}$
	[cos - sin]	$\circ - \frac{625}{9216}$	$\circ - \frac{81}{1120}$	\circ	\circ
	[cos]	H $- \frac{30625}{18432}$	H $- \frac{81}{35}$	H	H
	[sin]	H $- \frac{29375}{18432}$	H $- \frac{2511}{1120}$	H	H
x^1	[cos + sin]	$-\frac{125}{96} + \frac{2125}{768}$	$-\frac{27}{16} + \frac{1269}{320}$	$-\frac{16807}{7680}$	$-\frac{128}{45}$
	[cos - sin]	$\circ + \frac{125}{1536}$	$\circ + \frac{27}{320}$	\circ	\circ
	[cos]	H $+ \frac{4375}{3072}$	H $+ \frac{81}{40}$	H	H
	[sin]	H $+ \frac{1375}{1024}$	H $+ \frac{621}{320}$	H	H
x^2	[cos + sin]	$+\frac{25}{32} - \frac{425}{256}$	$+\frac{9}{8} - \frac{423}{160}$	$+\frac{2401}{1536}$	$+\frac{32}{15}$
	[cos - sin]	$\circ - \frac{25}{512}$	$\circ - \frac{9}{160}$	\circ	\circ
	[cos]	H $- \frac{875}{1024}$	H $- \frac{27}{20}$	H	H
	[sin]	H $- \frac{825}{1024}$	H $- \frac{207}{160}$	H	H
x^3	[cos + sin]	$-\frac{5}{16} + \frac{15}{32}$	$-\frac{9}{16} + \frac{9}{8}$	$-\frac{343}{384}$	$-\frac{4}{3}$
	[cos - sin]	$\circ - \frac{5}{384}$	$\circ \circ$	\circ	\circ
	[cos]	H $+ \frac{175}{768}$	H H	H	H
	[sin]	H $+ \frac{185}{768}$	H H	H	H

Table (x^4, x^5, x^6, x^7) $\frac{\cos}{\sin} f \dots$

$\frac{\cos}{\sin} f$	$[]^0$				$[]^1$			
	e	e^3	e^5	e^7	e^0	e^2	e^4	e^6
x^4	[cos + sin]		$-\frac{3}{8}$	o			$+\frac{3}{8}$	$-\frac{1}{8}$
	[cos - sin]		„	„			$+\frac{1}{4}$	$-\frac{5}{128}$
	[cos]		$-\frac{3}{8}$	o			$+\frac{5}{16}$	$-\frac{21}{256}$
	[sin]		o	o			$+\frac{1}{16}$	$-\frac{11}{256}$
x^5	[cos + sin]		$-\frac{5}{16}$	o				$+\frac{5}{16}$
	[cos - sin]		„	„				$+\frac{15}{64}$
	[cos]		$-\frac{5}{16}$	o				$+\frac{35}{128}$
	[sin]		o	o				$+\frac{5}{128}$
x^6	[cos + sin]		$-\frac{5}{16}$					$+\frac{5}{16}$
	[cos - sin]		„					$+\frac{15}{64}$
	[cos]		$-\frac{5}{16}$					$+\frac{35}{128}$
	[sin]		o					$+\frac{5}{128}$
x^7	[cos + sin]			$-\frac{35}{128}$				
	[cos - sin]			„				
	[cos]			$-\frac{35}{128}$				
	[sin]			o				

... Table $(x^4, x^5, x^6, x^7) \frac{\cos}{\sin} f \dots$

	$\cos f$ $\sin f$	$\overbrace{[]^2}_{e \quad e^3 \quad e^5 \quad e^7}$	$\overbrace{[]^3}_{e^2 \quad e^4 \quad e^6}$	$\overbrace{[]^4}_{e^3 \quad e^5 \quad e^7}$
x^4	[cos + sin]	$-\frac{1}{8} + \frac{1}{64}$	$+\frac{1}{4} - \frac{51}{128}$	$+\frac{5}{16} - \frac{9}{16}$
	[cos - sin]	$-\frac{1}{16} - \frac{1}{64}$	$+\frac{1}{16} - \frac{3}{32}$	$+\frac{1}{16} - \frac{5}{48}$
	[cos]	$-\frac{3}{32} \circ$	$+\frac{5}{32} - \frac{63}{256}$	$+\frac{3}{16} - \frac{1}{3}$
	[sin]	$-\frac{1}{32} + \frac{1}{64}$	$+\frac{3}{32} - \frac{39}{256}$	$+\frac{1}{8} - \frac{11}{48}$
x^5	[cos + sin]	$-\frac{5}{16} + \frac{35}{128}$	$-\frac{5}{64}$	$-\frac{5}{32} + \frac{7}{32}$
	[cos - sin]	$-\frac{5}{32} + \frac{13}{128}$	$-\frac{1}{32}$	$-\frac{1}{32} + \frac{1}{32}$
	[cos]	$-\frac{15}{64} + \frac{3}{16}$	$-\frac{7}{128}$	$-\frac{3}{32} + \frac{1}{8}$
	[sin]	$-\frac{5}{64} + \frac{11}{128}$	$-\frac{3}{128}$	$-\frac{1}{16} + \frac{3}{32}$
x^6	[cos + sin]	$-\frac{5}{32}$	$+\frac{15}{64}$	$+\frac{3}{16}$
	[cos - sin]	$-\frac{3}{32}$	$+\frac{3}{32}$	$+\frac{1}{16}$
	[cos]	$-\frac{1}{8}$	$+\frac{21}{128}$	$+\frac{1}{8}$
	[sin]	$-\frac{1}{32}$	$+\frac{9}{128}$	$+\frac{1}{16}$
x^7	[cos + sin]	$-\frac{35}{128}$		$-\frac{21}{128}$
	[cos - sin]	$-\frac{21}{128}$		$-\frac{7}{128}$
	[cos]	$-\frac{7}{32}$		$-\frac{7}{64}$
	[sin]	$-\frac{7}{128}$		$-\frac{7}{128}$

... Table (x^4, x^5, x^6, x^7) $\frac{\cos}{\sin} f.$

	$\frac{\cos}{\sin} f$	$[]^5$		$[]^6$		$[]^7$		$[]^8$	
		e^4	e^6	e^5	e^7	e^6		e^7	
x^4	$[\cos + \sin]$	$+\frac{1}{16}$	$+\frac{7}{3^2}$	$+\frac{3}{16}$	$-\frac{3}{64}$	$+\frac{49}{128}$		$+\frac{2}{3}$	
	$[\cos - \sin]$	\circ	$+\frac{7}{128}$	\circ	$+\frac{3}{64}$	\circ		\circ	
	$[\cos]$	H	$+\frac{35}{256}$	H	\circ	H		H	
	$[\sin]$	H	$+\frac{21}{256}$	H	$-\frac{3}{64}$	H		H	
x^5	$[\cos + \sin]$		$-\frac{9}{3^2}$	$-\frac{1}{3^2}$	$-\frac{41}{128}$	$-\frac{7}{64}$		$-\frac{1}{4}$	
	$[\cos - \sin]$		$-\frac{3}{64}$	\circ	$-\frac{7}{128}$	\circ		\circ	
	$[\cos]$		$-\frac{21}{128}$	H	$-\frac{3}{16}$	H		H	
	$[\sin]$		$-\frac{15}{128}$	H	$-\frac{17}{128}$	H		H	
x^6	$[\cos + \sin]$		$+\frac{3}{3^2}$		$+\frac{7}{3^2}$	$+\frac{1}{64}$		$+\frac{1}{16}$	
	$[\cos - \sin]$		$+\frac{1}{64}$		$+\frac{1}{3^2}$	\circ		\circ	
	$[\cos]$		$+\frac{7}{128}$		$+\frac{1}{8}$	H		H	
	$[\sin]$		$+\frac{5}{128}$		$+\frac{3}{3^2}$	H		H	
x^7	$[\cos + \sin]$				$-\frac{7}{128}$			$-\frac{1}{128}$	
	$[\cos - \sin]$				$-\frac{1}{128}$			\circ	
	$[\cos]$				$-\frac{1}{3^2}$			H	
	$[\sin]$				$-\frac{3}{128}$			H	

Table $(x^0, x^1, x^2, x^3) \frac{\cos}{\sin} 2f \dots$

		[] ⁰			[] ¹			[] ²							
		e^2	e^4	e^6	e	e^3	e^5	e^7	e^0	e^2	e^4	e^6			
x^0	[cos + sin]	$+\frac{3}{4}$	$+\frac{1}{8}$	$+\frac{3}{64}$	-2	$+\frac{7}{4}$	$-\frac{5}{96}$	$+\frac{271}{4608}$	$+1$	-4	$+\frac{55}{16}$	$-\frac{103}{144}$			
	[cos - sin]	„ „ „			o	$+\frac{1}{12}$	$+\frac{5}{192}$	$+\frac{49}{7680}$	o	o	$+\frac{1}{24}$	$+\frac{1}{60}$			
	[cos]	$+\frac{3}{4}$	$+\frac{1}{8}$	$+\frac{3}{64}$	H	$+\frac{11}{12}$	$-\frac{5}{384}$	$+\frac{751}{23040}$	H	H	$+\frac{167}{96}$	$-\frac{503}{1440}$			
	[sin]	o o o			H	$+\frac{5}{6}$	$-\frac{5}{128}$	$+\frac{151}{5760}$	H	H	$+\frac{163}{96}$	$-\frac{527}{1440}$			
x^1	[cos + sin]	$+\frac{3}{4}$	$-\frac{1}{8}$	$-\frac{3}{64}$	$-\frac{1}{2}$	$+\frac{5}{16}$	$-\frac{53}{384}$	$-\frac{203}{18432}$	$+\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{32}$				
	[cos - sin]	„ „ „			o	$-\frac{1}{16}$	$-\frac{61}{768}$	$-\frac{271}{6144}$	o	$-\frac{1}{48}$	$-\frac{17}{480}$				
	[cos]	$+\frac{3}{4}$	$-\frac{1}{8}$	$-\frac{3}{64}$	H	$+\frac{1}{8}$	$-\frac{167}{1536}$	$-\frac{127}{4608}$	H	$-\frac{25}{96}$	$-\frac{1}{30}$				
	[sin]	o o o			H	$+\frac{3}{16}$	$-\frac{15}{512}$	$+\frac{305}{18432}$	H	$-\frac{23}{96}$	$+\frac{1}{480}$				
x^2	[cos + sin]	$+\frac{1}{4}$	$+\frac{1}{8}$	$+\frac{3}{64}$		$-\frac{3}{4}$	$+\frac{17}{48}$	$+\frac{19}{512}$	$+\frac{1}{2}$	-1	$+\frac{53}{96}$				
	[cos - sin]	„ „ „				$-\frac{1}{4}$	$+\frac{1}{96}$	$+\frac{83}{7680}$	o	$-\frac{1}{16}$	$+\frac{1}{32}$				
	[cos]	$+\frac{1}{4}$	$+\frac{1}{8}$	$+\frac{3}{64}$		$-\frac{1}{2}$	$+\frac{35}{192}$	$+\frac{23}{960}$	H	$-\frac{17}{32}$	$+\frac{7}{24}$				
	[sin]	o o o				$-\frac{1}{4}$	$+\frac{11}{64}$	$+\frac{101}{7680}$	H	$-\frac{15}{32}$	$+\frac{25}{96}$				
x^3	[cos + sin]		$+\frac{1}{2}$	$-\frac{3}{64}$		$-\frac{3}{8}$	$+\frac{5}{32}$	$-\frac{49}{1024}$		$+\frac{3}{8}$	$-\frac{1}{4}$				
	[cos - sin]	„ „				$-\frac{1}{8}$	$-\frac{1}{64}$	$-\frac{39}{1024}$		$+\frac{1}{16}$	o				
	[cos]		$+\frac{1}{2}$	$-\frac{3}{64}$		$-\frac{1}{4}$	$+\frac{9}{128}$	$-\frac{11}{256}$		$+\frac{7}{32}$	$-\frac{1}{8}$				
	[sin]	o o				$-\frac{1}{8}$	$+\frac{11}{128}$	$-\frac{5}{1024}$		$+\frac{5}{32}$	$-\frac{1}{8}$				

... Table $(x^0, x^1, x^2, x^3) \frac{\cos}{\sin} 2f ...$

	$\begin{matrix} \cos \\ \sin \end{matrix} 2f$	[] ³				[] ⁴			[] ⁵		
		e	e^3	e^5	e^7	e^2	e^4	e^6	e^3	e^5	e^7
x^0	[cos + sin]	+ 2	- $\frac{27}{4}$	+ $\frac{207}{32}$	- $\frac{1147}{512}$	+ $\frac{13}{4}$	- $\frac{259}{24}$	+ $\frac{559}{48}$	+ $\frac{59}{12}$	- $\frac{3221}{192}$	+ $\frac{31087}{1536}$
	[cos - sin]	o	o	+ $\frac{9}{320}$	+ $\frac{27}{2560}$	o	o	+ $\frac{1}{45}$	o	o	+ $\frac{625}{32256}$
	[cos]	H	H	+ $\frac{2079}{640}$	- $\frac{1427}{1280}$	H	H	+ $\frac{8401}{1440}$	H	H	+ $\frac{163363}{16128}$
	[sin]	H	H	+ $\frac{2061}{640}$	- $\frac{2881}{2560}$	H	H	+ $\frac{8369}{1440}$	H	H	+ $\frac{326101}{32256}$
x^1	[cos + sin]	- $\frac{1}{2}$	+ $\frac{33}{16}$	- $\frac{255}{128}$	+ $\frac{1081}{2048}$	- $\frac{5}{4}$	+ $\frac{107}{24}$	- $\frac{229}{48}$	- $\frac{37}{16}$	+ $\frac{6259}{768}$	- $\frac{59723}{6144}$
	[cos - sin]	o	o	- $\frac{3}{256}$	- $\frac{231}{10240}$	o	o	- $\frac{1}{120}$	o	o	- $\frac{125}{18432}$
	[cos]	H	H	- $\frac{513}{512}$	+ $\frac{2587}{10240}$	H	H	- $\frac{1147}{480}$	H	H	- $\frac{89647}{18432}$
	[sin]	H	H	- $\frac{507}{512}$	+ $\frac{1409}{5120}$	H	H	- $\frac{381}{160}$	H	H	- $\frac{44761}{9216}$
x^2	[cos + sin]	+ $\frac{1}{4}$	- $\frac{13}{16}$	+ $\frac{367}{512}$		+ $\frac{1}{4}$	- $\frac{5}{8}$	o	+ $\frac{3}{4}$	- $\frac{223}{96}$	+ $\frac{2987}{1536}$
	[cos - sin]	o	- $\frac{1}{32}$	+ $\frac{71}{2560}$		o	o	- $\frac{1}{48}$	o	o	- $\frac{25}{1536}$
	[cos]	H	- $\frac{27}{64}$	+ $\frac{953}{2560}$		H	H	- $\frac{1}{96}$	H	H	+ $\frac{1481}{1536}$
	[sin]	H	- $\frac{25}{64}$	+ $\frac{441}{1280}$		H	H	+ $\frac{1}{96}$	H	H	+ $\frac{251}{256}$
x^3	[cos + sin]	- $\frac{3}{8}$	+ $\frac{33}{32}$	- $\frac{753}{1024}$		- $\frac{1}{2}$	+ $\frac{95}{64}$		- $\frac{1}{8}$	- $\frac{13}{64}$	+ $\frac{1577}{1024}$
	[cos - sin]	o	+ $\frac{3}{64}$	- $\frac{13}{1024}$		o	+ $\frac{7}{192}$		o	o	+ $\frac{95}{3072}$
	[cos]	H	+ $\frac{69}{128}$	- $\frac{383}{1024}$		H	+ $\frac{73}{96}$		H	H	+ $\frac{2413}{3072}$
	[sin]	H	+ $\frac{63}{128}$	- $\frac{185}{512}$		H	+ $\frac{139}{192}$		H	H	+ $\frac{1159}{1536}$

... Table $(x^0, x^1, x^2, x^3) \frac{\cos}{\sin} 2f$.

	$\cos 2f$ \sin	$[]^6$		$[]^7$		$[]^8$		$[]^9$	
		e^4	e^6	e^5	e^7	e^6		e^7	
x^0	[cos + sin]	$+\frac{115}{16}$	$-\frac{2049}{80}$	$+\frac{9893}{960}$	$-\frac{889303}{23040}$	$+\frac{42037}{2880}$		$+\frac{367439}{17920}$	
	[cos - sin]	o	o	o	o	o		o	
	[cos]	H	H	H	H	H		H	
	[sin]	H	H	H	H	H		H	
x	[cos + sin]	$-\frac{61}{16}$	$+\frac{69}{5}$	$-\frac{4553}{768}$	$+\frac{2061389}{92160}$	$-\frac{8551}{960}$		$-\frac{26809}{2048}$	
	[cos - sin]	o	o	o	o	o		o	
	[cos]	H	H	H	H	H		H	
	[sin]	H	H	H	H	H		H	
x^2	[cos + sin]	$+\frac{25}{16}$	$-\frac{21}{4}$	$+\frac{269}{96}$	$-\frac{77143}{7680}$	$+\frac{297}{64}$		$+\frac{18749}{2560}$	
	[cos - sin]	o	o	o	o	o		o	
	[cos]	H	H	H	H	H		H	
	[sin]	H	H	H	H	H		H	
x^3	[cos + sin]	$-\frac{7}{16}$	$+\frac{3}{4}$	$-\frac{65}{64}$	$+\frac{8425}{3072}$	$-\frac{379}{192}$		$-\frac{3563}{1024}$	
	[cos - sin]	o	o	o	o	o		o	
	[cos]	H	H	H	H	H		H	
	[sin]	H	H	H	H	H		H	

Table (x^4, x^5, x^6, x^7) $\frac{\cos}{\sin} 2f \dots$

$\cos \sin 2f$	[] ⁰			[] ¹				[] ²			
	e^2	e^4	e^6	e	e^3	e^5	e^7	e^0	e^2	e^4	e^6
x^4	[cos + sin]	$+\frac{1}{4}$	$+\frac{3}{64}$			$-\frac{1}{2}$	$+\frac{5}{32}$		$+\frac{3}{8}$	$-\frac{1}{2}$	
	[cos - sin]	„	„			$-\frac{1}{4}$	o		$+\frac{1}{16}$	$-\frac{1}{16}$	
	[cos]	$+\frac{1}{4}$	$+\frac{3}{64}$			$-\frac{3}{8}$	$+\frac{5}{64}$		$+\frac{7}{32}$	$-\frac{9}{32}$	
	[sin]	o	o			$-\frac{1}{8}$	$+\frac{5}{64}$		$+\frac{5}{32}$	$-\frac{7}{32}$	
x^5	[cos + sin]		$+\frac{25}{64}$			$-\frac{5}{16}$	$+\frac{25}{256}$			$+\frac{5}{16}$	
	[cos - sin]		„			$-\frac{5}{32}$	$-\frac{1}{256}$			$+\frac{3}{32}$	
	[cos]		$+\frac{25}{64}$			$-\frac{15}{64}$	$+\frac{3}{64}$			$+\frac{13}{64}$	
	[sin]		o			$-\frac{5}{64}$	$+\frac{13}{256}$			$+\frac{7}{64}$	
x^6	[cos + sin]		$+\frac{15}{64}$				$-\frac{25}{64}$			$+\frac{5}{16}$	
	[cos - sin]		„				$-\frac{15}{64}$			$+\frac{3}{32}$	
	[cos]		$+\frac{15}{64}$				$-\frac{5}{16}$			$+\frac{13}{64}$	
	[sin]		o				$-\frac{5}{64}$			$+\frac{7}{64}$	
x^7	[cos + sin]						$-\frac{35}{128}$				
	[cos - sin]						$-\frac{21}{128}$				
	[cos]						$-\frac{7}{32}$				
	[sin]						$-\frac{7}{128}$				

... Table $(x^4, x^5, x^6, x^7) \frac{\cos}{\sin} 2f \dots$

	$\cos \sin 2f$	$[]^3$			$[]^4$			$[]^5$			
		e	e^3	e^5	e^2	e^4	e^6	e^3	e^5	e^7	
x^4	$[\cos + \sin]$	o		$-\frac{3}{16}$		$+\frac{1}{4}$	$-\frac{43}{64}$		$+\frac{1}{2}$	$-\frac{47}{32}$	
	$[\cos - \sin]$	o		$-\frac{1}{32}$		o	$-\frac{1}{64}$		o	$-\frac{1}{48}$	
	$[\cos]$	o		$-\frac{7}{64}$	H		$-\frac{11}{32}$	H		$-\frac{143}{192}$	
	$[\sin]$	o		$-\frac{5}{64}$	H		$-\frac{21}{64}$	H		$-\frac{139}{192}$	
x^5	$[\cos + \sin]$		$-\frac{5}{16}$	$+\frac{165}{256}$			$-\frac{15}{64}$		$-\frac{5}{32}$	$+\frac{59}{256}$	
	$[\cos - \sin]$		$-\frac{1}{32}$	$+\frac{17}{256}$			$-\frac{1}{64}$		o	$-\frac{1}{256}$	
	$[\cos]$		$-\frac{11}{64}$	$+\frac{91}{256}$			$-\frac{1}{8}$	H		$+\frac{29}{256}$	
	$[\sin]$		$-\frac{9}{64}$	$+\frac{37}{128}$			$-\frac{7}{64}$	H		$+\frac{15}{128}$	
x^6	$[\cos + \sin]$			$-\frac{5}{64}$			$+\frac{15}{64}$			$+\frac{21}{64}$	
	$[\cos - \sin]$			$-\frac{1}{64}$			$+\frac{1}{64}$			$+\frac{1}{64}$	
	$[\cos]$			$-\frac{3}{64}$			$+\frac{1}{8}$			$+\frac{11}{64}$	
	$[\sin]$			$-\frac{1}{32}$			$+\frac{7}{64}$			$+\frac{5}{32}$	
x^7	$[\cos + \sin]$				$-\frac{35}{128}$					$-\frac{21}{128}$	
	$[\cos - \sin]$				$-\frac{7}{128}$					$-\frac{1}{128}$	
	$[\cos]$				$-\frac{21}{128}$					$-\frac{11}{128}$	
	$[\sin]$				$-\frac{7}{64}$					$-\frac{5}{64}$	

... Table $(x^4, x^5, x^6, x^7) \frac{\cos}{\sin} 2f$

		$[]^6$		$[]^7$		$[]^8$		$[]^9$	
		e^4	e^6	e^5	e^7	e^6		e^7	
x^4	$[\cos + \sin]$	$+\frac{1}{16}$	$+\frac{9}{16}$	$+\frac{1}{4}$	$+\frac{5}{24}$	$+\frac{41}{64}$		$+\frac{43}{32}$	
	$[\cos - \sin]$	o	o	o	o	o		o	
	$[\cos]$	H	H	H	H	H		H	
	$[\sin]$	H	H	H	H	H		H	
x^5	$[\cos + \sin]$		$-\frac{13}{32}$	$-\frac{1}{32}$	$-\frac{163}{256}$	$-\frac{9}{64}$		$-\frac{101}{256}$	
	$[\cos - \sin]$		o	o	o	o		o	
	$[\cos]$		H	H	H	H		H	
	$[\sin]$		H	H	H	H		H	
x^6	$[\cos + \sin]$		$+\frac{3}{32}$		$+\frac{19}{64}$	$+\frac{1}{64}$		$+\frac{5}{64}$	
	$[\cos - \sin]$		o		o	o		o	
	$[\cos]$		H		H	H		H	
	$[\sin]$		H		H	H		H	
x^7	$[\cos + \sin]$				$-\frac{7}{128}$			$-\frac{1}{128}$	
	$[\cos - \sin]$				o			o	
	$[\cos]$				H			H	
	$[\sin]$				H			H	

Table $(x^0, x^1, x^2, x^3) \frac{\cos}{\sin} 3f \dots$

		$[]^0$			$[]^1$			$[]^2$			
		e^3	e^5	e^7	e^2	e^4	e^6	e	e^3	e^5	e^7
x^0	[cos + sin]	$-\frac{1}{2}$	$-\frac{3}{16}$	$-\frac{3}{32}$	$+\frac{21}{8}$	$-\frac{31}{16}$	$-\frac{103}{1024}$	-3	$+\frac{33}{4}$	$-\frac{89}{16}$	$+\frac{163}{192}$
	[cos - sin]	„	„	„	o	$-\frac{5}{128}$	$-\frac{11}{320}$	o	o	$-\frac{1}{80}$	$-\frac{17}{960}$
	[cos]	$-\frac{1}{2}$	$-\frac{3}{16}$	$-\frac{3}{32}$	H	$-\frac{253}{256}$	$-\frac{691}{10240}$	H	H	$-\frac{223}{80}$	$+\frac{133}{320}$
	[sin]	o	o	o	H	$-\frac{243}{256}$	$-\frac{339}{10240}$	H	H	$-\frac{111}{40}$	$+\frac{13}{30}$
x^1	[cos + sin]	$-\frac{3}{4}$	$+\frac{1}{32}$	$+\frac{3}{64}$	$+\frac{5}{4}$	$-\frac{61}{48}$	$+\frac{129}{512}$	$-\frac{1}{2}$	$+\frac{9}{8}$	$-\frac{125}{96}$	$+\frac{709}{1152}$
	[cos - sin]	„	„	„	o	$-\frac{1}{96}$	$+\frac{59}{1280}$	o	o	$-\frac{1}{96}$	$+\frac{89}{5760}$
	[cos]	$-\frac{3}{4}$	$+\frac{1}{32}$	$+\frac{3}{64}$	H	$-\frac{41}{64}$	$+\frac{763}{5120}$	H	H	$-\frac{21}{32}$	$+\frac{1817}{5760}$
	[sin]	o	o	o	H	$-\frac{121}{192}$	$+\frac{527}{5120}$	H	H	$-\frac{31}{48}$	$+\frac{3}{10}$
x^2	[cos + sin]	$-\frac{1}{2}$	$+\frac{1}{8}$	o	$+\frac{1}{4}$	$+\frac{5}{16}$	$-\frac{85}{1536}$	-1	$+\frac{19}{12}$	$-\frac{49}{96}$	
	[cos - sin]	„	„	„	o	$+\frac{5}{32}$	$+\frac{53}{768}$	o	$+\frac{1}{24}$	$+\frac{7}{480}$	
	[cos]	$-\frac{1}{2}$	$+\frac{1}{8}$	o	H	$+\frac{15}{64}$	$+\frac{7}{1024}$	H	$+\frac{13}{16}$	$-\frac{119}{480}$	
	[sin]	o	o	o	H	$+\frac{5}{64}$	$-\frac{191}{3072}$	H	$+\frac{37}{48}$	$-\frac{21}{80}$	
x^3	[cos + sin]	$-\frac{1}{8}$	$-\frac{9}{32}$	$-\frac{3}{64}$	$+\frac{3}{4}$	$-\frac{65}{128}$		$-\frac{3}{8}$	$+\frac{9}{16}$	$-\frac{61}{128}$	
	[cos - sin]	„	„	„	$+\frac{3}{16}$	$-\frac{1}{32}$		o	o	$-\frac{5}{128}$	
	[cos]	$-\frac{1}{8}$	$-\frac{9}{32}$	$-\frac{3}{64}$	$+\frac{15}{32}$	$-\frac{69}{256}$		H	H	$-\frac{33}{128}$	
	[sin]	o	o	o	$+\frac{9}{32}$	$-\frac{61}{256}$		H	H	$-\frac{7}{32}$	

... Table $(x^0, x^1, x^2, x^3) \frac{\cos}{\sin} 3f \dots$

		$[]^3$				$[]^4$				$[]^5$			
		e^0	e^2	e^4	e^6	e	e^3	e^5	e^7	e^2	e^4	e^6	
x^0	$[\cos + \sin]$	$+ 1 - 9 + \frac{1215}{64} - \frac{449}{32}$				$+ 3 - \frac{39}{2} + \frac{155}{4} - \frac{767}{24}$				$+ \frac{51}{8} - \frac{593}{16} + \frac{75643}{1024}$			
	$[\cos - \sin]$	$\circ \quad \circ \quad \circ \quad - \frac{27}{5120}$				$\circ \quad \circ \quad \circ \quad - \frac{1}{420}$				$\circ \quad \circ \quad \circ$			
	$[\cos]$	$H \quad H \quad H \quad - \frac{71867}{10240}$				$H \quad H \quad H \quad - \frac{8949}{560}$				$H \quad H \quad H$			
	$[\sin]$	$H \quad H \quad H \quad - \frac{71813}{10240}$				$H \quad H \quad H \quad - \frac{26843}{1680}$				$H \quad H \quad H$			
x^1	$[\cos + \sin]$	$+ \frac{1}{2} - \frac{9}{8} - \frac{9}{128}$				$- \frac{1}{2} + \frac{15}{4} - \frac{173}{24} + \frac{161}{36}$				$- \frac{7}{4} + \frac{509}{48} - \frac{10439}{512}$			
	$[\cos - \sin]$	$\circ \quad \circ \quad - \frac{21}{2560}$				$\circ \quad \circ \quad \circ \quad - \frac{1}{144}$				$\circ \quad \circ \quad \circ$			
	$[\cos]$	$H \quad H \quad - \frac{201}{5120}$				$H \quad H \quad H \quad + \frac{643}{288}$				$H \quad H \quad H$			
	$[\sin]$	$H \quad H \quad - \frac{159}{5120}$				$H \quad H \quad H \quad + \frac{215}{96}$				$H \quad H \quad H$			
x^2	$[\cos + \sin]$	$+ \frac{1}{2} - \frac{9}{4} + \frac{399}{128}$				$+ \frac{1}{2} - \frac{8}{3} + \frac{437}{96}$				$+ \frac{1}{4} - \frac{13}{16} - \frac{1393}{1536}$			
	$[\cos - \sin]$	$\circ \quad \circ \quad + \frac{11}{512}$				$\circ \quad \circ \quad + \frac{7}{480}$				$\circ \quad \circ \quad \circ$			
	$[\cos]$	$H \quad H \quad + \frac{1607}{1024}$				$H \quad H \quad + \frac{137}{60}$				$H \quad H \quad H$			
	$[\sin]$	$H \quad H \quad + \frac{1585}{1024}$				$H \quad H \quad + \frac{363}{160}$				$H \quad H \quad H$			
x^3	$[\cos + \sin]$	$+ \frac{3}{8} - \frac{9}{16}$				$- \frac{3}{8} + \frac{15}{8} - \frac{343}{128}$				$- \frac{3}{4} + \frac{469}{128}$			
	$[\cos - \sin]$	$\circ \quad - \frac{1}{128}$				$\circ \quad \circ \quad - \frac{1}{128}$				$\circ \quad \circ$			
	$[\cos]$	$H \quad - \frac{73}{256}$				$H \quad H \quad - \frac{43}{32}$				$H \quad H$			
	$[\sin]$	$H \quad - \frac{71}{256}$				$H \quad H \quad - \frac{171}{128}$				$H \quad H$			

... Table $(x^0, x^1, x^2, x^3) \frac{\cos}{\sin} 3f$.

	$\cos \sin 3f$	$[]^6$			$[]^7$		$[]^8$		$[]^9$		$[]^{10}$	
		e^8	e^5	e^7	e^4	e^6	e^5	e^7	e^6		e^7	
x^0	$[\cos + \sin]$	$+\frac{47}{4} - \frac{525}{8} + \frac{43041}{320}$			$+\frac{2567}{128} - \frac{35563}{320}$		$+\frac{2611}{80} - \frac{87599}{480}$		$+\frac{263351}{5120}$		$+\frac{106469}{1344}$	
	$[\cos - \sin]$	o	o	o	o	o	o	o	o	o	o	
	$[\cos]$	H	H	H	H	H	H	H	H	H	H	
	$[\sin]$	H	H	H	H	H	H	H	H	H	H	
x^1	$[\cos + \sin]$	$-\frac{33}{8} + \frac{373}{16} - \frac{5919}{128}$			$-\frac{787}{96} + \frac{58143}{1280}$		$-\frac{1423}{96} + \frac{237373}{2880}$		$-\frac{64653}{2560}$		$-\frac{47603}{1152}$	
	$[\cos - \sin]$	o	o	o	o	o	o	o	o	o	o	
	$[\cos]$	H	H	H	H	H	H	H	H	H	H	
	$[\sin]$	H	H	H	H	H	H	H	H	H	H	
x^2	$[\cos + \sin]$	$+1 - \frac{37}{8} + \frac{891}{160}$			$+\frac{83}{32} - \frac{9871}{768}$		$+\frac{133}{24} - \frac{857}{30}$		$+\frac{5431}{512}$		$+\frac{1817}{96}$	
	$[\cos - \sin]$	o	o	o	o	o	o	o	o	o	o	
	$[\cos]$	H	H	H	H	H	H	H	H	H	H	
	$[\sin]$	H	H	H	H	H	H	H	H	H	H	
x^3	$[\cos + \sin]$	$-\frac{1}{8} - \frac{9}{16} + \frac{627}{128}$			$-\frac{9}{16} + \frac{35}{32}$		$-\frac{51}{32} + \frac{367}{64}$		$-\frac{467}{128}$		$-\frac{945}{128}$	
	$[\cos - \sin]$	o	o	o	o	o	o	o	o	o	o	
	$[\cos]$	H	H	H	H	H	H	H	H	H	H	
	$[\sin]$	H	H	H	H	H	H	H	H	H	H	

Table $(x^4, x^5, x^6, x^7) \frac{\cos}{\sin} 3f \dots$

$\cos 3f$ $\sin 3f$	$[]^0$			$[]^1$			$[]^2$			
	e^3	e^5	e^7	e^2	e^4	e^6	e	e^3	e^5	e^7
x^4	[cos + sin]	$-\frac{7}{16}$	$+\frac{3}{32}$		$+\frac{1}{4}$	$+\frac{9}{128}$			$-\frac{5}{8}$	$+\frac{43}{64}$
	[cos - sin]	"	"		$+\frac{1}{16}$	$+\frac{3}{32}$			$-\frac{1}{16}$	$+\frac{5}{192}$
	[cos]	$-\frac{7}{16}$	$+\frac{3}{32}$		$+\frac{5}{32}$	$+\frac{21}{256}$			$-\frac{11}{32}$	$+\frac{67}{192}$
	[sin]	o	o		$+\frac{3}{32}$	$-\frac{3}{256}$			$-\frac{9}{32}$	$+\frac{31}{96}$
x^5	[cos + sin]		$-\frac{5}{32}$	$-\frac{9}{64}$		$+\frac{35}{64}$			$-\frac{5}{16}$	$+\frac{45}{128}$
	[cos - sin]	"	"			$+\frac{7}{32}$			$-\frac{1}{32}$	$+\frac{1}{128}$
	[cos]		$-\frac{5}{32}$	$-\frac{9}{64}$		$+\frac{49}{128}$			$-\frac{11}{64}$	$+\frac{23}{128}$
	[sin]	o	o			$+\frac{21}{128}$			$-\frac{9}{64}$	$+\frac{11}{64}$
x^6	[cos + sin]			$-\frac{3}{8}$		$+\frac{15}{64}$				$-\frac{15}{32}$
	[cos - sin]			"		$+\frac{3}{32}$				$-\frac{3}{32}$
	[cos]			$-\frac{3}{8}$		$+\frac{21}{128}$				$-\frac{9}{32}$
	[sin]			o		$+\frac{9}{128}$				$-\frac{3}{16}$
x^7	[cos + sin]			$-\frac{21}{128}$						$-\frac{35}{128}$
	[cos - sin]			"						$-\frac{7}{128}$
	[cos]			$-\frac{21}{128}$						$-\frac{21}{128}$
	[sin]			o						$-\frac{7}{64}$

... Table $(x^4, x^5, x^6, x^7) \frac{\cos}{\sin} 3f \dots$

	$\cos 3f$ $\sin 3f$	$[]^3$ $e^0 \quad e^2 \quad e^4 \quad e^6$	$[]^4$ $e \quad e^3 \quad e^5 \quad e^7$	$[]^5$ $e^2 \quad e^4 \quad e^6$
x^4	[cos + sin]	$+\frac{3}{8} - \frac{9}{8}$	$+\frac{1}{8} - \frac{25}{32}$	$+\frac{1}{4} - \frac{129}{128}$
	[cos - sin]	$\circ - \frac{3}{128}$	$\circ - \frac{1}{96}$	$\circ \circ \circ$
	[cos]	$H - \frac{147}{256}$	$H - \frac{19}{48}$	$H \quad H$
	[sin]	$H - \frac{141}{256}$	$H - \frac{37}{96}$	$H \quad H$
x^5	[cos + sin]	$+\frac{5}{16}$	$-\frac{5}{16} + \frac{75}{64}$	$-\frac{25}{64}$
	[cos - sin]	$+\frac{1}{64}$	$\circ + \frac{1}{64}$	\circ
	[cos]	$+\frac{21}{128}$	$H + \frac{19}{32}$	H
	[sin]	$+\frac{19}{128}$	$H + \frac{37}{64}$	H
x^6	[cos + sin]	$+\frac{5}{16}$	\circ	$+\frac{15}{64}$
	[cos - sin]	$+\frac{1}{64}$	\circ	\circ
	[cos]	$+\frac{21}{128}$	\circ	H
	[sin]	$+\frac{19}{128}$	\circ	H
x^7	[cos + sin]		$-\frac{35}{128}$	
	[cos - sin]		$-\frac{1}{128}$	
	[cos]		$-\frac{9}{64}$	
	[sin]		$-\frac{17}{128}$	

... Table $(x^4, x^5, x^6, x^7) \frac{\cos}{\sin} 3f$.

	$\frac{\cos}{\sin} 3f$	$[]^6$			$[]^7$			$[]^8$			$[]^9$			$[]^{10}$		
		e^3	e^5	e^7	e^4	e^6		e^5	e^7		e^6		e^7		e^7	
x^4	$[\cos + \sin]$		$+\frac{11}{16} - \frac{195}{64}$			$+\frac{1}{16} + \frac{33}{32}$		$+\frac{5}{16} + \frac{67}{96}$			$+\frac{123}{128}$		$+\frac{451}{192}$			
	$[\cos - \sin]$		o o			o o		o o			o		o			
	$[\cos]$		H H			H H		H H			H		H			
	$[\sin]$		H H			H H		H H			H		H			
x^5	$[\cos + \sin]$		$-\frac{5}{32} + \frac{27}{128}$			$-\frac{17}{32}$		$-\frac{1}{32} - \frac{67}{64}$			$-\frac{11}{64}$		$-\frac{73}{128}$			
	$[\cos - \sin]$		o o			o		o o			o		o			
	$[\cos]$		H H			H		H H			H		H			
	$[\sin]$		H H			H		H H			H		H			
x^6	$[\cos + \sin]$			$+\frac{15}{32}$			$+\frac{3}{32}$			$+\frac{3}{8}$		$+\frac{1}{64}$		$+\frac{3}{32}$		
	$[\cos - \sin]$			o			o			o		o		o		
	$[\cos]$			H			H			H		H		H		
	$[\sin]$			H			H			H		H		H		
x^7	$[\cos + \sin]$				$-\frac{21}{128}$					$-\frac{7}{128}$				$-\frac{1}{128}$		
	$[\cos - \sin]$				o					o				o		
	$[\cos]$				H					H			H		H	
	$[\sin]$				H					H			H		H	

Table $(x^0, x^1, x^2, x^3) \frac{\cos}{\sin} 4f \dots$

$\frac{\cos}{\sin} 4f$		$[]^0$		$[]^1$				$[]^2$			
		e^4	e^6	e	e^3	e^5	e^7	e^2	e^4	e^6	
x^0	$[\cos + \sin]$	$+\frac{5}{16}$	$+\frac{3}{16}$	$-\frac{17}{6}$	$+\frac{161}{96}$	$+\frac{907}{3840}$		$+\frac{11}{2}$	$-\frac{149}{12}$	$+\frac{325}{48}$	
	$[\cos - \sin]$	„	„	o	$+\frac{7}{480}$	$+\frac{277}{11520}$		o	o	$+\frac{1}{720}$	
	$[\cos]$	$+\frac{5}{16}$	$+\frac{3}{16}$	H	$+\frac{203}{240}$	$+\frac{1499}{11520}$		H	H	$+\frac{1219}{360}$	
	$[\sin]$	o	o	H	$+\frac{133}{160}$	$+\frac{611}{5760}$		H	H	$+\frac{2437}{720}$	
x^1	$[\cos + \sin]$	$+\frac{5}{8}$	$+\frac{3}{32}$	$-\frac{31}{16}$	$+\frac{1447}{768}$	$-\frac{4537}{30720}$		$+\frac{7}{4}$	$-\frac{53}{12}$	$+\frac{743}{192}$	
	$[\cos - \sin]$	„	„	o	$+\frac{19}{768}$	$-\frac{793}{92160}$		o	o	$+\frac{11}{960}$	
	$[\cos]$	$+\frac{5}{8}$	$+\frac{3}{32}$	H	$+\frac{733}{768}$	$-\frac{3601}{46080}$		H	H	$+\frac{621}{320}$	
	$[\sin]$	o	o	H	$+\frac{119}{128}$	$-\frac{6409}{92160}$		H	H	$+\frac{463}{240}$	
x^2	$[\cos + \sin]$	$+\frac{5}{8}$	$-\frac{5}{32}$	$-\frac{3}{4}$	$+\frac{53}{96}$	$-\frac{2123}{7680}$		$+\frac{1}{4}$	$+\frac{1}{2}$	$-\frac{49}{64}$	
	$[\cos - \sin]$	„	„	o	$-\frac{7}{96}$	$-\frac{649}{7680}$		o	o	$-\frac{1}{64}$	
	$[\cos]$	$+\frac{5}{8}$	$-\frac{5}{32}$	H	$+\frac{23}{96}$	$-\frac{231}{1280}$		H	H	$-\frac{25}{64}$	
	$[\sin]$	o	o	H	$+\frac{5}{16}$	$-\frac{737}{7680}$		H	H	$-\frac{3}{8}$	
x^3	$[\cos + \sin]$	$+\frac{5}{16}$	o	$-\frac{1}{8}$	$-\frac{43}{64}$	$+\frac{389}{1024}$		+ I	-	$-\frac{7}{4}$	
	$[\cos - \sin]$	„	„	o	$-\frac{11}{64}$	$-\frac{59}{3072}$		o	-	$-\frac{1}{48}$	
	$[\cos]$	$+\frac{5}{16}$	o	H	$-\frac{27}{64}$	$+\frac{277}{1536}$		H	-	$\frac{85}{96}$	
	$[\sin]$	o	o	H	$-\frac{1}{4}$	$+\frac{613}{3072}$		H	-	$-\frac{83}{96}$	

... Table $(x^0, x^1, x^2, x^3) \frac{\cos}{\sin} 4f \dots$

$\frac{\cos}{\sin} 4f$		$[]^3$				$[]^4$			
		e	e^3	e^5	e^7	e^0	e^2	e^4	e^6
x^0	$[\cos + \sin]$	- 4	+ $\frac{45}{2}$	- $\frac{591}{16}$	+ $\frac{5737}{256}$	+ 1	- 16	+ $\frac{247}{4}$	- $\frac{3355}{36}$
	$[\cos - \sin]$	o	o	o	- $\frac{9}{8960}$	o	o	o	o
	$[\cos]$	H	H	H	+ $\frac{100393}{8960}$	H	H	H	H
	$[\sin]$	H	H	H	+ $\frac{50201}{4480}$	H	H	H	H
x^1	$[\cos + \sin]$	- $\frac{1}{2}$	+ $\frac{39}{16}$	- $\frac{687}{128}$	+ $\frac{12355}{2048}$	+ $\frac{1}{2}$	- 2	- $\frac{1}{8}$	
	$[\cos - \sin]$	o	o	o	+ $\frac{69}{10240}$	o	o	o	o
	$[\cos]$	H	H	H	+ $\frac{15461}{5120}$	H	H	H	H
	$[\sin]$	H	H	H	+ $\frac{30853}{10240}$	H	H	H	H
x^2	$[\cos + \sin]$	- $\frac{5}{4}$	+ $\frac{67}{16}$	- $\frac{2119}{512}$		+ $\frac{1}{2}$	- 4	+ $\frac{245}{24}$	
	$[\cos - \sin]$	o	o	- $\frac{17}{2560}$		o	o	o	
	$[\cos]$	H	H	- $\frac{2653}{1280}$		H	H	H	
	$[\sin]$	H	H	- $\frac{5289}{2560}$		H	H	H	
x^3	$[\cos + \sin]$	- $\frac{3}{8}$	+ $\frac{39}{32}$	- $\frac{2037}{1024}$		+ $\frac{3}{8}$	- 1		
	$[\cos - \sin]$	o	o	- $\frac{7}{1024}$		o	o	o	
	$[\cos]$	H	H	- $\frac{511}{512}$		H	H	H	
	$[\sin]$	H	H	- $\frac{1015}{1024}$		H	H	H	

... Table $(x^0, x^1, x^2, x^3) \frac{\cos}{\sin} 4f \dots$

	$\frac{\cos}{\sin} 4f$	$[]^5$				$[]^6$			$[]^7$		
		e	e^3	e^5	e^7	e^2	e^4	e^6	e^3	e^5	e^7
x^0	$[\cos + \sin]$	+ 4	- $\frac{85}{2}$	+ $\frac{6845}{48}$	- $\frac{490585}{2304}$	+ $\frac{21}{2}$	- $\frac{377}{4}$	+ $\frac{597}{2}$	+ $\frac{137}{6}$	- $\frac{18113}{96}$	+ $\frac{2249741}{3840}$
	$[\cos - \sin]$	o	o	o	o	o	o	o	o	o	o
	$[\cos]$	H	H	H	H	H	H	H	H	H	H
	$[\sin]$	H	H	H	H	H	H	H	H	H	H
x^1	$[\cos + \sin]$	- $\frac{1}{2}$	+ $\frac{95}{16}$	- $\frac{7325}{384}$	+ $\frac{407635}{18432}$	- $\frac{9}{4}$	+ $\frac{83}{4}$	- $\frac{4029}{64}$	- $\frac{103}{16}$	+ $\frac{40967}{768}$	- $\frac{4894577}{30720}$
	$[\cos - \sin]$	o	o	o	o	o	o	o	o	o	o
	$[\cos]$	H	H	H	H	H	H	H	H	H	H
	$[\sin]$	H	H	H	H	H	H	H	H	H	H
x^2	$[\cos + \sin]$	+ $\frac{3}{4}$	- $\frac{295}{48}$	+ $\frac{8785}{512}$	+ $\frac{1}{4}$	- 1	- $\frac{215}{64}$	+ $\frac{5}{4}$	- $\frac{779}{96}$	+ $\frac{103901}{7680}$	
	$[\cos - \sin]$	o	o	o	o	o	o	o	o	o	o
	$[\cos]$	H	H	H	H	H	H	H	H	H	H
	$[\sin]$	H	H	H	H	H	H	H	H	H	H
x^3	$[\cos + \sin]$	- $\frac{3}{8}$	+ $\frac{95}{32}$	- $\frac{7285}{1024}$	- 1	+ $\frac{117}{16}$	- $\frac{1}{8}$	- $\frac{67}{64}$	+ $\frac{12069}{1024}$		
	$[\cos - \sin]$	o	o	o	o	o	o	o	o	o	o
	$[\cos]$	H	H	H	H	H	H	H	H	H	H
	$[\sin]$	H	H	H	H	H	H	H	H	H	H

... Table $(x^0, x^1, x^2, x^3) \frac{\cos}{\sin} 4f$.

		[] ⁸		[] ⁹		[] ¹⁰		[] ¹¹	
		e^4	e^6	e^5	e^7	e^6		e^7	
x^0	[cos + sin]	$+\frac{2141}{48}$	$-\frac{84857}{240}$	$+\frac{13011}{160}$	$-\frac{809813}{1280}$	$+\frac{5087}{36}$		$+\frac{19116073}{80640}$	
	[cos - sin]	o	o	o	o	o		o	
	[cos]	H	H	H	H	H		H	
	[sin]	H	H	H	H	H		H	
x^1	[cos + sin]	$-\frac{359}{24}$	$+\frac{56527}{480}$	$-\frac{7887}{256}$	$+\frac{2427367}{10240}$	$-\frac{11263}{192}$		$-\frac{9742939}{92160}$	
	[cos - sin]	o	o	o	o	o		o	
	[cos]	H	H	H	H	H		H	
	[sin]	H	H	H	H	H		H	
x^2	[cos + sin]	$+\frac{31}{8}$	$-\frac{855}{32}$	$+\frac{307}{32}$	$-\frac{172907}{2560}$	$+\frac{3995}{192}$		$+\frac{317653}{7680}$	
	[cos - sin]	o	o	o	o	o		o	
	[cos]	H	H	H	H	H		H	
	[sin]	H	H	H	H	H		H	
x^3	[cos + sin]	$-\frac{11}{16}$	$+\frac{3}{2}$	$-\frac{147}{64}$	$+\frac{10999}{1024}$	$-\frac{145}{24}$		$-\frac{42325}{3072}$	
	[cos - sin]	o	o	o	o	o		o	
	[cos]	H	H	H	H	H		H	
	[sin]	H	H	H	H	H		H	

Table $(x^4, x^5, x^6, x^7) \frac{\cos}{\sin} 4f \dots$

$\frac{\cos}{\sin} 4f$		$[]^0$		$[]^1$		$[]^2$		$[]^3$					
		e^4	e^6	e^3	e^5	e^7	e^2	e^4	e^6	e^1	e^3	e^5	e^7
x^4	$[\cos + \sin]$	$+\frac{1}{16}$	$+\frac{3}{8}$	$-\frac{5}{8}$	$+\frac{29}{64}$		$+\frac{1}{4}$	$+\frac{1}{32}$		$-\frac{3}{4}$	$+\frac{111}{64}$		
	$[\cos - \sin]$	"	"	$-\frac{1}{8}$	$+\frac{1}{192}$		o	$+\frac{1}{32}$		o	$+\frac{1}{64}$		
	$[\cos]$	$+\frac{1}{16}$	$+\frac{3}{8}$	$-\frac{3}{8}$	$+\frac{11}{48}$		H	$+\frac{1}{32}$		H	$+\frac{7}{8}$		
	$[\sin]$	o	o	$-\frac{1}{4}$	$+\frac{43}{192}$		H	o		H	$+\frac{55}{64}$		
x^5	$[\cos + \sin]$		$+\frac{11}{32}$	$-\frac{5}{32}$	$-\frac{79}{256}$			$+\frac{45}{64}$		$-\frac{5}{16}$	$+\frac{195}{256}$		
	$[\cos - \sin]$		"	$-\frac{1}{32}$	$-\frac{37}{256}$			$+\frac{3}{64}$		o	$+\frac{1}{256}$		
	$[\cos]$		$+\frac{11}{32}$	$-\frac{3}{32}$	$-\frac{29}{128}$			$+\frac{3}{8}$		H	$+\frac{49}{128}$		
	$[\sin]$		o	$-\frac{1}{16}$	$-\frac{21}{256}$			$+\frac{21}{64}$		H	$+\frac{97}{256}$		
x^6	$[\cos + \sin]$		$+\frac{3}{32}$		$-\frac{33}{64}$			$+\frac{15}{64}$			$-\frac{35}{64}$		
	$[\cos - \sin]$		"		$-\frac{11}{64}$			$+\frac{1}{64}$			$-\frac{1}{64}$		
	$[\cos]$		$+\frac{3}{32}$		$-\frac{11}{32}$			$+\frac{1}{8}$			$-\frac{9}{32}$		
	$[\sin]$		o		$-\frac{11}{64}$			$+\frac{7}{64}$			$-\frac{17}{64}$		
x^7	$[\cos + \sin]$				$-\frac{21}{128}$						$-\frac{35}{128}$		
	$[\cos - \sin]$				$-\frac{7}{128}$						$-\frac{1}{128}$		
	$[\cos]$				$-\frac{7}{64}$						$-\frac{9}{64}$		
	$[\sin]$				$-\frac{7}{128}$						$-\frac{17}{128}$		

... Table (x^4, x^5, x^6, x^7) $\frac{\cos}{\sin} 4f \dots$

		$[]^4$				$[]^5$				$[]^6$			
		e^0	e^2	e^4	e^6	e	e^3	e^5	e^7	e^2	e^4	e^6	
x^4	$[\cos + \sin]$			$+\frac{3}{8}$	-2			$+\frac{1}{4}$	$-\frac{125}{64}$		$+\frac{1}{4}$	$-\frac{45}{32}$	
	$[\cos - \sin]$			o	o			o	o		o	o	
	$[\cos]$		H	H			H	H		H	H		
	$[\sin]$		H	H			H	H		H	H		
x^5	$[\cos + \sin]$				$-\frac{5}{16}$			$-\frac{5}{16}$	$+\frac{475}{256}$			$-\frac{35}{64}$	
	$[\cos - \sin]$				o			o	o			o	
	$[\cos]$			H			H	H				H	
	$[\sin]$			H			H	H				H	
x^6	$[\cos + \sin]$				$+\frac{5}{16}$				$+\frac{5}{64}$			$+\frac{15}{64}$	
	$[\cos - \sin]$				o				o			o	
	$[\cos]$		H				H					H	
	$[\sin]$		H				H					H	
x^7	$[\cos + \sin]$								$-\frac{35}{128}$				
	$[\cos - \sin]$								o				
	$[\cos]$							H					
	$[\sin]$							H					

... Table (x^4, x^5, x^6, x^7) $\frac{\cos}{\sin} 4f$.

	$\frac{\cos}{\sin} 4f$	$[]^7$			$[]^8$			$[]^9$			$[]^{10}$			$[]^{11}$		
		e^3	e^5	e^7	e^4	e^6		e^5	e^7	e^6		e^7		e^7		
x^4	$[\cos + \sin]$		$+\frac{7}{8}$	$-\frac{351}{64}$		$+\frac{1}{16}$		$+\frac{13}{8}$		$+\frac{3}{8}$	$+\frac{95}{64}$		$+\frac{43}{32}$		$+\frac{719}{192}$	
	$[\cos - \sin]$		o	o		o		o	o	o	o		o		o	
	$[\cos]$	H	H		H	H		H	H	H	H		H		H	
	$[\sin]$	H	H		H	H		H	H	H	H		H		H	
x^5	$[\cos + \sin]$		$-\frac{5}{32}$	$+\frac{41}{256}$		$-\frac{21}{32}$		$-\frac{1}{32}$	$-\frac{397}{256}$		$-\frac{13}{64}$		$-\frac{199}{256}$			
	$[\cos - \sin]$		o	o		o		o	o	o	o		o		o	
	$[\cos]$	H	H		H	H		H	H	H	H		H		H	
	$[\sin]$	H	H		H	H		H	H	H	H		H		H	
x^6	$[\cos + \sin]$			$+\frac{39}{64}$			$+\frac{3}{32}$			$+\frac{29}{64}$		$+\frac{1}{64}$		$+\frac{7}{64}$		
	$[\cos - \sin]$			o			o			o		o		o		
	$[\cos]$		H		H			H		H		H		H		H
	$[\sin]$		H		H			H		H		H		H		H
x^7	$[\cos + \sin]$				$-\frac{21}{128}$					$-\frac{7}{128}$				$-\frac{1}{128}$		
	$[\cos - \sin]$				o					o				o		
	$[\cos]$		H							H				H		H
	$[\sin]$		H							H				H		H

Table $(x^0, x^1, x^2, x^3) \frac{\cos}{\sin} 5f \dots$

$\cos \sin 5f$		$[]^0$ $e^5 \quad e^7$		$[]^1$ $e^4 \quad e^6$		$[]^2$ $e^3 \quad e^5 \quad e^7$		$[]^3$ $e^2 \quad e^4 \quad e^6$		
x^0	[cos + sin]	$-\frac{3}{16} - \frac{5}{32}$	$+\frac{1045}{384} - \frac{229}{192}$	$-\frac{95}{12} + \frac{185}{12} - \frac{1295}{192}$		$+\frac{75}{8} - \frac{665}{16} + \frac{58515}{1024}$				
	[cos - sin]	„ „	o - $\frac{37}{9216}$	o o + $\frac{5}{4032}$		o o o				
	[cos]	$-\frac{3}{16} - \frac{5}{32}$	H - $\frac{11029}{18432}$	H H - $\frac{13595}{4032}$		H H H				
	[sin]	o o	H - $\frac{10955}{18432}$	H H - $\frac{425}{126}$		H H H				
x^1	[cos + sin]	$-\frac{15}{32} - \frac{11}{64}$	$+\frac{229}{96} - \frac{7747}{3840}$	$-\frac{29}{8} + \frac{205}{24} - \frac{11831}{1920}$		$+\frac{9}{4} - \frac{169}{16} + \frac{9609}{512}$				
	[cos - sin]	„ „	o - $\frac{151}{7680}$	o o - $\frac{37}{5760}$		o o o				
	[cos]	$-\frac{15}{32} - \frac{11}{64}$	H - $\frac{1043}{1024}$	H H - $\frac{3553}{1152}$		H H H				
	[sin]	o o	H - $\frac{15343}{15360}$	H H - $\frac{277}{90}$		H H H				
x^2	[cos + sin]	$-\frac{5}{8} + \frac{1}{16}$	$+\frac{43}{32} - \frac{357}{256}$	$-1 + \frac{37}{24} - \frac{691}{480}$		$+\frac{1}{4} + \frac{11}{16} - \frac{1435}{512}$				
	[cos - sin]	„ „	o + $\frac{37}{1536}$	o o + $\frac{1}{480}$		o o o				
	[cos]	$-\frac{5}{8} + \frac{1}{16}$	H - $\frac{2105}{3072}$	H H - $\frac{23}{32}$		H H H				
	[sin]	o o	H - $\frac{2179}{3072}$	H H - $\frac{173}{240}$		H H H				
x^3	[cos + sin]	$-\frac{15}{32} + \frac{5}{32}$	$+\frac{7}{16} + \frac{3}{32}$	$-\frac{1}{8} - \frac{19}{16} + \frac{261}{128}$		$+\frac{5}{4} - \frac{531}{128}$				
	[cos - sin]	„ „	o + $\frac{47}{384}$	o o + $\frac{7}{384}$		o o				
	[cos]	$-\frac{15}{32} + \frac{5}{32}$	H + $\frac{83}{768}$	H H + $\frac{395}{384}$		H H				
	[sin]	o o	H - $\frac{11}{768}$	H H + $\frac{97}{96}$		H H				

... Table $(x^0, x^1, x^2, x^3) \frac{\cos}{\sin} 5f \dots$

		$[]^4$				$[]^5$			
		e	e^3	e^5	e^7	e^0	e^2	e^4	e^6
x^0	[cos + sin]	- 5	+ $\frac{95}{2}$	- $\frac{1675}{12}$	+ $\frac{25085}{144}$	+ 1	- 25	+ $\frac{9775}{64}$	- $\frac{110225}{288}$
	[cos - sin]	o	o	o	o	o	o	o	o
	[cos]	H	H	H	H	H	H	H	H
	[sin]	H	H	H	H	H	H	H	H
x^1	[cos + sin]	- $\frac{1}{2}$	+ $\frac{17}{4}$	- $\frac{365}{24}$	+ $\frac{8413}{288}$	+ $\frac{1}{2}$	- $\frac{25}{8}$	- $\frac{25}{128}$	
	[cos - sin]	o	o	o	o	o	o	o	o
	[cos]	H	H	H	H	H	H	H	H
	[sin]	H	H	H	H	H	H	H	H
x^2	[cos + sin]	- $\frac{3}{2}$	+ $\frac{26}{3}$	- $\frac{527}{3^2}$		+ $\frac{1}{2}$	- $\frac{25}{4}$	+ $\frac{9725}{384}$	
	[cos - sin]	o	o	o		o	o	o	
	[cos]	H	H	H		H	H	H	
	[sin]	H	H	H		H	H	H	
x^3	[cos + sin]	- $\frac{3}{8}$	+ $\frac{17}{8}$	- $\frac{725}{128}$		+ $\frac{3}{8}$	- $\frac{25}{16}$		
	[cos - sin]	o	o	o		o	o	o	
	[cos]	H	H	H		H	H	H	
	[sin]	H	H	H		H	H	H	

...Table $(x^0, x^1, x^2, x^3) \frac{\cos}{\sin} 5f...$

		$[]^6$				$[]^7$			$[]^8$		
		e	e^3	e^5	e^7	e^2	e^4	e^6	e^3	e^5	e^7
x^0	$[\cos + \sin]$	$+ 5 - \frac{315}{4} + \frac{6375}{16} - \frac{59585}{64}$				$+ \frac{125}{8} - \frac{9605}{48} + \frac{2825155}{3072}$			$+ \frac{235}{6} - \frac{21515}{48} + \frac{187915}{96}$		
	$[\cos - \sin]$	o o o o				o o o			o o o		
	$[\cos]$	H H H H				H H H			H H H		
	$[\sin]$	H H H H				H H H			H H H		
x^1	$[\cos + \sin]$	$- \frac{1}{2} + \frac{69}{8} - \frac{1335}{32} + \frac{10003}{128}$				$- \frac{11}{4} + \frac{1723}{48} - \frac{242573}{1536}$			$- \frac{37}{4} + \frac{10145}{96} - \frac{425783}{960}$		
	$[\cos - \sin]$	o o o o				o o o			o o o		
	$[\cos]$	H H H H				H H H			H H H		
	$[\sin]$	H H H H				H H H			H H H		
x^2	$[\cos + \sin]$	$+ 1 - \frac{47}{4} + \frac{1553}{32}$				$+ \frac{1}{4} - \frac{19}{16} - \frac{4295}{512}$			$+ \frac{3}{2} - \frac{313}{24} + \frac{6977}{240}$		
	$[\cos - \sin]$	o o o				o o o			o o o		
	$[\cos]$	H H H				H H H			H H H		
	$[\sin]$	H H H				H H H			H H H		
x^3	$[\cos + \sin]$	$- \frac{3}{8} + \frac{69}{16} - \frac{1995}{128}$				$- \frac{5}{4} + \frac{1639}{128}$			$- \frac{1}{8} - \frac{53}{32} + \frac{769}{32}$		
	$[\cos - \sin]$	o o o				o o o			o o o		
	$[\cos]$	H H H				H H H			H H H		
	$[\sin]$	H H H				H H H			H H H		

... Table (x^0, x^1, x^2, x^3) $\frac{\cos}{\sin} 5f$.

	$\cos \sin 5f$	$[]^9$		$[]^{10}$		$[]^{11}$		$[]^{12}$	
		e^4	e^6	e^5	e^7	e^6		e^7	
x^0	[cos + sin]	$+\frac{11035}{128}$	$-\frac{59127}{64}$	$+\frac{8359}{48}$	$-\frac{1033885}{576}$	$+\frac{3050417}{9216}$		$+\frac{67335}{112}$	
	[cos - sin]	o	o	o	o	o		o	
	[cos]	H	H	H	H	H		H	
	[sin]	H	H	H	H	H		H	
x^1	[cos + sin]	$-\frac{787}{32}$	$+\frac{334011}{1280}$	$-\frac{5455}{96}$	$+\frac{665623}{1152}$	$-\frac{918191}{7680}$		$-\frac{37647}{160}$	
	[cos - sin]	o	o	o	o	o		o	
	[cos]	H	H	H	H	H		H	
	[sin]	H	H	H	H	H		H	
x^2	[cos + sin]	$+\frac{173}{32}$	$-\frac{12697}{256}$	$+\frac{365}{24}$	$-\frac{13481}{96}$	$+\frac{18859}{512}$		$+\frac{12907}{160}$	
	[cos - sin]	o	o	o	o	o		o	
	[cos]	H	H	H	H	H		H	
	[sin]	H	H	H	H	H		H	
x^3	[cos + sin]	$-\frac{13}{16}$	$+\frac{63}{32}$	$-\frac{25}{8}$	$+\frac{7115}{384}$	$-\frac{3563}{384}$		$-\frac{3011}{128}$	
	[cos - sin]	o	o	o	o	o		o	
	[cos]	H	H	H	H	H		H	
	[sin]	H	H	H	H	H		H	

Table $(x^4, x^5, x^6, x^7) \frac{\cos}{\sin} 5f \dots$

$\frac{\cos}{\sin} 5f$	$[]^0$		$[]^1$		$[]^2$			$[]^3$		
	e^5	e^7	e^4	e^6	e^3	e^5	e^7	e^2	e^4	e^6
x^4	$[\cos + \sin]$	$-\frac{3}{16} - \frac{5}{32}$	$+\frac{1}{16}$	$+\frac{25}{32}$		$-\frac{13}{16}$	$+\frac{81}{64}$		$+\frac{1}{4}$	$-\frac{9}{128}$
	$[\cos - \sin]$	„ „	o	$+\frac{19}{128}$		o	$-\frac{1}{192}$		o	o
	$[\cos]$	$-\frac{3}{16} - \frac{5}{32}$	H	$+\frac{119}{256}$		H	$+\frac{121}{192}$		H	H
	$[\sin]$	o o	H	$+\frac{81}{256}$		H	$+\frac{61}{96}$		H	H
x^5	$[\cos + \sin]$	$-\frac{1}{32} - \frac{25}{64}$		$+\frac{15}{32}$		$-\frac{5}{32}$	$-\frac{65}{128}$			$+\frac{55}{64}$
	$[\cos - \sin]$	„ „		$+\frac{5}{64}$		o	$-\frac{1}{128}$			o
	$[\cos]$	$-\frac{1}{32} - \frac{25}{64}$		$+\frac{35}{128}$		H	$-\frac{35}{128}$			H
	$[\sin]$	o o		$+\frac{25}{128}$		H	$-\frac{15}{64}$			H
x^6	$[\cos + \sin]$		$-\frac{1}{4}$		$+\frac{3}{32}$			$-\frac{21}{32}$		$+\frac{15}{64}$
	$[\cos - \sin]$		„		$+\frac{1}{64}$			$-\frac{1}{32}$		o
	$[\cos]$		$-\frac{1}{4}$		$+\frac{7}{128}$			$-\frac{11}{32}$		H
	$[\sin]$		o		$+\frac{5}{128}$			$-\frac{5}{16}$		H
x^7	$[\cos + \sin]$			$-\frac{7}{128}$				$-\frac{21}{128}$		
	$[\cos - \sin]$			„				$-\frac{1}{128}$		
	$[\cos]$			$-\frac{7}{128}$				$-\frac{11}{128}$		
	$[\sin]$			o				$-\frac{5}{64}$		

... Table $(x^4, x^5, x^6, x^7) \frac{\cos}{\sin} 5f ...$

	$\frac{\cos}{\sin} 5f$	$[]^4$			$[]^5$			$[]^6$			$[]^7$					
		e	e^3	e^5	e^7	e^0	e^2	e^4	e^6	e	e^3	e^5	e^7	e^2	e^4	e^6
x^4	[cos + sin]		$-\frac{7}{8}$	$+\frac{113}{32}$			$+\frac{3}{8}$	$-\frac{25}{8}$			$+\frac{3}{8}$	$-\frac{249}{64}$		$+\frac{1}{4}$	$-\frac{239}{128}$	
	[cos - sin]		o	o			o	o			o	o		o	o	
	[cos]	H	H			H	H			H	H		H	H		
	[sin]	H	H			H	H			H	H		H	H		
x^5	[cos + sin]		$-\frac{5}{16}$	$+\frac{85}{64}$			$+\frac{5}{16}$				$-\frac{5}{16}$	$+\frac{345}{128}$			$-\frac{45}{64}$	
	[cos - sin]		o	o			o				o	o		o		
	[cos]	H	H				H			H	H		H			
	[sin]	H	H				H			H	H		H			
x^6	[cos + sin]			$-\frac{5}{8}$			$+\frac{5}{16}$				$+\frac{5}{32}$			$+\frac{15}{64}$		
	[cos - sin]			o			o				o			o		
	[cos]		H				H			H			H			
	[sin]		H				H			H			H			
x^7	[cos + sin]				$-\frac{35}{128}$						$-\frac{35}{128}$					
	[cos - sin]				o						o					
	[cos]			H						H			H			
	[sin]			H						H			H			

... Table $(x^4, x^5, x^6, x^7) \frac{\cos}{\sin} 5f$.

	$\frac{\cos}{\sin} 5f$	$[]^8$			$[]^9$		$[]^{10}$		$[]^{11}$		$[]^{12}$	
		e^3	e^5	e^7	e^4	e^6	e^5	e^7	e^6		e^7	
x^4	[cos + sin]		$+\frac{17}{16} - \frac{287}{32}$		$+\frac{1}{16}$	$+\frac{75}{32}$	$+\frac{7}{16}$	$+\frac{505}{192}$		$+\frac{229}{128}$	$+\frac{179}{32}$	
	[cos - sin]		o	o		o	o	o	o		o	
	[cos]	H	H		H	H	H	H	H		H	
	[sin]	H	H		H	H	H	H	H		H	
x^5	[cos + sin]		$-\frac{5}{32} + \frac{5}{64}$			$-\frac{25}{32}$	$-\frac{1}{32}$	$-\frac{275}{128}$		$-\frac{15}{64}$	$-\frac{65}{64}$	
	[cos - sin]		o	o		o	o	o	o		o	
	[cos]	H	H		H	H	H	H	H		H	
	[sin]	H	H		H	H	H	H	H		H	
x^6	[cos + sin]			$+\frac{3}{4}$		$+\frac{3}{32}$		$+\frac{17}{32}$		$+\frac{1}{64}$	$+\frac{1}{8}$	
	[cos - sin]			o		o		o	o		o	
	[cos]		H		H		H	H	H		H	
	[sin]		H		H		H	H	H		H	
x^7	[cos + sin]			$-\frac{21}{128}$				$-\frac{7}{128}$			$-\frac{1}{128}$	
	[cos - sin]			o				o			o	
	[cos]		H				H				H	
	[sin]		H				H				H	

Table $(x^0, x^1, x^2, x^3) \frac{\cos}{\sin} 6f \dots$

$\frac{\cos}{\sin} 6f$		$[]^0$ e^6		$[]^1$ $e^5 \quad e^7$		$[]^2$ $e^4 \quad e^6$		$[]^3$ $e^3 \quad e^5 \quad e^7$		
x^0	[cos + sin]	$+\frac{7}{64}$	$-\frac{773}{320}$	$+\frac{5167}{7680}$		$+\frac{157}{16} - \frac{1339}{80}$		$-\frac{67}{4} + \frac{4053}{64} - \frac{192531}{2560}$		
	[cos - sin]	"	o	$+\frac{17}{53760}$		o o		o o o		
	[cos]	$+\frac{7}{64}$	H	$+\frac{6031}{17920}$		H H		H H H		
	[sin]	o	H	$+\frac{4519}{13440}$		H H		H H H		
x^1	[cos + sin]	$+\frac{21}{64}$	$-\frac{1961}{768}$	$+\frac{162773}{92160}$		$+\frac{277}{48} - \frac{993}{80}$		$-\frac{93}{16} + \frac{6289}{256} - \frac{384789}{10240}$		
	[cos - sin]	"	o	$+\frac{1091}{92160}$		o o		o o o		
	[cos]	$+\frac{21}{64}$	H	$+\frac{20483}{23040}$		H H		H H H		
	[sin]	o	H	$+\frac{26947}{30720}$		H H		H H H		
x^2	[cos + sin]	$+\frac{35}{64}$	$-\frac{179}{96}$	$+\frac{14689}{7680}$		$+\frac{37}{16} - \frac{235}{48}$		$-\frac{5}{4} + \frac{107}{32} - \frac{12123}{2560}$		
	[cos - sin]	"	o	$-\frac{17}{7680}$		o o		o o o		
	[cos]	$+\frac{35}{64}$	H	$+\frac{917}{960}$		H H		H H H		
	[sin]	o	H	$+\frac{2451}{2560}$		H H		H H H		
x^3	[cos + sin]	$+\frac{35}{64}$	$-\frac{57}{64}$	$+\frac{739}{1024}$		$+\frac{9}{16} + \frac{1}{4}$		$-\frac{1}{8} - \frac{117}{64} + \frac{6249}{1024}$		
	[cos - sin]	"	o	$-\frac{75}{1024}$		o o		o o o		
	[cos]	$+\frac{35}{64}$	H	$+\frac{83}{256}$		H H		H H H		
	[sin]	o	H	$+\frac{407}{1024}$		H H		H H H		

... Table $(x^0, x^1, x^2, x^3) \frac{\cos}{\sin} 6f \dots$

		$\left[\begin{smallmatrix} & 4 \\ e^2 & e^4 & e^6 \end{smallmatrix} \right]$			$\left[\begin{smallmatrix} & 5 \\ e & e^3 & e^5 & e^7 \end{smallmatrix} \right]$			
$\cos \sin 6f$								
x^0	[cos + sin]	$+\frac{57}{4}$	$-\frac{829}{8}$	$+\frac{8117}{32}$	-6	$+\frac{345}{4}$	$-\frac{12605}{32}$	$+\frac{1246315}{1536}$
	[cos - sin]	o	o	o	o	o	o	o
	[cos]	H	H	H	H	H	H	H
	[sin]	H	H	H	H	H	H	H
x^1	[cos + sin]	$+\frac{11}{4}$	$-\frac{497}{24}$	$+\frac{1933}{32}$	$-\frac{1}{2}$	$+\frac{105}{16}$	$-\frac{13325}{384}$	$+\frac{1838665}{18432}$
	[cos - sin]	o	o	o	o	o	o	o
	[cos]	H	H	H	H	H	H	H
	[sin]	H	H	H	H	H	H	H
x^2	[cos + sin]	$+\frac{1}{4}$	$+\frac{7}{8}$	$-\frac{685}{96}$	$-\frac{7}{4}$	$+\frac{745}{48}$	$-\frac{72955}{1536}$	
	[cos - sin]	o	o	o	o	o	o	o
	[cos]	H	H	H	H	H	H	H
	[sin]	H	H	H	H	H	H	H
x^3	[cos + sin]		$+\frac{3}{2}$	$-\frac{517}{64}$	$-\frac{3}{8}$	$+\frac{105}{32}$	$-\frac{13265}{1024}$	
	[cos - sin]		o	o	o	o	o	o
	[cos]		H	H	H	H	H	H
	[sin]		H	H	H	H	H	H

... Table $(x^0, x^1, x^2, x^3) \frac{\cos}{\sin} 6f \dots$

		$[]^6$				$[]^7$			
		e^0	e^2	e^4	e^6	e	e^3	e^5	e^7
x^0	$[\cos + \sin]$	$+ 1$	$- 36$	$+ \frac{5103}{16}$	$- \frac{19015}{16}$	$+ 6$	$- \frac{525}{4}$	$+ \frac{29813}{32}$	$- \frac{4795063}{1536}$
	$[\cos - \sin]$	o	o	o	o	o	o	o	o
	$[\cos]$	H	H	H	H	H	H	H	H
	$[\sin]$	H	H	H	H	H	H	H	H
x^1	$[\cos + \sin]$	$+ \frac{1}{2}$	$- \frac{9}{2}$	$- \frac{9}{32}$		$- \frac{1}{2}$	$+ \frac{189}{16}$	$- \frac{30821}{384}$	$+ \frac{4080685}{18432}$
	$[\cos - \sin]$	o	o	o		o	o	o	o
	$[\cos]$	H	H	H		H	H	H	H
	$[\sin]$	H	H	H		H	H	H	H
x^2	$[\cos + \sin]$	$+ \frac{1}{2}$	$- 9$	$+ \frac{1695}{32}$		$+ \frac{5}{4}$	$- \frac{959}{48}$	$+ \frac{175553}{1536}$	
	$[\cos - \sin]$	o	o	o		o	o	o	
	$[\cos]$	H	H	H		H	H	H	
	$[\sin]$	H	H	H		H	H	H	
x^3	$[\cos + \sin]$		$+ \frac{3}{8}$	$- \frac{9}{4}$		$- \frac{3}{8}$	$+ \frac{189}{32}$	$- \frac{30737}{1024}$	
	$[\cos - \sin]$		o	o		o	o	o	
	$[\cos]$		H	H		H	H	H	
	$[\sin]$		H	H		H	H	H	

... Table $(x^0, x^1, x^2, x^3) \frac{\cos}{\sin} 6f \dots$

		$\left[\begin{smallmatrix} & \\ & \end{smallmatrix} \right]^8$			$\left[\begin{smallmatrix} & \\ & \end{smallmatrix} \right]^9$				
		e^2	e^4	e^6	e^3	e^5	e^7		
x^0	$[\cos + \sin]$	$+\frac{87}{4}$	$-\frac{3011}{8}$	$+\frac{151205}{64}$	$+\frac{247}{4}$	$-\frac{59781}{64}$	$+\frac{13947303}{2560}$		
	$[\cos - \sin]$	o	o	o	o	o	o		
	$[\cos]$	H	H	H	H	H	H		
	$[\sin]$	H	H	H	H	H	H		
x^1	$[\cos + \sin]$	$-\frac{13}{4}$	$+\frac{1369}{24}$	$-\frac{21969}{64}$	$-\frac{201}{16}$	$+\frac{48449}{256}$	$-\frac{10874049}{10240}$		
	$[\cos - \sin]$	o	o	o	o	o	o		
	$[\cos]$	H	H	H	H	H	H		
	$[\sin]$	H	H	H	H	H	H		
x^2	$[\cos + \sin]$	$+\frac{1}{4}$	$-\frac{11}{8}$	$-\frac{3317}{192}$	$+\frac{7}{4}$	$-\frac{629}{32}$	$+\frac{145449}{2560}$		
	$[\cos - \sin]$	o	o	o	o	o	o		
	$[\cos]$	H	H	H	H	H	H		
	$[\sin]$	H	H	H	H	H	H		
x^3	$[\cos + \sin]$		$-\frac{3}{2}$	$+\frac{1313}{64}$	$-\frac{1}{8}$	$-\frac{153}{64}$	$+\frac{44889}{1024}$		
	$[\cos - \sin]$		o	o	o	o	o		
	$[\cos]$	H	H	H	H	H	H		
	$[\sin]$	H	H	H	H	H	H		

... Table $(x^0, x^1, x^2, x^3) \frac{\cos}{\sin} 6f$.

	$\frac{\cos}{\sin} 6f$	$\overbrace{[]^{10}}_{e^4 \quad e^6}$	$\overbrace{[]^{11}}_{e^5 \quad e^7}$	$\overbrace{[]^{12}}_{e^6}$	$\overbrace{[]^{13}}_{e^7}$
x^0	[cos + sin]	$+\frac{605}{4} - \frac{16829}{8}$	$+\frac{107333}{320} - \frac{33928027}{7680}$	$+\frac{110793}{160}$	$+\frac{72815971}{53760}$
	[cos - sin]	o o	o o	o	o
	[cos]	H H	H H	H	H
	[sin]	H H	H H	H	H
x^1	[cos + sin]	$-\frac{1805}{48} + \frac{16547}{32}$	$-\frac{73961}{768} + \frac{115203737}{92160}$	$-\frac{35499}{160}$	$-\frac{43634809}{92160}$
	[cos - sin]	o o	o o	o	o
	[cos]	H H	H H	H	H
	[sin]	H H	H H	H	H
x^2	[cos + sin]	$+\frac{115}{16} - \frac{8135}{96}$	$+\frac{2173}{96} - \frac{2039459}{7680}$	$+\frac{1937}{32}$	$+\frac{1111819}{7680}$
	[cos - sin]	o o	o o	o	o
	[cos]	H H	H H	H	H
	[sin]	H H	H H	H	H
x^3	[cos + sin]	$-\frac{15}{16} + \frac{5}{2}$	$-\frac{261}{64} + \frac{30707}{1024}$	$-\frac{863}{64}$	$-\frac{38507}{1024}$
	[cos - sin]	o o	o o	o	o
	[cos]	H H	H H	H	H
	[sin]	H H	H H	H	H

Table $(x^4, x^5, x^6, x^7) \frac{\cos}{\sin} 6f \dots$

$\cos \sin 6f$	$[]^0$ e^6	$[]^1$ $e^5 e^7$	$[]^2$ $e^4 e^6$	$[]^3$ $e^3 e^5 e^7$	$[]^4$ $e^2 e^4 e^6$
x^4	$[\cos + \sin]$ +	$\frac{21}{64}$ $-\frac{1}{4} - \frac{25}{48}$	$+\frac{1}{16} + \frac{21}{16}$	$-1 + \frac{87}{32}$	$+\frac{1}{4} - \frac{15}{64}$
	$[\cos - \sin]$,,	$\circ - \frac{13}{96}$	$\circ \circ$	$\circ \circ$	$\circ \circ$
	$[\cos]$ +	$\frac{21}{64}$ $H - \frac{21}{64}$	$H H$	$H H$	$H H$
	$[\sin]$ o	$H - \frac{37}{192}$	$H H$	$H H$	$H H$
x^5	$[\cos + \sin]$ +	$\frac{7}{64}$ $-\frac{1}{32} - \frac{187}{256}$	$+\frac{19}{32}$	$-\frac{5}{32} - \frac{189}{256}$	$+\frac{65}{64}$
	$[\cos - \sin]$,,	$\circ - \frac{29}{256}$	\circ	$\circ \circ$	\circ
	$[\cos]$ +	$\frac{7}{64}$ $H - \frac{27}{64}$	H	$H H$	H
	$[\sin]$ o	$H - \frac{79}{256}$	H	$H H$	H
x^6	$[\cos + \sin]$ +	$\frac{1}{64}$ $-\frac{21}{64}$	$+\frac{3}{32}$	$-\frac{51}{64}$	$+\frac{15}{64}$
	$[\cos - \sin]$,,	$-\frac{3}{64}$	\circ	\circ	\circ
	$[\cos]$ +	$\frac{1}{64}$ $-\frac{3}{16}$	H	H	H
	$[\sin]$ o	$-\frac{9}{64}$	H	H	H
x^7	$[\cos + \sin]$	$-\frac{7}{128}$		$-\frac{21}{128}$	
	$[\cos - \sin]$	$-\frac{1}{128}$		\circ	
	$[\cos]$	$-\frac{1}{32}$		H	
	$[\sin]$	$-\frac{3}{128}$		H	

... Table $(x^4, x^5, x^6, x^7) \frac{\cos}{\sin} 6f \dots$

	$\cos \sin 6f$	$[]^5$			$[]^6$			$[]^7$			$[]^8$		
		e^3	e^5	e^7	e^4	e^6		e^3	e^5	e^7	e^4	e^6	
x^4	[cos + sin]	- 1	+ $\frac{25}{4}$		+ $\frac{3}{8}$	- $\frac{9}{2}$		+ $\frac{1}{2}$	- $\frac{217}{32}$		+ $\frac{1}{4}$	- $\frac{153}{64}$	
	[cos - sin]	o	o		o	o		o	o		o	o	
	[cos]	H	H		H	H		H	H		H	H	
	[sin]	H	H		H	H		H	H		H	H	
x^5	[cos + sin]	- $\frac{5}{16}$	+ $\frac{525}{256}$		+ $\frac{5}{16}$			- $\frac{5}{16}$	+ $\frac{945}{256}$		- $\frac{55}{64}$		
	[cos - sin]	o	o		o			o	o		o		
	[cos]	H	H		H			H	H		H		
	[sin]	H	H		H			H	H		H		
x^6	[cos + sin]	- $\frac{45}{64}$		+ $\frac{5}{16}$				+ $\frac{15}{64}$			+ $\frac{15}{64}$		
	[cos - sin]	o		o				o			o		
	[cos]	H		H				H			H		
	[sin]	H		H				H			H		
x^7	[cos + sin]	- $\frac{35}{128}$						- $\frac{35}{128}$					
	[cos - sin]	o						o					
	[cos]	H						H					
	[sin]	H						H					

... Table (x^4, x^5, x^6, x^7) $\frac{\cos}{\sin} 6f$.

	$\cos 6f$ $\sin 6f$	$[\]^9$			$[\]^{10}$			$[\]^{11}$			$[\]^{12}$		$[\]^{13}$
		e^3	e^5	e^7	e^4	e^6		e^5	e^7	e^6		e^7	
x^4	[cos + sin]		$+\frac{5}{4}$	$-\frac{219}{16}$		$+\frac{1}{16}$	$+\frac{51}{16}$		$+\frac{1}{2}$	$+\frac{403}{96}$	$+\frac{147}{64}$		$+\frac{191}{24}$
	[cos - sin]		o	o		o	o		o	o	o		o
	[cos]		H	H		H	H		H	H	H		H
	[sin]		H	H		H	H		H	H	H		H
x^5	[cos + sin]		$-\frac{5}{3^2}$	$-\frac{9}{256}$			$-\frac{29}{3^2}$		$-\frac{1}{3^2}$	$-\frac{727}{256}$	$-\frac{17}{64}$		$-\frac{329}{256}$
	[cos - sin]		o	o			o		o	o	o		o
	[cos]		H	H			H		H	H	H		H
	[sin]		H	H			H		H	H	H		H
x^6	[cos + sin]			$+\frac{57}{64}$			$+\frac{3}{3^2}$			$+\frac{39}{64}$	$+\frac{1}{64}$		$+\frac{9}{64}$
	[cos - sin]			o			o			o	o		o
	[cos]			H			H			H	H		H
	[sin]			H			H			H	H		H
x^7	[cos + sin]				$-\frac{21}{128}$					$-\frac{7}{128}$			$-\frac{1}{128}$
	[cos - sin]				o					o			o
	[cos]				H					H			H
	[sin]				H					H			H

Table $(x^0, x^1, x^2, x^3) \frac{\cos}{\sin} 7f \dots$

$\frac{\cos}{\sin} 7f$	$\overbrace{[]^0}_{e^7}$	$\overbrace{[]^1}_{e^6}$	$\overbrace{[]^2}_{e^5 \quad e^7}$		$\overbrace{[]^3}_{e^4 \quad e^6}$		
x^0	[cos + sin] -	$\frac{1}{16}$ $\frac{93289}{46080}$	$-\frac{329}{30}$	$+\frac{11809}{720}$	$+\frac{3227}{128}$	$-\frac{26943}{320}$	
	[cos - sin] "	o	o	o	o	o	
	[cos] -	$\frac{1}{16}$	H	H	H	H	
	[sin] o	H	H	H	H	H	
x^1	[cos + sin] -	$\frac{7}{32}$ $\frac{19081}{7680}$	$-\frac{187}{24}$	$+\frac{10927}{720}$	$+\frac{361}{32}$	$-\frac{55317}{1280}$	
	[cos - sin] "	o	o	o	o	o	
	[cos] -	$\frac{7}{32}$	H	H	H	H	
	[sin] o	H	H	H	H	H	
x^2	[cos + sin] -	$\frac{7}{16}$ $\frac{1131}{512}$	$-\frac{97}{24}$	$+\frac{4223}{480}$	$+\frac{113}{32}$	$-\frac{3177}{256}$	
	[cos - sin] "	o	o	o	o	o	
	[cos] -	$\frac{7}{16}$	H	H	H	H	
	[sin] o	H	H	H	H	H	
x^3	[cos + sin] -	$\frac{35}{64}$ $\frac{529}{384}$	$-\frac{23}{16}$	$+\frac{391}{192}$	$+\frac{11}{16}$	$+\frac{15}{32}$	
	[cos - sin] "	o	o	o	o	o	
	[cos] -	$\frac{35}{64}$	H	H	H	H	
	[sin] o	H	H	H	H	H	

... Table $(x^0, x^1, x^2, x^3) \frac{\cos}{\sin} 7f \dots$

		$\left[\begin{smallmatrix} e^3 & e^5 & e^7 \end{smallmatrix} \right]^4$			$\left[\begin{smallmatrix} e^2 & e^4 & e^6 \end{smallmatrix} \right]^5$				
$\cos \sin 7f$									
x^0	[cos + sin]	$-\frac{91}{3}$	$+\frac{4445}{24}$	$-\frac{47299}{120}$	$+\frac{161}{8}$	$-\frac{10409}{48}$	$+\frac{2509759}{3072}$		
	[cos - sin]	o	o	o	o	o	o		
	[cos]	H	H	H	H	H	H		
	[sin]	H	H	H	H	H	H		
x^1	[cos + sin]	$-\frac{17}{2}$	$+\frac{2689}{48}$	$-\frac{3395}{24}$	$+\frac{13}{4}$	$-\frac{1721}{48}$	$+\frac{236923}{1536}$		
	[cos - sin]	o	o	o	o	o	o		
	[cos]	H	H	H	H	H	H		
	[sin]	H	H	H	H	H	H		
x^2	[cos + sin]	$-\frac{3}{2}$	$+\frac{149}{24}$	$-\frac{5897}{480}$	$+\frac{1}{4}$	$+\frac{17}{16}$	$-\frac{7671}{512}$		
	[cos - sin]	o	o	o	o	o	o		
	[cos]	H	H	H	H	H	H		
	[sin]	H	H	H	H	H	H		
x^3	[cos + sin]	$-\frac{1}{8}$	$-\frac{83}{32}$	$+\frac{1807}{128}$		$+\frac{7}{4}$	$-\frac{1781}{128}$		
	[cos - sin]	o	o	o		o	o		
	[cos]	H	H	H		H	H		
	[sin]	H	H	H		H	H		

... Table $(x^0, x^1, x^2, x^3) \frac{\cos}{\sin} 7f \dots$

		$[]^6$				$[]^7$			
		e^1	e^3	e^5	e^7	e^0	e^2	e^4	e^6
x^0	$[\cos + \sin]$	-7	$+\frac{567}{4}$	$-\frac{14805}{16}$	$+\frac{89999}{32}$	$+1$	-49	$+\frac{37975}{64}$	$-\frac{883421}{288}$
	$[\cos - \sin]$	o	o	o	o	o	o	o	o
	$[\cos]$	H	H	H	H	H	H	H	H
	$[\sin]$	H	H	H	H	H	H	H	H
x^1	$[\cos + \sin]$	$-\frac{1}{2}$	$+\frac{75}{8}$	$-\frac{2199}{32}$	$+\frac{17549}{64}$	$+\frac{1}{2}$	$-\frac{49}{8}$	$-\frac{49}{128}$	
	$[\cos - \sin]$	o	o	o	o	o	o	o	
	$[\cos]$	H	H	H	H	H	H	H	H
	$[\sin]$	H	H	H	H	H	H	H	H
x^2	$[\cos + \sin]$		-2	$+\frac{101}{4}$	$-\frac{3611}{32}$	$+\frac{1}{2}$	$-\frac{49}{4}$	$+\frac{37877}{384}$	
	$[\cos - \sin]$	o	o	o		o	o	o	
	$[\cos]$	H	H	H		H	H	H	H
	$[\sin]$	H	H	H		H	H	H	H
x^3	$[\cos + \sin]$		$-\frac{3}{8}$	$+\frac{75}{16}$	$-\frac{411}{16}$		$+\frac{3}{8}$	$-\frac{49}{16}$	
	$[\cos - \sin]$	o	o	o		o	o	o	
	$[\cos]$	H	H	H		H	H	H	H
	$[\sin]$	H	H	H		H	H	H	H

... Table $(x^0, x^1, x^2, x^3) \frac{\cos}{\sin} 7f \dots$

		$[]^8$				$[]^9$		
		e	e^3	e^5	e^7	e^2	e^4	e^6
x^0	$[\cos + \sin]$	$+ 7$	$- 203$	$+ \frac{5768}{3}$	$- \frac{1255219}{144}$	$+ \frac{231}{8}$	$- \frac{10381}{16}$	$+ \frac{5454687}{1024}$
	$[\cos - \sin]$	o	o	o	o	o	o	o
	$[\cos]$	H	H	H	H	H	H	H
	$[\sin]$	H	H	H	H	H	H	H
x^1	$[\cos + \sin]$	$- \frac{1}{2}$	$+ \frac{31}{2}$	$- \frac{845}{6}$	$+ \frac{154651}{288}$	$- \frac{15}{4}$	$+ \frac{1363}{16}$	$- \frac{344031}{512}$
	$[\cos - \sin]$	o	o	o	o	o	o	o
	$[\cos]$	H	H	H	H	H	H	H
	$[\sin]$	H	H	H	H	H	H	H
x^2	$[\cos + \sin]$	$+ \frac{3}{2}$	$- \frac{94}{3}$	$+ \frac{3791}{16}$		$+ \frac{1}{4}$	$- \frac{25}{16}$	$- \frac{16155}{512}$
	$[\cos - \sin]$	o	o	o		o	o	o
	$[\cos]$	H	H	H		H	H	H
	$[\sin]$	H	H	H		H	H	H
x^3	$[\cos + \sin]$	$- \frac{3}{8}$	$+ \frac{31}{4}$	$- \frac{3373}{64}$		$- \frac{7}{4}$	$+ \frac{3945}{118}$	
	$[\cos - \sin]$	o	o	o		o	o	
	$[\cos]$	H	H	H		H	H	
	$[\sin]$	H	H	H		H	H	

... Table $(x^0, x^1, x^2, x^3) \frac{\cos}{\sin} 7f \dots$

		$[]^{10}$			$[]^{11}$	
		e^3	e^5	e^7	e^4	e^6
x^0	$[\cos + \sin]$	$+\frac{1099}{12}$	$-\frac{84889}{48}$	$+\frac{1273433}{96}$	$+\frac{94885}{384}$	$-\frac{4151693}{960}$
	$[\cos - \sin]$	o	o	o	o	o
	$[\cos]$	H	H	H	H	H
	$[\sin]$	H	H	H	H	H
x^1	$[\cos + \sin]$	$-\frac{131}{8}$	$+\frac{30193}{96}$	$-\frac{436457}{192}$	$-\frac{5231}{96}$	$+\frac{3618557}{3840}$
	$[\cos - \sin]$	o	o	o	o	o
	$[\cos]$	H	H	H	H	H
	$[\sin]$	H	H	H	H	H
x^2	$[\cos + \sin]$	$+\frac{2}{2}$	$-\frac{677}{24}$	$+\frac{9889}{96}$	$+\frac{295}{32}$	$-\frac{34789}{256}$
	$[\cos - \sin]$	o	o	o	o	o
	$[\cos]$	H	H	H	H	H
	$[\sin]$	H	H	H	H	H
x^3	$[\cos + \sin]$	$-\frac{1}{8}$	$-\frac{13}{4}$	$+\frac{2361}{32}$	$-\frac{17}{16}$	$+\frac{99}{32}$
	$[\cos - \sin]$	o	o	o	o	o
	$[\cos]$	H	H	H	H	H
	$[\sin]$	H	H	H	H	H

Table $(x^0, x^1, x^2, x^3) \frac{\cos}{\sin} 7f \dots$

		$[]^{12}$		$[]^{13}$	$[]^{14}$
		e^5	e^7	e^6	e^7
x^0	[cos + sin]	$+\frac{23877}{40}$	$-\frac{390873}{40}$	$+\frac{61307827}{46080}$	$+\frac{1004057}{360}$
	[cos - sin]	o	o	o	o
	[cos]	H	H	H	H
	[sin]	H	H	H	H
x^1	[cos + sin]	$-\frac{2451}{16}$	$+\frac{49413}{20}$	$-\frac{2945503}{7680}$	$-\frac{254051}{288}$
	[cos - sin]	o	o	o	o
	[cos]	H	H	H	H
	[sin]	H	H	H	H
x^2	[cos + sin]	$+\frac{257}{8}$	$-\frac{74701}{160}$	$+\frac{144373}{1536}$	$+\frac{116977}{480}$
	[cos - sin]	o	o	o	o
	[cos]	H	H	H	H
	[sin]	H	H	H	H
x^3	[cos + sin]	$-\frac{165}{32}$	$+\frac{5905}{128}$	$-\frac{7213}{384}$	$-\frac{10969}{192}$
	[cos - sin]	o	o	o	o
	[cos]	H	H	H	H
	[sin]	H	H	H	H

Table $(x^4, x^5, x^6, x^7) \frac{\cos}{\sin} 7f \dots$

$\frac{\cos}{\sin} 7f$	$\underbrace{[]^0}_{e^7}$	$\underbrace{[]^1}_{e^6}$	$\underbrace{[]^2}_{e^5 e^7}$	$\underbrace{[]^3}_{e^4 e^6}$	$\underbrace{[]^4}_{e^3 e^5 e^7}$	$\underbrace{[]^5}_{e^4 e^6}$	
x^4	[cos + sin] -	$-\frac{7}{16}$	$+\frac{73}{128}$	$-\frac{5}{16} - \frac{221}{192}$	$+\frac{1}{16} + \frac{63}{32}$	$-\frac{19}{16} + 5$	$+\frac{1}{4} - \frac{59}{128}$
	[cos - sin] ,,	o	o o	o o	o o	o o	o o
	[cos] -	$-\frac{7}{16}$	H H H	H H	H H	H H	H H
	[sin] o	H H H	H H	H H	H H	H H	H H
x^5	[cos + sin] -	$-\frac{7}{3^2}$	$+\frac{9}{64}$	$-\frac{1}{3^2} - \frac{149}{128}$	$+\frac{23}{3^2}$	$-\frac{5}{3^2} - 1$	$+\frac{75}{64}$
	[cos - sin] ,,	o	o o	o	o	o o	o
	[cos] -	$-\frac{7}{3^2}$	H H H	H	H H	H	H
	[sin] o	H H H	H H	H	H H	H	H
x^6	[cos + sin] -	$-\frac{1}{16}$	$+\frac{1}{64}$	$-\frac{13}{3^2}$	$+\frac{3}{3^2}$	$-\frac{15}{16}$	$+\frac{15}{64}$
	[cos - sin] ,,	o	o	o	o	o	o
	[cos] -	$-\frac{1}{16}$	H H	H H	H H	H H	H H
	[sin] o	H H	H H	H H	H H	H H	H H
x^7	[cos + sin] -	$-\frac{1}{128}$		$-\frac{7}{128}$		$-\frac{21}{128}$	
	[cos - sin] ,,			o		o	
	[cos] -	$-\frac{1}{128}$		H		H	
	[sin] o			H		H	

... Table $(x^4, x^5, x^6, x^7) \frac{\cos}{\sin} 7f \dots$

	$\frac{\cos}{\sin} 7f$	$[]^6$		$[]^7$		$[]^8$			$[]^9$			
		e^5	e^7	e^4	e^6	e	e^3	e^5	e^7	e^2	e^4	e^6
x^4	$[\cos + \sin]$	$-\frac{9}{8}$	$+\frac{645}{64}$	$+\frac{3}{8}$	$-\frac{49}{8}$			$+\frac{5}{8}$	$-\frac{173}{16}$	$+\frac{1}{4}$	$-\frac{381}{128}$	
	$[\cos - \sin]$	o	o	o	o			o	o	o	o	
	$[\cos]$	H	H	H	H			H	H	H	H	
	$[\sin]$	H	H	H	H			H	H	H	H	
x^5	$[\cos + \sin]$	$-\frac{5}{16}$	$+\frac{375}{128}$			$+\frac{5}{16}$		$-\frac{5}{16}$	$+\frac{155}{32}$		$-\frac{65}{64}$	
	$[\cos - \sin]$	o	o			o		o	o		o	
	$[\cos]$	H	H			H		H	H		H	
	$[\sin]$	H	H			H		H	H		H	
x^6	$[\cos + \sin]$		$-\frac{25}{32}$			$+\frac{5}{16}$			$+\frac{5}{16}$		$+\frac{15}{64}$	
	$[\cos - \sin]$		o			o			o		o	
	$[\cos]$		H			H			H		H	
	$[\sin]$		H			H			H		H	
x^7	$[\cos + \sin]$		$-\frac{35}{128}$						$-\frac{35}{128}$			
	$[\cos - \sin]$		o						o			
	$[\cos]$		H						H			
	$[\sin]$		H						H			

... Table (x^4, x^5, x^6, x^7) $\frac{\cos}{\sin} 7f.$

	$\cos \sin 7f$	$[]^{10}$		$[]^{11}$		$[]^{12}$		$[]^{13}$		$[]^{14}$	
		e^5	e^7	e^4	e^6	e^5	e^7	e^6		e^7	
x^4	$[\cos + \sin]$	$+\frac{23}{16}$	$-\frac{3807}{192}$	$+\frac{1}{16}$	$+\frac{133}{32}$	$+\frac{9}{16}$	$+\frac{25}{4}$	$+\frac{367}{128}$		$+\frac{2093}{192}$	
	$[\cos - \sin]$	o	o	o	o	o	o	o		o	
	$[\cos]$	H	H	H	H	H	H	H		H	
	$[\sin]$	H	H	H	H	H	H	H		H	
x^5	$[\cos + \sin]$	$-\frac{5}{32}$	$-\frac{23}{128}$		$-\frac{33}{32}$	$-\frac{1}{32}$	$-\frac{29}{8}$	$-\frac{19}{64}$		$-\frac{203}{128}$	
	$[\cos - \sin]$	o	o		o	o	o	o		o	
	$[\cos]$	H	H		H	H	H	H		H	
	$[\sin]$	H	H		H	H	H	H		H	
x^6	$[\cos + \sin]$		$+\frac{33}{32}$		$+\frac{3}{32}$		$+\frac{11}{16}$	$+\frac{1}{64}$		$+\frac{5}{32}$	
	$[\cos - \sin]$		o		o		o	o		o	
	$[\cos]$		H		H		H	H		H	
	$[\sin]$		H		H		H	H		H	
x^7	$[\cos + \sin]$		$-\frac{21}{128}$				$-\frac{7}{128}$			$-\frac{1}{128}$	
	$[\cos - \sin]$		o				o			o	
	$[\cos]$		H				H			H	
	$[\sin]$		H				H			H	

Table $\left(\left(\frac{r}{a}\right)^{-5} \dots \left(\frac{r}{a}\right)^{-1}, \log \frac{r}{a}, \left(\frac{r}{a}\right)^{-1} \dots \left(\frac{r}{a}\right)^4 \right) \dots$

	[] ⁰				[] ¹			
	e^0	e^2	e^4	e^6	e	e^3	e^5	e^7
$\left(\frac{r}{a}\right)^{-5}$	+ 1	+ 5	+ $\frac{105}{8}$	+ $\frac{105}{4}$	+ $\frac{5}{2}$	+ $\frac{135}{16}$	+ $\frac{7285}{384}$	+ $\frac{643015}{18432}$
$\left(\frac{r}{a}\right)^{-4}$	+ 1	+ 3	+ $\frac{45}{8}$	+ $\frac{35}{4}$	+ 2	+ $\frac{17}{4}$	+ $\frac{685}{96}$	+ $\frac{48293}{4608}$
$\left(\frac{r}{a}\right)^{-3}$	+ 1	+ $\frac{3}{2}$	+ $\frac{15}{8}$	+ $\frac{35}{16}$	+ $\frac{3}{2}$	+ $\frac{27}{16}$	+ $\frac{261}{128}$	+ $\frac{14309}{6144}$
$\left(\frac{r}{a}\right)^{-2}$	+ 1	+ $\frac{1}{2}$	+ $\frac{3}{8}$	+ $\frac{5}{16}$	+ 1	+ $\frac{3}{8}$	+ $\frac{65}{192}$	+ $\frac{2675}{9216}$
$\left(\frac{r}{a}\right)^{-1}$	+ 1	o	o	o	+ $\frac{1}{2}$	- $\frac{1}{16}$	+ $\frac{1}{384}$	- $\frac{1}{18432}$
[cos]								
$\log \frac{r}{a}$	o	+ $\frac{1}{4}$	+ $\frac{1}{32}$	+ $\frac{1}{96}$	- $\frac{1}{2}$	+ $\frac{3}{16}$	+ $\frac{1}{128}$	+ $\frac{127}{18432}$
$\left(\frac{r}{a}\right)^1$	+ 1	+ $\frac{1}{2}$	o	o	- $\frac{1}{2}$	+ $\frac{3}{16}$	- $\frac{5}{384}$	+ $\frac{7}{18432}$
$\left(\frac{r}{a}\right)^2$	+ 1	+ $\frac{3}{2}$	o	o	- 1	+ $\frac{1}{8}$	- $\frac{1}{192}$	+ $\frac{1}{9216}$
$\left(\frac{r}{a}\right)^3$	+ 1	+ 3	+ $\frac{3}{8}$	o	- $\frac{3}{2}$	- $\frac{9}{16}$	+ $\frac{15}{128}$	- $\frac{35}{6144}$
$\left(\frac{r}{a}\right)^4$	+ 1	+ 5	+ $\frac{15}{8}$	o	- 2	- $\frac{9}{4}$	+ $\frac{19}{96}$	- $\frac{29}{4608}$

... Table $\left(\left(\frac{r}{a}\right)^{-5} \dots \left(\frac{r}{a}\right)^{-1}, \log \frac{r}{a}, \left(\frac{r}{a}\right)^1 \dots \left(\frac{r}{a}\right)^4\right) \dots$

		$[]^2$			$[]^3$			$[]^4$	
		e^2	e^4	e^6	e^3	e^5	e^7	e^4	e^6
$\left(\frac{r}{a}\right)^{-5}$		+ 5	+ $\frac{155}{12}$	+ $\frac{835}{32}$	+ $\frac{145}{16}$	+ $\frac{4715}{256}$	+ $\frac{70893}{2048}$	+ $\frac{745}{48}$	+ $\frac{197}{8}$
$\left(\frac{r}{a}\right)^{-4}$		+ $\frac{7}{2}$	+ $\frac{67}{12}$	+ $\frac{421}{48}$	+ $\frac{23}{4}$	+ $\frac{435}{64}$	+ $\frac{27027}{2560}$	+ $\frac{437}{48}$	+ $\frac{899}{120}$
$\left(\frac{r}{a}\right)^{-3}$		+ $\frac{9}{4}$	+ $\frac{7}{4}$	+ $\frac{141}{64}$	+ $\frac{53}{16}$	+ $\frac{393}{256}$	+ $\frac{24753}{10240}$	+ $\frac{77}{16}$	+ $\frac{129}{160}$
$\left(\frac{r}{a}\right)^{-2}$		+ $\frac{5}{4}$	+ $\frac{1}{6}$	+ $\frac{21}{64}$	+ $\frac{13}{8}$	- $\frac{25}{128}$	+ $\frac{393}{1024}$	+ $\frac{103}{48}$	- $\frac{129}{160}$
$\left(\frac{r}{a}\right)^{-1}$		+ $\frac{1}{2}$	- $\frac{1}{6}$	+ $\frac{1}{48}$	+ $\frac{9}{16}$	- $\frac{81}{256}$	+ $\frac{729}{10240}$	+ $\frac{2}{3}$	- $\frac{8}{15}$
[cos]									
$\log \frac{r}{a}$		- $\frac{3}{8}$	+ $\frac{11}{48}$	- $\frac{3}{128}$	- $\frac{17}{48}$	+ $\frac{77}{256}$	- $\frac{743}{10240}$	- $\frac{71}{192}$	+ $\frac{129}{320}$
$\left(\frac{r}{a}\right)^1$		- $\frac{1}{4}$	+ $\frac{1}{6}$	- $\frac{1}{32}$	- $\frac{3}{16}$	+ $\frac{45}{256}$	- $\frac{567}{10240}$	- $\frac{1}{6}$	+ $\frac{1}{5}$
$\left(\frac{r}{a}\right)^2$		- $\frac{1}{4}$	+ $\frac{1}{12}$	- $\frac{1}{96}$	- $\frac{1}{8}$	+ $\frac{9}{128}$	- $\frac{81}{5120}$	- $\frac{1}{12}$	+ $\frac{1}{15}$
$\left(\frac{r}{a}\right)^3$		o	- $\frac{1}{4}$	+ $\frac{3}{32}$	+ $\frac{1}{16}$	- $\frac{45}{256}$	+ $\frac{189}{2048}$	+ $\frac{1}{16}$	- $\frac{3}{20}$
$\left(\frac{r}{a}\right)^4$		+ $\frac{1}{2}$	- $\frac{7}{12}$	+ $\frac{1}{8}$	+ $\frac{1}{4}$	- $\frac{19}{64}$	+ $\frac{261}{2560}$	+ $\frac{7}{48}$	- $\frac{1}{5}$

... Table $\left(\left(\frac{r}{a}\right)^{-5} \dots \left(\frac{r}{a}\right)^{-1}, \log \frac{r}{a} \dots \left(\frac{r}{a}\right)^1 \dots \left(\frac{r}{a}\right)^4 \right)$.

		[] ⁵		[] ⁶		[] ⁷	
		e^5	e^7	e^6	e^7		
$\left(\frac{r}{a}\right)^{-5}$		$+ \frac{19669}{768}$	$+ \frac{565135}{18432}$	$+ \frac{1317}{32}$		$+ \frac{1195093}{18432}$	
$\left(\frac{r}{a}\right)^{-4}$		$+ \frac{2701}{192}$	$+ \frac{31997}{4608}$	$+ \frac{1709}{80}$		$+ \frac{737707}{23040}$	
$\left(\frac{r}{a}\right)^{-3}$		$+ \frac{1773}{256}$	$- \frac{4987}{6144}$	$+ \frac{3167}{320}$		$+ \frac{432091}{30720}$	
$\left(\frac{r}{a}\right)^{-2}$		$+ \frac{1097}{384}$	$- \frac{16621}{9216}$	$+ \frac{1223}{320}$		$+ \frac{47273}{9216}$	
$\left(\frac{r}{a}\right)^{-1}$		$+ \frac{625}{768}$	$- \frac{15625}{18432}$	$+ \frac{81}{80}$		$+ \frac{117649}{92160}$	
[cos]							
$\log \frac{r}{a}$		$- \frac{523}{1280}$	$+ \frac{10039}{18432}$	$- \frac{899}{1920}$		$- \frac{355081}{645120}$	
$\left(\frac{r}{a}\right)^1$		$- \frac{125}{768}$	$+ \frac{4375}{18432}$	$- \frac{27}{160}$		$- \frac{16807}{92160}$	
$\left(\frac{r}{a}\right)^2$		$- \frac{25}{384}$	$+ \frac{615}{9216}$	$- \frac{9}{160}$		$- \frac{2401}{46080}$	
$\left(\frac{r}{a}\right)^3$		$+ \frac{15}{256}$	$- \frac{875}{6144}$	$+ \frac{9}{160}$		$+ \frac{343}{6144}$	
$\left(\frac{r}{a}\right)^4$		$+ \frac{19}{192}$	$- \frac{725}{4608}$	$+ \frac{3}{40}$		$+ \frac{1421}{23040}$	

Table $\left(\left(\frac{r}{a}\right)^1 \dots \left(\frac{r}{a}\right)^4\right) \cos f \dots$

$\cos f$ $\sin f$	[] ⁰				[] ¹				
	e	e^3	e^5	e^7	e	e^2	e^4	e^6	
$\left(\frac{r}{a}\right)^1$	[cos + sin] -	$\frac{3}{2}$	o	o	o	+ I	- $\frac{1}{2}$	- $\frac{1}{64}$	- $\frac{29}{1152}$
	[cos - sin] "	"	"	"	o	+ $\frac{1}{8}$	+ $\frac{1}{24}$	+ $\frac{25}{1024}$	
	[cos] -	$\frac{3}{2}$	o	o	o	H	- $\frac{3}{16}$	+ $\frac{5}{384}$	- $\frac{7}{18432}$
	[sin] o	o	o	o	o	H	- $\frac{5}{16}$	- $\frac{11}{384}$	- $\frac{457}{18432}$
$\left(\frac{r}{a}\right)^2$	[cos + sin] -	2	- $\frac{1}{2}$	o	o	+ I	+ $\frac{1}{2}$	- $\frac{25}{64}$	- $\frac{23}{1152}$
	[cos - sin] "	"	"	"	o	+ $\frac{5}{8}$	o	+ $\frac{143}{3072}$	
	[cos] -	2	- $\frac{1}{2}$	o	o	H	+ $\frac{9}{16}$	- $\frac{25}{128}$	+ $\frac{245}{18432}$
	[sin] o	o	o	o	o	H	- $\frac{1}{16}$	- $\frac{25}{128}$	- $\frac{613}{18432}$
$\left(\frac{r}{a}\right)^3$	[cos + sin] -	$\frac{5}{2}$	- $\frac{15}{8}$	o	o	+ I	+ 2	- $\frac{41}{64}$	- $\frac{37}{576}$
	[cos - sin] "	"	"	"	o	+ $\frac{11}{8}$	+ $\frac{7}{48}$	+ $\frac{265}{3072}$	
	[cos] -	$\frac{5}{2}$	- $\frac{15}{8}$	o	o	H	+ $\frac{27}{16}$	- $\frac{95}{384}$	+ $\frac{203}{18432}$
	[sin] o	o	o	o	o	H	+ $\frac{5}{16}$	- $\frac{151}{384}$	- $\frac{1387}{18432}$
$\left(\frac{r}{a}\right)^4$	[cos + sin] -	3	- $\frac{9}{2}$	- $\frac{3}{8}$	o	+ I	+ 4	- $\frac{1}{64}$	- $\frac{199}{576}$
	[cos - sin] "	"	"	"	o	+ $\frac{19}{8}$	+ $\frac{47}{48}$	+ $\frac{115}{1024}$	
	[cos] -	3	- $\frac{9}{2}$	- $\frac{3}{8}$	o	H	+ $\frac{51}{16}$	+ $\frac{185}{384}$	- $\frac{2149}{18432}$
	[sin] o	o	o	o	o	H	+ $\frac{13}{16}$	- $\frac{191}{384}$	- $\frac{4219}{18432}$

...Table $\left(\left(\frac{r}{a}\right)^1 \dots \left(\frac{r}{a}\right)^4\right) \cos f \dots$

	$\cos f$ $\sin f$	$[]^2$				$[]^3$			$[]^4$		
		e	e^3	e^5	e^7	e^2	e^4	e^6	e^3	e^5	e^7
$\left(\frac{r}{a}\right)^1$	$[\cos + \sin]$	$+\frac{1}{2}$	$-\frac{3}{8}$	$+\frac{5}{96}$	$-\frac{1}{72}$	$+\frac{3}{8}$	$-\frac{3}{8}$	$+\frac{111}{1024}$	$+\frac{1}{3}$	$-\frac{5}{12}$	$+\frac{43}{240}$
	$[\cos - \sin]$	\circ	$+\frac{1}{24}$	$+\frac{1}{96}$	$+\frac{1}{120}$	\circ	$+\frac{3}{128}$	$+\frac{3}{1280}$	\circ	$+\frac{1}{60}$	$-\frac{1}{720}$
	$[\cos]$	H	$-\frac{1}{6}$	$+\frac{1}{32}$	$-\frac{1}{360}$	H	$-\frac{45}{256}$	$+\frac{567}{10240}$	H	$-\frac{1}{5}$	$+\frac{4}{45}$
	$[\sin]$	H	$-\frac{5}{24}$	$+\frac{1}{48}$	$-\frac{1}{90}$	H	$-\frac{51}{256}$	$+\frac{543}{10240}$	H	$-\frac{13}{60}$	$+\frac{13}{144}$
$\left(\frac{r}{a}\right)^2$	$[\cos + \sin]$	\circ	$+\frac{1}{2}$	$-\frac{1}{3}$	$+\frac{5}{96}$	$-\frac{1}{8}$	$+\frac{1}{2}$	$-\frac{397}{1024}$	$-\frac{1}{6}$	$+\frac{13}{24}$	$-\frac{47}{96}$
	$[\cos - \sin]$	\circ	$+\frac{1}{6}$	$-\frac{1}{24}$	$+\frac{7}{480}$	\circ	$+\frac{11}{128}$	$-\frac{11}{256}$	\circ	$+\frac{7}{120}$	$-\frac{7}{160}$
	$[\cos]$	\circ	$+\frac{1}{3}$	$-\frac{3}{16}$	$+\frac{1}{30}$	H	$+\frac{75}{256}$	$-\frac{441}{2048}$	H	$+\frac{3}{10}$	$-\frac{4}{15}$
	$[\sin]$	\circ	$+\frac{1}{6}$	$-\frac{7}{48}$	$+\frac{3}{160}$	H	$+\frac{53}{256}$	$-\frac{353}{2048}$	H	$+\frac{29}{120}$	$-\frac{107}{480}$
$\left(\frac{r}{a}\right)^3$	$[\cos + \sin]$	$-\frac{1}{2}$	$+\frac{1}{1}$	$-\frac{35}{96}$	$+\frac{23}{576}$	$-\frac{3}{8}$	$+\frac{11}{16}$	$-\frac{351}{1024}$	$-\frac{7}{24}$	$+\frac{55}{96}$	$-\frac{701}{1920}$
	$[\cos - \sin]$	\circ	$+\frac{1}{6}$	$-\frac{1}{96}$	$+\frac{7}{960}$	\circ	$+\frac{7}{128}$	$-\frac{9}{640}$	\circ	$+\frac{13}{480}$	$-\frac{73}{5760}$
	$[\cos]$	H	$+\frac{7}{12}$	$-\frac{3}{16}$	$+\frac{17}{720}$	H	$+\frac{95}{256}$	$-\frac{1827}{10240}$	H	$+\frac{3}{10}$	$-\frac{17}{90}$
	$[\sin]$	H	$+\frac{5}{12}$	$-\frac{17}{96}$	$+\frac{47}{2880}$	H	$+\frac{81}{256}$	$-\frac{1683}{10240}$	H	$+\frac{131}{480}$	$-\frac{203}{1152}$
$\left(\frac{r}{a}\right)^4$	$[\cos + \sin]$	-1	$+\frac{3}{4}$	$+\frac{13}{48}$	$-\frac{83}{576}$	$-\frac{3}{8}$	$+\frac{3}{16}$	$+\frac{249}{1024}$	$-\frac{1}{6}$	$+\frac{1}{48}$	$+\frac{119}{480}$
	$[\cos - \sin]$	\circ	$-\frac{1}{12}$	$+\frac{1}{6}$	$-\frac{43}{960}$	\circ	$-\frac{9}{128}$	$+\frac{57}{640}$	\circ	$-\frac{11}{240}$	$+\frac{91}{1440}$
	$[\cos]$	H	$+\frac{1}{3}$	$+\frac{7}{32}$	$-\frac{17}{180}$	H	$+\frac{15}{256}$	$+\frac{1701}{10240}$	H	$-\frac{1}{80}$	$+\frac{7}{45}$
	$[\sin]$	H	$+\frac{5}{12}$	$+\frac{5}{96}$	$-\frac{143}{2880}$	H	$+\frac{33}{256}$	$+\frac{789}{10240}$	H	$+\frac{1}{30}$	$+\frac{133}{1440}$

... Table $\left(\left(\frac{r}{a} \right)^1 \cdots \left(\frac{r}{a} \right)^4 \right) \frac{\cos}{\sin} f.$

	$\cos f$ $\sin f$	[] ⁵		[] ⁶		[] ⁷		[] ⁸	
		e^4	e^6	e^5	e^7	e^6	e^7	e^7	
$\left(\frac{r}{a} \right)^1$	[cos + sin]	$+\frac{125}{384}$	$-\frac{125}{256}$	$+\frac{27}{80}$	$-\frac{189}{320}$	$+\frac{16807}{46080}$	$+\frac{128}{315}$		
	[cos - sin]	o	$+\frac{125}{9216}$	o	$+\frac{27}{2240}$	o	o		
	[cos]	H	$-\frac{4375}{18432}$	H	$-\frac{81}{280}$	H	H	H	
	[sin]	H	$-\frac{4625}{18432}$	H	$-\frac{135}{448}$	H	H	H	
$\left(\frac{r}{a} \right)^2$	[cos + sin]	$-\frac{25}{128}$	$+\frac{475}{768}$	$-\frac{9}{40}$	$+\frac{117}{160}$	$-\frac{2401}{9216}$	$-\frac{32}{105}$		
	[cos - sin]	o	$+\frac{425}{9216}$	o	$+\frac{9}{224}$	o	o		
	[cos]	H	$+\frac{6125}{18432}$	H	$+\frac{27}{70}$	H	H	H	
	[sin]	H	$+\frac{5275}{18432}$	H	$+\frac{387}{1120}$	H	H	H	
$\left(\frac{r}{a} \right)^3$	[cos + sin]	$-\frac{95}{384}$	$+\frac{205}{384}$	$-\frac{9}{40}$	$+\frac{171}{320}$	$-\frac{9947}{46080}$	$-\frac{68}{315}$		
	[cos - sin]	o	$+\frac{155}{9216}$	o	$+\frac{27}{2240}$	o	o		
	[cos]	H	$+\frac{5075}{18432}$	H	$+\frac{153}{560}$	H	H	H	
	[sin]	H	$+\frac{4765}{18432}$	H	$+\frac{117}{448}$	H	H	H	
$\left(\frac{r}{a} \right)^4$	[cos + sin]	$-\frac{31}{384}$	$-\frac{7}{128}$	$-\frac{3}{80}$	$-\frac{33}{320}$	$-\frac{539}{46080}$	$+\frac{2}{315}$		
	[cos - sin]	o	$-\frac{301}{9216}$	o	$-\frac{57}{2240}$	o	o		
	[cos]	H	$-\frac{805}{18432}$	H	$-\frac{9}{140}$	H	H	H	
	[sin]	H	$-\frac{203}{18432}$	H	$-\frac{87}{2240}$	H	H	H	

... Table $\left(\left(\frac{r}{a} \right)^{-1} \dots \left(\frac{r}{a} \right)^{-5} \right) \cos f \dots$

$\frac{\cos f}{\sin f}$	[] ⁰				[] ¹			
	e	e^3	e^5	e^7	e^0	e^2	e^4	e^6
$\left(\frac{r}{a} \right)^{-1}$	[cos + sin]	$-\frac{1}{2}$	$-\frac{1}{8}$	$-\frac{1}{16}$	$-\frac{5}{128}$	$+1$	-1	$-\frac{1}{64}$
	[cos - sin]	"	"	"	"	o	$-\frac{1}{8}$	$-\frac{1}{16}$
	[cos]	$-\frac{1}{2}$	$-\frac{1}{8}$	$-\frac{1}{16}$	$-\frac{5}{128}$	H	$-\frac{9}{16}$	$-\frac{5}{128}$
	[sin]	o	o	o	o	H	$-\frac{7}{16}$	$+\frac{3}{128}$
$\left(\frac{r}{a} \right)^{-2}$	[cos + sin]	o	o	o	o	+1	$-\frac{1}{2}$	$-\frac{1}{64}$
	[cos - sin]	"	"	"	"	o	$+\frac{1}{8}$	$+\frac{1}{24}$
	[cos]	o	o	o	o	H	$-\frac{3}{16}$	$+\frac{5}{384}$
	[sin]	o	o	o	o	H	$-\frac{5}{16}$	$-\frac{11}{384}$
$\left(\frac{r}{a} \right)^{-3}$	[cos + sin]	$+\frac{1}{2}$	$+\frac{3}{4}$	$+\frac{15}{16}$	$+\frac{35}{32}$	+1	$+\frac{1}{2}$	$+\frac{55}{64}$
	[cos - sin]	"	"	"	"	o	$+\frac{5}{8}$	$+\frac{5}{6}$
	[cos]	$+\frac{1}{2}$	$+\frac{3}{4}$	$+\frac{15}{16}$	$+\frac{35}{32}$	H	$+\frac{9}{16}$	$+\frac{325}{384}$
	[sin]	o	o	o	o	H	$-\frac{1}{16}$	$+\frac{5}{384}$
$\left(\frac{r}{a} \right)^{-4}$	[cos + sin]	$+1$	$+\frac{5}{2}$	$+\frac{35}{8}$	$+\frac{105}{16}$	+1	$+\frac{239}{64}$	$+\frac{3323}{576}$
	[cos - sin]	"	"	"	"	o	$+\frac{11}{8}$	$+\frac{49}{16}$
	[cos]	$+1$	$+\frac{5}{2}$	$+\frac{35}{8}$	$+\frac{105}{16}$	H	$+\frac{27}{16}$	$+\frac{435}{128}$
	[sin]	o	o	o	o	H	$+\frac{5}{16}$	$+\frac{43}{128}$
$\left(\frac{r}{a} \right)^{-5}$	[cos + sin]	$+\frac{3}{2}$	$+\frac{45}{8}$	$+\frac{105}{8}$	$+\frac{1575}{64}$	+1	$+\frac{647}{64}$	$+\frac{11465}{576}$
	[cos - sin]	"	"	"	"	o	$+\frac{19}{8}$	$+\frac{371}{48}$
	[cos]	$+\frac{3}{2}$	$+\frac{45}{8}$	$+\frac{105}{8}$	$+\frac{1575}{64}$	H	$+\frac{51}{16}$	$+\frac{3425}{384}$
	[sin]	o	o	o	o	H	$+\frac{13}{16}$	$+\frac{457}{384}$

... Table $\left(\left(\frac{r}{a}\right)^{-1} \dots \left(\frac{r}{a}\right)^{-5}\right) \cos f \dots$

$\frac{\cos}{\sin} f$	$[]^3$				$[]^4$		
	e	e^3	e^5	e^7	e^2	e^4	e^6
$\left(\frac{r}{a}\right)^{-1}$	[cos + sin]	$+\frac{3}{2}$	$-\frac{7}{4}$	$+\frac{31}{96}$	$-$	$\frac{1}{12}$	$+\frac{1069}{1024}$
	[cos - sin]	o	$-\frac{1}{12}$	$-\frac{1}{24}$	$-$	$\frac{7}{240}$	$-\frac{9}{320}$
	[cos]	H	$-\frac{11}{12}$	$+\frac{9}{64}$	$-$	$\frac{9}{160}$	$-\frac{385}{256} + \frac{5201}{10240}$
	[sin]	H	$-\frac{5}{6}$	$+\frac{35}{192}$	$-$	$\frac{13}{480}$	$-\frac{367}{256} + \frac{5489}{10240}$
$\left(\frac{r}{a}\right)^{-2}$	[cos + sin]	$+2$	$-\frac{3}{2}$	$+\frac{5}{24}$	$-$	$\frac{1}{18}$	$+\frac{27}{8} - \frac{27}{8} + \frac{999}{1024}$
	[cos - sin]	o	$+\frac{1}{6}$	$+\frac{1}{24}$	$+$	$\frac{1}{30}$	$o + \frac{27}{128} + \frac{27}{1280}$
	[cos]	H	$-\frac{2}{3}$	$+\frac{1}{8}$	$-$	$\frac{1}{90}$	$H - \frac{405}{256} + \frac{5103}{10240}$
	[sin]	H	$-\frac{5}{6}$	$+\frac{1}{12}$	$-$	$\frac{2}{45}$	$H - \frac{459}{256} + \frac{4887}{10240}$
$\left(\frac{r}{a}\right)^{-3}$	[cos + sin]	$+\frac{5}{2}$	$-\frac{1}{8}$	$+\frac{103}{96}$	$+$	$\frac{1255}{1152}$	$+\frac{39}{8} - 2 + \frac{1803}{1024}$
	[cos - sin]	o	$+\frac{19}{24}$	$+\frac{43}{48}$	$+$	$\frac{2063}{1920}$	$o + \frac{131}{128} + \frac{237}{256}$
	[cos]	H	$+\frac{1}{3}$	$+\frac{63}{64}$	$+$	$\frac{779}{720}$	$H - \frac{125}{256} + \frac{2751}{2048}$
	[sin]	H	$-\frac{11}{24}$	$+\frac{17}{192}$	$+$	$\frac{43}{5760}$	$H - \frac{387}{256} + \frac{855}{2048}$
$\left(\frac{r}{a}\right)^{-4}$	[cos + sin]	$+3$	$+\frac{11}{4}$	$+\frac{245}{48}$	$+$	$\frac{463}{64}$	$+\frac{53}{8} + \frac{39}{16} + \frac{7041}{1024}$
	[cos - sin]	o	$+\frac{23}{12}$	$+\frac{89}{24}$	$+$	$\frac{5663}{960}$	$o + \frac{343}{128} + \frac{2819}{640}$
	[cos]	H	$+\frac{7}{3}$	$+\frac{141}{32}$	$+$	$\frac{197}{30}$	$H + \frac{655}{256} + \frac{57757}{10240}$
	[sin]	H	$+\frac{5}{12}$	$+\frac{67}{96}$	$+$	$\frac{641}{960}$	$H - \frac{31}{256} + \frac{12653}{10240}$
$\left(\frac{r}{a}\right)^{-5}$	[cos + sin]	$+\frac{7}{2}$	$+\frac{15}{2}$	$+\frac{1529}{96}$	$+$	$\frac{16207}{576}$	$+\frac{69}{8} + \frac{183}{16} + \frac{23865}{1024}$
	[cos - sin]	o	$+\frac{11}{3}$	$+\frac{997}{96}$	$+$	$\frac{20279}{960}$	$o + \frac{711}{128} + \frac{8733}{640}$
	[cos]	H	$+\frac{67}{12}$	$+\frac{421}{32}$	$+$	$\frac{8867}{360}$	$H + \frac{2175}{256} + \frac{189189}{10240}$
	[sin]	H	$+\frac{23}{12}$	$+\frac{133}{48}$	$+$	$\frac{10099}{2880}$	$H + \frac{753}{256} + \frac{49461}{10240}$

... Table $\left(\frac{r}{a} \right)^{-1} \dots \left(\frac{r}{a} \right)^{-5} \frac{\cos f}{\sin f} \dots$

	$\cos f$ $\sin f$	$[]^4$			$[]^5$		
		e^3	e^5	e^7	e^4	e^6	
$\left(\frac{r}{a} \right)^{-1}$	$[\cos + \sin]$	$+\frac{71}{24}$	$-\frac{229}{48}$	$+\frac{1169}{480}$	$+\frac{523}{128}$	$-\frac{1451}{192}$	
	$[\cos - \sin]$	o	$-\frac{1}{15}$	$-\frac{1}{60}$	o	$-\frac{625}{9216}$	
	$[\cos]$	H	$-\frac{387}{160}$	$+\frac{387}{320}$	H	$-\frac{70273}{18432}$	
	$[\sin]$	H	$-\frac{1129}{480}$	$+\frac{1177}{960}$	H	$-\frac{69023}{18432}$	
$\left(\frac{r}{a} \right)^{-2}$	$[\cos + \sin]$	$+\frac{16}{3}$	$-\frac{20}{3}$	$+\frac{43}{15}$	$+\frac{3125}{384}$	$-\frac{3125}{256}$	
	$[\cos - \sin]$	o	$+\frac{4}{15}$	$-\frac{1}{45}$	o	$+\frac{3125}{9216}$	
	$[\cos]$	H	$-\frac{16}{5}$	$+\frac{64}{45}$	H	$-\frac{109375}{18432}$	
	$[\sin]$	H	$-\frac{52}{15}$	$+\frac{13}{9}$	H	$-\frac{115625}{18432}$	
$\left(\frac{r}{a} \right)^{-3}$	$[\cos + \sin]$	$+\frac{103}{12}$	$-\frac{593}{96}$	$+\frac{721}{192}$	$+\frac{5485}{384}$	$-\frac{11053}{768}$	
	$[\cos - \sin]$	o	$+\frac{643}{480}$	$+\frac{2569}{2880}$	o	$+\frac{16289}{9216}$	
	$[\cos]$	H	$-\frac{387}{160}$	$+\frac{1673}{720}$	H	$-\frac{116347}{18432}$	
	$[\sin]$	H	$-\frac{451}{120}$	$+\frac{4123}{2880}$	H	$-\frac{148925}{18432}$	
$\left(\frac{r}{a} \right)^{-4}$	$[\cos + \sin]$	$+\frac{77}{6}$	$-\frac{25}{48}$	$+\frac{4751}{480}$	$+\frac{2955}{128}$	$-\frac{3463}{384}$	
	$[\cos - \sin]$	o	$+\frac{899}{240}$	$+\frac{2441}{480}$	o	$+\frac{48203}{9216}$	
	$[\cos]$	H	$+\frac{129}{80}$	$+\frac{899}{120}$	H	$-\frac{34909}{18432}$	
	$[\sin]$	H	$-\frac{32}{15}$	$+\frac{77}{32}$	H	$-\frac{131315}{18432}$	
$\left(\frac{r}{a} \right)^{-5}$	$[\cos + \sin]$	$+\frac{437}{24}$	$+\frac{1361}{96}$	$+\frac{63341}{1920}$	$+\frac{13505}{384}$	$+\frac{1541}{128}$	
	$[\cos - \sin]$	o	$+\frac{3983}{480}$	$+\frac{100849}{5760}$	o	$+\frac{113027}{9216}$	
	$[\cos]$	H	$+\frac{899}{80}$	$+\frac{36359}{1440}$	H	$+\frac{223979}{18432}$	
	$[\sin]$	H	$+\frac{1411}{480}$	$+\frac{44587}{5760}$	H	$-\frac{2075}{18432}$	

... Table $\left(\left(\frac{r}{a}\right)^{-1} \dots \left(\frac{r}{a}\right)^{-5} \right) \frac{\cos}{\sin} f.$

	$\cos f$ $\sin f$	[] ⁶		[] ⁷	[] ⁸
		e^5	e^7	e^6	e^7
$\left(\frac{r}{a}\right)^{-1}$	[cos + sin]	+ $\frac{899}{160}$	- $\frac{1879}{160}$	+ $\frac{355081}{46080}$	+ $\frac{47259}{4480}$
	[cos - sin]	o	- $\frac{81}{1120}$	o	o
	[cos]	H	- $\frac{6617}{1120}$	H	H
	[sin]	H	- $\frac{817}{140}$	H	H
$\left(\frac{r}{a}\right)^{-2}$	[cos + sin]	+ $\frac{243}{20}$	- $\frac{1701}{80}$	+ $\frac{823543}{46080}$	+ $\frac{8192}{315}$
	[cos - sin]	o	+ $\frac{243}{560}$	o	o
	[cos]	H	- $\frac{729}{70}$	H	H
	[sin]	H	- $\frac{1215}{112}$	H	H
$\left(\frac{r}{a}\right)^{-3}$	[cos + sin]	+ $\frac{3669}{160}$	- $\frac{18847}{640}$	+ $\frac{330911}{9216}$	+ $\frac{556403}{10080}$
	[cos - sin]	o	+ $\frac{2101}{896}$	o	o
	[cos]	H	- $\frac{7589}{560}$	H	H
	[sin]	H	- $\frac{71217}{4480}$	H	H
$\left(\frac{r}{a}\right)^{-4}$	[cos + sin]	+ $\frac{3167}{80}$	- $\frac{8999}{320}$	+ $\frac{3024637}{46080}$	+ $\frac{178331}{1680}$
	[cos - sin]	o	+ $\frac{16337}{2240}$	o	o
	[cos]	H	- $\frac{729}{70}$	H	H
	[sin]	H	- $\frac{7933}{448}$	H	H
$\left(\frac{r}{a}\right)^{-5}$	[cos + sin]	+ $\frac{5127}{80}$	- $\frac{753}{320}$	+ $\frac{5163949}{46080}$	+ $\frac{3830591}{20160}$
	[cos - sin]	o	+ $\frac{40263}{2240}$	o	o
	[cos]	H	+ $\frac{2187}{280}$	H	H
	[sin]	H	- $\frac{22767}{2240}$	H	H

Table $\left(\left(\frac{r}{a}\right)^1 \dots \left(\frac{r}{a}\right)^4\right) \frac{\cos}{\sin} 2f \dots$

$\cos 2f$ $\sin 2f$	$[]^0$			$[]^1$				
	e^2	e^4	e^6	e	e^3	e^5	e^7	
$\left(\frac{r}{a}\right)^1$	[cos + sin]	$+\frac{3}{2}$	○	○	$-\frac{5}{2}$	$+\frac{33}{16}$	$-\frac{73}{384}$	$+\frac{881}{18432}$
	[cos - sin]	„	„	„	○	$+\frac{1}{48}$	$-\frac{41}{768}$	$-\frac{1159}{30720}$
	[cos]	$+\frac{3}{2}$	○	○	H	$+\frac{25}{24}$	$-\frac{187}{1536}$	$+\frac{29}{5760}$
	[sin]	○	○	○	H	$+\frac{49}{48}$	$-\frac{35}{512}$	$+\frac{3941}{92160}$
$\left(\frac{r}{a}\right)^2$	[cos + sin]	$+\frac{5}{2}$	○	○	-3	$+\frac{13}{8}$	$+\frac{5}{192}$	$+\frac{227}{3072}$
	[cos - sin]	„	„	„	○	$-\frac{7}{24}$	$-\frac{47}{384}$	$-\frac{1091}{15360}$
	[cos]	$+\frac{5}{2}$	○	○	H	$+\frac{2}{3}$	$-\frac{37}{768}$	$+\frac{11}{7680}$
	[sin]	○	○	○	H	$+\frac{23}{24}$	$+\frac{19}{256}$	$+\frac{371}{5120}$
$\left(\frac{r}{a}\right)^3$	[cos + sin]	$+\frac{15}{4}$	$+\frac{5}{8}$	○	$-\frac{7}{2}$	$+\frac{1}{16}$	$+\frac{289}{384}$	$+\frac{1645}{18432}$
	[cos - sin]	„	„	„	„	$-\frac{47}{48}$	$-\frac{151}{768}$	$-\frac{4043}{30720}$
	[cos]	$+\frac{15}{4}$	$+\frac{5}{8}$	○	H	$-\frac{11}{24}$	$+\frac{427}{1536}$	$-\frac{61}{2880}$
	[sin]	○	○	○	H	$+\frac{25}{48}$	$+\frac{243}{512}$	$+\frac{10177}{92160}$
$\left(\frac{r}{a}\right)^4$	[cos + sin]	$+\frac{21}{4}$	$+\frac{21}{8}$	○	-4	-3	$+\frac{79}{48}$	$+\frac{233}{1152}$
	[cos - sin]	„	„	„	„	$-\frac{13}{6}$	$-\frac{13}{24}$	$-\frac{989}{3840}$
	[cos]	$+\frac{21}{4}$	$+\frac{21}{8}$	○	H	$-\frac{31}{12}$	$+\frac{53}{96}$	$-\frac{637}{23040}$
	[sin]	○	○	○	H	$-\frac{5}{12}$	$+\frac{35}{32}$	$+\frac{5297}{23040}$

... Table $\left(\left(\frac{r}{a}\right)^1 \dots \left(\frac{r}{a}\right)^4\right) \cos 2f \dots$

$\cos \sin 2f$	$[]^2$				$[]^3$				
	e^0	e^2	e^4	e^6	e	e^3	e^5	e^7	
$\left(\frac{r}{a}\right)^1$	[cos + sin]	+ 1	- $\frac{7}{2}$	+ $\frac{47}{16}$	- $\frac{215}{288}$	+ $\frac{3}{2}$	- $\frac{75}{16}$	+ $\frac{573}{128}$	- $\frac{3507}{2048}$
	[cos - sin]	o	o	+ $\frac{1}{48}$	- $\frac{3}{160}$	o	o	+ $\frac{21}{1280}$	- $\frac{123}{10240}$
	[cos]	H	H	+ $\frac{71}{48}$	- $\frac{551}{1440}$	H	H	+ $\frac{5751}{2560}$	- $\frac{8829}{10240}$
	[sin]	H	H	+ $\frac{35}{24}$	- $\frac{131}{360}$	H	H	+ $\frac{5709}{2560}$	- $\frac{4353}{5120}$
$\left(\frac{r}{a}\right)^2$	[cos + sin]	+ 1	- $\frac{5}{2}$	+ $\frac{23}{16}$	- $\frac{65}{288}$	+ 1	- $\frac{19}{8}$	+ $\frac{107}{64}$	- $\frac{479}{1024}$
	[cos - sin]	o	o	- $\frac{1}{16}$	- $\frac{11}{480}$	o	o	- $\frac{17}{640}$	- $\frac{7}{1024}$
	[cos]	H	H	+ $\frac{11}{16}$	- $\frac{179}{1440}$	H	H	+ $\frac{1053}{1280}$	- $\frac{243}{1024}$
	[sin]	H	H	+ $\frac{3}{4}$	- $\frac{73}{720}$	H	H	+ $\frac{1087}{1280}$	- $\frac{59}{256}$
$\left(\frac{r}{a}\right)^3$	[cos + sin]	+ 1	- 1	- $\frac{11}{16}$	+ $\frac{43}{72}$	+ $\frac{1}{2}$	- $\frac{3}{16}$	- $\frac{117}{128}$	+ $\frac{1553}{2048}$
	[cos - sin]	o	o	- $\frac{7}{48}$	+ $\frac{1}{240}$	o	o	- $\frac{69}{1280}$	+ $\frac{137}{10240}$
	[cos]	H	H	- $\frac{5}{12}$	+ $\frac{433}{1440}$	H	H	- $\frac{1239}{2560}$	+ $\frac{3951}{10240}$
	[sin]	H	H	- $\frac{13}{48}$	+ $\frac{427}{1440}$	H	H	- $\frac{1101}{2560}$	+ $\frac{1907}{5120}$
$\left(\frac{r}{a}\right)^4$	[cos + sin]	+ 1	+ 1	- $\frac{43}{16}$	+ $\frac{35}{36}$	+ $\frac{3}{2}$	- $\frac{9}{4}$	+ $\frac{267}{256}$	
	[cos - sin]	o	o	- $\frac{5}{48}$	o	o	- $\frac{3}{160}$	+ $\frac{3}{640}$	
	[cos]	H	H	- $\frac{67}{48}$	+ $\frac{35}{72}$	H	- $\frac{363}{320}$	+ $\frac{1341}{2560}$	
	[sin]	H	H	- $\frac{31}{24}$	+ $\frac{35}{72}$	H	- $\frac{357}{320}$	+ $\frac{1329}{2560}$	

... Table $\left(\left(\frac{r}{a}\right)^1 \dots \left(\frac{r}{a}\right)^4 \right) \cos 2f \dots$

	$\cos 2f$ $\sin 2f$	$\left[\quad \right]^4$			$\left[\quad \right]^5$		
		e^2	e^4	e^6	e^3	e^5	e^7
$\left(\frac{r}{a}\right)^1$	[cos + sin]	+ 2	- $\frac{19}{3}$	+ $\frac{55}{8}$	+ $\frac{125}{48}$	- $\frac{6625}{768}$	+ $\frac{64625}{6144}$
	[cos - sin]	o	o	+ $\frac{1}{72}$	o	o	+ $\frac{1625}{129024}$
	[cos]	H	H	+ $\frac{31}{9}$	H	H	+ $\frac{679375}{129024}$
	[sin]	H	H	+ $\frac{247}{72}$	H	H	+ $\frac{338875}{64512}$
$\left(\frac{r}{a}\right)^2$	[cos + sin]	+ 1	- $\frac{5}{2}$	+ $\frac{101}{48}$	+ $\frac{25}{24}$	- $\frac{1075}{384}$	+ $\frac{8425}{3072}$
	[cos - sin]	o	o	- $\frac{11}{720}$	o	o	- $\frac{75}{7168}$
	[cos]	H	H	+ $\frac{47}{45}$	H	H	+ $\frac{29375}{21504}$
	[sin]	H	H	+ $\frac{763}{720}$	H	H	+ $\frac{925}{672}$
$\left(\frac{r}{a}\right)^3$	[cos + sin]	+ $\frac{1}{4}$	+ $\frac{5}{24}$	- $\frac{227}{192}$	+ $\frac{5}{48}$	+ $\frac{385}{768}$	- $\frac{9515}{6144}$
	[cos - sin]	o	o	- $\frac{83}{2880}$	o	o	- $\frac{2435}{129024}$
	[cos]	H	H	- $\frac{109}{180}$	H	H	- $\frac{101125}{129024}$
	[sin]	H	H	- $\frac{1661}{2880}$	H	H	- $\frac{49345}{64512}$
$\left(\frac{r}{a}\right)^4$	[cos + sin]	- $\frac{1}{4}$	+ $\frac{37}{24}$	- $\frac{139}{64}$	- $\frac{1}{3}$	+ $\frac{151}{96}$	- $\frac{877}{384}$
	[cos - sin]	o	o	- $\frac{17}{2880}$	o	o	- $\frac{41}{16128}$
	[cos]	H	H	- $\frac{49}{45}$	H	H	- $\frac{36875}{32256}$
	[sin]	H	H	- $\frac{3119}{2880}$	H	H	- $\frac{36793}{32256}$

... Table $\left(\left(\frac{r}{a}\right)^1 \dots \left(\frac{r}{a}\right)^4\right) \cos 2f$

	$\cos \sin^2 f$	$[]^6$		$[]^7$		$[]^8$		$[]^9$	
		e^4	e^6	e^5	e^7	e^6		e^7	
$\left(\frac{r}{a}\right)^1$	[\cos + \sin]	$+\frac{27}{8}$	$-\frac{189}{16}$	$+\frac{16807}{3840}$	$-\frac{1495823}{92160}$	$+\frac{256}{45}$		$+\frac{531441}{71680}$	
	[\cos - \sin]	o	o	o	o	o		o	
	[\cos]	H	H	H	H	H		H	
	[\sin]	H	H	H	H	H		H	
$\left(\frac{r}{a}\right)^2$	[\cos + \sin]	$+\frac{9}{8}$	$-\frac{261}{80}$	$+\frac{2401}{1920}$	$-\frac{12005}{3072}$	$+\frac{64}{45}$		$+\frac{59049}{35840}$	
	[\cos - \sin]	o	o	o	o	o		o	
	[\cos]	H	H	H	H	H		H	
	[\sin]	H	H	H	H	H		H	
$\left(\frac{r}{a}\right)^3$	[\cos + \sin]	o	$+\frac{63}{80}$	$-\frac{343}{3840}$	$+\frac{102557}{92160}$	$-\frac{8}{45}$		$-\frac{19683}{71680}$	
	[\cos - \sin]	o	o	o	o	o		o	
	[\cos]	o	H	H	H	H		H	
	[\sin]	o	H	H	H	H		H	
$\left(\frac{r}{a}\right)^4$	[\cos + \sin]	$-\frac{3}{8}$	$+\frac{33}{20}$	$-\frac{49}{120}$	$+\frac{20531}{11520}$	$-\frac{4}{9}$		$-\frac{2187}{4480}$	
	[\cos - \sin]	o	o	o	o	o		o	
	[\cos]	H	H	H	H	H		H	
	[\sin]	H	H	H	H	H		H	

Table $\left(\frac{r}{a}\right)^{-1} \dots \left(\frac{r}{a}\right)^{-5}$ $\cos 2f \dots$

$\cos 2f$ $\sin 2f$	$[]^0$			$[]^1$				
	e^2	e^4	e^6	e	e^3	e^5	e^7	
$\left(\frac{r}{a}\right)^{-1}$	[cos + sin] +	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{5}{64}$	$-\frac{3}{2}$	$+\frac{17}{16}$	$+\frac{37}{384}$	$+\frac{197}{2048}$
	[cos - sin] ,,	,,	,,	o	$+\frac{1}{48}$	$+\frac{29}{768}$	$+\frac{1013}{30720}$	
	[cos] +	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{5}{64}$	H	$+\frac{13}{24}$	$+\frac{103}{1536}$	$+\frac{31}{480}$
	[sin] o	o	o	o	H	$+\frac{25}{48}$	$+\frac{15}{512}$	$+\frac{971}{30720}$
$\left(\frac{r}{a}\right)^{-2}$	[cos + sin] o	o	o	o	- I	$+\frac{3}{8}$	$+\frac{7}{192}$	$+\frac{295}{9216}$
	[cos - sin] ,,	,,	,,	o	$-\frac{1}{24}$	$-\frac{13}{384}$	$-\frac{389}{15360}$	
	[cos] o	o	o	H	$+\frac{1}{6}$	$+\frac{1}{768}$	$+\frac{77}{23040}$	
	[sin] o	o	o	H	$+\frac{5}{24}$	$+\frac{9}{256}$	$+\frac{1321}{46080}$	
$\left(\frac{r}{a}\right)^{-3}$	[cos + sin] o	o	o	o	$-\frac{1}{2}$	$+\frac{1}{16}$	$-\frac{5}{384}$	$-\frac{143}{18432}$
	[cos - sin] ,,	,,	,,	o	$+\frac{1}{48}$	$+\frac{11}{768}$	$+\frac{313}{30720}$	
	[cos] o	o	o	H	$+\frac{1}{24}$	$+\frac{1}{1536}$	$+\frac{7}{5760}$	
	[sin] o	o	o	H	$+\frac{1}{48}$	$-\frac{7}{512}$	$-\frac{827}{92160}$	
$\left(\frac{r}{a}\right)^{-4}$	[cos + sin] +	$\frac{1}{4}$	$\frac{5}{8}$	$\frac{35}{32}$	o	$+\frac{1}{2}$	$+\frac{11}{12}$	$+\frac{1099}{768}$
	[cos - sin] ,,	,,	,,	o	$+\frac{1}{3}$	$+\frac{73}{96}$	$+\frac{2441}{1920}$	
	[cos] +	$\frac{1}{4}$	$\frac{5}{8}$	$\frac{35}{32}$	o	$+\frac{5}{12}$	$+\frac{161}{192}$	$+\frac{3459}{2560}$
	[sin] o	o	o	o	$+\frac{1}{12}$	$+\frac{5}{64}$	$+\frac{613}{7680}$	
$\left(\frac{r}{a}\right)^{-5}$	[cos + sin] +	$\frac{3}{4}$	$\frac{21}{8}$	$\frac{189}{32}$	$+\frac{1}{2}$	$+\frac{33}{16}$	$+\frac{1865}{384}$	$+\frac{169229}{18432}$
	[cos - sin] ,,	,,	,,	o	$+\frac{49}{48}$	$+\frac{2545}{768}$	$+\frac{219221}{30720}$	
	[cos] +	$\frac{3}{4}$	$\frac{21}{8}$	$\frac{189}{32}$	H	$+\frac{37}{24}$	$+\frac{6275}{1536}$	$+\frac{23497}{2880}$
	[sin] o	o	o	H	$+\frac{25}{48}$	$+\frac{395}{512}$	$+\frac{94241}{92160}$	

... Table $\left(\left(\frac{r}{a}\right)^{-1} \dots \left(\frac{r}{a}\right)^{-5}\right) \cos 2f \dots$

	\cos \sin $2f$	$[]^2$				$[]^3$			
		e^0	e^2	e^4	e^6	e	e^3	e^5	e^7
$\left(\frac{r}{a}\right)^{-1}$	$[\cos + \sin]$	+ 1	- 4	+ $\frac{47}{16}$	- $\frac{55}{144}$	+ $\frac{5}{2}$	- $\frac{131}{16}$	+ $\frac{887}{128}$	- $\frac{3999}{2048}$
	$[\cos - \sin]$	o	o	o	+ $\frac{1}{48}$	o	o	- $\frac{9}{1280}$	+ $\frac{153}{10240}$
	$[\cos]$	H	H	H	- $\frac{13}{72}$	H	H	+ $\frac{8861}{2560}$	- $\frac{9921}{10240}$
	$[\sin]$	H	H	H	- $\frac{29}{144}$	H	H	+ $\frac{8879}{2560}$	- $\frac{5037}{5120}$
$\left(\frac{r}{a}\right)^{-2}$	$[\cos + \sin]$	+ 1	- $\frac{7}{2}$	+ $\frac{29}{16}$	- $\frac{53}{288}$	+ 3	- $\frac{69}{8}$	+ $\frac{369}{64}$	- $\frac{1401}{1024}$
	$[\cos - \sin]$	o	o	- $\frac{1}{24}$	- $\frac{3}{80}$	o	o	- $\frac{27}{640}$	- $\frac{189}{5120}$
	$[\cos]$	H	H	+ $\frac{85}{96}$	- $\frac{319}{2880}$	H	H	+ $\frac{3663}{1280}$	- $\frac{3597}{5120}$
	$[\sin]$	H	H	+ $\frac{89}{96}$	- $\frac{211}{2880}$	H	H	+ $\frac{3717}{1280}$	- $\frac{213}{320}$
$\left(\frac{r}{a}\right)^{-3}$	$[\cos + \sin]$	+ 1	- $\frac{5}{2}$	+ $\frac{13}{16}$	- $\frac{35}{288}$	+ $\frac{7}{2}$	- $\frac{123}{16}$	+ $\frac{489}{128}$	- $\frac{1763}{2048}$
	$[\cos - \sin]$	o	o	+ $\frac{1}{24}$	+ $\frac{7}{240}$	o	o	+ $\frac{81}{1280}$	+ $\frac{81}{2048}$
	$[\cos]$	H	H	+ $\frac{41}{96}$	- $\frac{133}{2880}$	H	H	+ $\frac{4971}{2560}$	- $\frac{841}{2048}$
	$[\sin]$	H	H	+ $\frac{37}{96}$	- $\frac{217}{2880}$	H	H	+ $\frac{4809}{2560}$	- $\frac{461}{1024}$
$\left(\frac{r}{a}\right)^{-4}$	$[\cos + \sin]$	+ 1	- 1	+ $\frac{17}{16}$	+ $\frac{85}{72}$	+ 4	- 5	+ $\frac{51}{16}$	+ $\frac{143}{128}$
	$[\cos - \sin]$	o	o	+ $\frac{7}{16}$	+ $\frac{109}{120}$	o	o	+ $\frac{23}{40}$	+ $\frac{1369}{1280}$
	$[\cos]$	H	H	+ $\frac{3}{4}$	+ $\frac{47}{45}$	H	H	+ $\frac{301}{160}$	+ $\frac{2799}{2560}$
	$[\sin]$	H	H	+ $\frac{5}{16}$	+ $\frac{49}{360}$	H	H	+ $\frac{209}{160}$	+ $\frac{61}{2560}$
$\left(\frac{r}{a}\right)^{-5}$	$[\cos + \sin]$	+ 1	+ 1	+ $\frac{65}{16}$	+ $\frac{139}{18}$	+ $\frac{9}{2}$	- $\frac{3}{16}$	+ $\frac{963}{128}$	+ $\frac{22737}{2048}$
	$[\cos - \sin]$	o	o	+ $\frac{67}{48}$	+ $\frac{333}{80}$	o	o	+ $\frac{2451}{1280}$	+ $\frac{53193}{10240}$
	$[\cos]$	H	H	+ $\frac{131}{48}$	+ $\frac{8557}{1440}$	H	H	+ $\frac{12081}{2560}$	+ $\frac{83439}{10240}$
	$[\sin]$	H	H	+ $\frac{4}{3}$	+ $\frac{2563}{1440}$	H	H	+ $\frac{7179}{2560}$	+ $\frac{15123}{5120}$

... Table $\left(\left(\frac{r}{a} \right)^{-1} \dots \left(\frac{r}{a} \right)^{-5} \right) \cos 2f \dots$

	\cos $\sin 2f$	[] ⁴			[] ⁵			
		e^2	e^4	e^6	e^3	e^5	e^7	
$\left(\frac{r}{a} \right)^{-1}$	[cos + sin]	$+\frac{19}{4}$	$- \frac{121}{8}$	$+\frac{707}{48}$	$+\frac{389}{48}$	$-\frac{20267}{768}$	$+\frac{179141}{6144}$	
	[cos - sin]	o	o	$-\frac{1}{90}$	o	o	$-\frac{625}{43008}$	
	[cos]	H	H	$+\frac{10597}{1440}$	H	H	$+\frac{626681}{43008}$	
	[sin]	H	H	$+\frac{10613}{1440}$	H	H	$+\frac{104551}{7168}$	
$\left(\frac{r}{a} \right)^{-2}$	[cos + sin]	$+\frac{13}{2}$	$-\frac{55}{3}$	$+\frac{239}{16}$	$+\frac{295}{24}$	$-\frac{13745}{384}$	$+\frac{105175}{3072}$	
	[cos - sin]	o	o	$-\frac{2}{45}$	o	o	$-\frac{3125}{64512}$	
	[cos]	H	H	$+\frac{10723}{1440}$	H	H	$+\frac{1102775}{64512}$	
	[sin]	H	H	$+\frac{10787}{1440}$	H	H	$+\frac{276475}{16128}$	
$\left(\frac{r}{a} \right)^{-3}$	[cos + sin]	$+\frac{17}{2}$	$-\frac{115}{6}$	$+\frac{601}{48}$	$+\frac{845}{48}$	$-\frac{32525}{768}$	$+\frac{208225}{6144}$	
	[cos - sin]	o	o	$+\frac{4}{45}$	o	o	$+\frac{15625}{129024}$	
	[cos]	H	H	$+\frac{9079}{1440}$	H	H	$+\frac{2194175}{129024}$	
	[sin]	H	H	$+\frac{8951}{1440}$	H	H	$+\frac{1089275}{64512}$	
$\left(\frac{r}{a} \right)^{-4}$	[cos + sin]	$+\frac{43}{4}$	$-\frac{129}{8}$	$+\frac{1985}{192}$	$+\frac{145}{6}$	$-\frac{1015}{24}$	$+\frac{23581}{768}$	
	[cos - sin]	o	o	$+\frac{437}{576}$	o	o	$+\frac{2701}{2688}$	
	[cos]	H	H	$+\frac{799}{144}$	H	H	$+\frac{56823}{3584}$	
	[sin]	H	H	$+\frac{2759}{576}$	H	H	$+\frac{159665}{10752}$	
$\left(\frac{r}{a} \right)^{-5}$	[cos + sin]	$+\frac{53}{4}$	$-\frac{179}{24}$	$+\frac{977}{64}$	$+\frac{1541}{48}$	$-\frac{23479}{768}$	$+\frac{216917}{6144}$	
	[cos - sin]	o	o	$+\frac{7579}{2880}$	o	o	$+\frac{467069}{129024}$	
	[cos]	H	H	$+\frac{6443}{720}$	H	H	$+\frac{2511163}{129024}$	
	[sin]	H	H	$+\frac{18193}{2880}$	H	H	$+\frac{1022047}{64512}$	

... Table $\left(\frac{r}{a}\right)^{-1} \dots \left(\frac{r}{a}\right)^{-5}$ $\frac{\cos}{\sin} 2f \dots$

	$\frac{\cos}{\sin} 2f$	[] ⁶		[] ⁷		[] ⁸		[] ⁹	
		e^4	e^6	e^5	e^7	e^6	e^7	e^7	e^7
$\left(\frac{r}{a}\right)^{-1}$	[cos + sin]	+ $\frac{209}{16}$	- $\frac{887}{20}$	+ $\frac{78077}{3840}$	- $\frac{2228929}{30720}$	+ $\frac{17807}{576}$		+ $\frac{3313213}{71680}$	
	[cos - sin]	o	o	o	o	o		o	
	[cos]	H	H	H	H	H		H	
	[sin]	H	H	H	H	H		H	
$\left(\frac{r}{a}\right)^{-2}$	[cos + sin]	+ $\frac{345}{16}$	- $\frac{10569}{160}$	+ $\frac{69251}{1920}$	- $\frac{5394109}{46080}$	+ $\frac{42037}{720}$		+ $\frac{3306951}{35840}$	
	[cos - sin]	o	o	o	o	o		o	
	[cos]	H	H	H	H	H		H	
	[sin]	H	H	H	H	H		H	
$\left(\frac{r}{a}\right)^{-3}$	[cos + sin]	+ $\frac{533}{16}$	- $\frac{13827}{160}$	+ $\frac{228347}{3840}$	- $\frac{3071075}{18432}$	+ $\frac{73369}{720}$		+ $\frac{12144273}{71680}$	
	[cos - sin]	o	o	o	o	o		o	
	[cos]	H	H	H	H	H		H	
	[sin]	H	H	H	H	H		H	
$\left(\frac{r}{a}\right)^{-4}$	[cos + sin]	+ 49	- 98	+ $\frac{44569}{480}$	- $\frac{133707}{640}$	+ $\frac{241517}{1440}$		+ $\frac{2619177}{8960}$	
	[cos - sin]	o	o	o	o	o		o	
	[cos]	H	H	H	H	H		H	
	[sin]	H	H	H	H	H		H	
$\left(\frac{r}{a}\right)^{-5}$	[cos + sin]	+ $\frac{555}{8}$	- $\frac{7149}{80}$	+ $\frac{533617}{3840}$	- $\frac{20449667}{92160}$	+ $\frac{379691}{1440}$		+ $\frac{34431741}{71680}$	
	[cos - sin]	o	o	o	o	o		o	
	[cos]	H	H	H	H	H		H	
	[sin]	H	H	H	H	H		H	

Table $\left(\frac{r}{a}\right) \dots \left(\frac{r}{a}\right)^4 \cos \sin 3f \dots$

\cos $\sin 3f$	$[]^0$			$[]^1$		
	e^3	e^5	e^7	e^2	e^4	e^6
$\left(\frac{r}{a}\right)^1$	[cos + sin] [cos - sin] [cos] [sin]	$-\frac{5}{4}$,, $-\frac{5}{4}$ o	$-\frac{5}{3^2}$,, $-\frac{5}{3^2}$ o	$-\frac{3}{64}$,, $-\frac{3}{64}$ o	$+\frac{31}{8}$ o H H	$-\frac{77}{24}$ $-\frac{19}{384}$ $-\frac{417}{256}$ $-\frac{1213}{768}$
						$+\frac{155}{1024}$ $+\frac{3}{256}$ $+\frac{167}{2048}$ $+\frac{143}{2048}$
$\left(\frac{r}{a}\right)^2$	[cos + sin] [cos - sin] [cos] [sin]	$-\frac{5}{2}$,, $-\frac{5}{2}$ o	o ,, o o	o ,, o o	$+\frac{43}{8}$ o H H	$-\frac{25}{6}$ $+\frac{37}{384}$ $-\frac{521}{256}$ $-\frac{1637}{768}$
						$+\frac{1069}{3072}$ $+\frac{487}{3840}$ $+\frac{2431}{10240}$ $+\frac{3397}{30720}$
$\left(\frac{r}{a}\right)^3$	[cos + sin] [cos - sin] [cos] [sin]	$-\frac{35}{8}$,, $-\frac{35}{8}$ o	o ,, o o	o ,, o o	$+\frac{57}{8}$ o H H	$-\frac{65}{16}$ $+\frac{75}{128}$ $-\frac{445}{256}$ $-\frac{595}{256}$
						$-\frac{19}{1024}$ $+\frac{179}{640}$ $+\frac{1337}{10240}$ $-\frac{1527}{10240}$
$\left(\frac{r}{a}\right)^4$	[cos + sin] [cos - sin] [cos] [sin]	- 7 ,, - 7 o	$-\frac{7}{8}$,, $-\frac{7}{8}$ o	o	$+\frac{73}{8}$ o H H	$-\frac{91}{48}$ $+\frac{641}{384}$ $-\frac{29}{256}$ $-\frac{1369}{768}$
						$-\frac{1419}{1024}$ $+\frac{341}{640}$ $-\frac{4367}{10240}$ $-\frac{9823}{10240}$

...Table $\left(\left(\frac{r}{a}\right) \dots \left(\frac{r}{a}\right)^4\right)$ $\cos \sin 3f\dots$

	$\cos \sin 3f$	$[]^2$				$[]^3$			
		e	e^3	e^5	e^7	e^0	e^2	e^4	e^6
$\left(\frac{r}{a}\right)^1$	[cos + sin]	$-\frac{7}{2}$	$+\frac{75}{8}$	$-\frac{659}{96}$	$+\frac{1687}{1152}$	$+1$	$-\frac{17}{2}$	$+\frac{1143}{64}$	$+\frac{1805}{128}$
	[cos - sin]	o	o	$-\frac{11}{480}$	$-\frac{13}{5760}$	o	o	o	$-\frac{69}{5120}$
	[cos]	H	H	$-\frac{551}{160}$	$+\frac{4211}{5760}$	H	H	H	$-\frac{72269}{10240}$
	[sin]	H	H	$-\frac{821}{240}$	$+\frac{11}{15}$	H	H	H	$-\frac{72131}{10240}$
$\left(\frac{r}{a}\right)^2$	[cos + sin]	-4	$+\frac{19}{2}$	$-\frac{79}{12}$	$+\frac{113}{72}$	$+1$	$-\frac{15}{2}$	$+\frac{927}{64}$	$-\frac{1415}{128}$
	[cos - sin]	o	o	$+\frac{1}{120}$	$+\frac{1}{36}$	o	o	o	$-\frac{1}{5120}$
	[cos]	H	H	$-\frac{263}{80}$	$+\frac{115}{144}$	H	H	H	$-\frac{56601}{10240}$
	[sin]	H	H	$-\frac{791}{240}$	$+\frac{37}{48}$	H	H	H	$-\frac{56599}{10240}$
$\left(\frac{r}{a}\right)^3$	[cos + sin]	$-\frac{9}{2}$	$+\frac{33}{4}$	$-\frac{133}{32}$	$+\frac{11}{16}$	$+1$	-6	$+\frac{591}{64}$	$-\frac{349}{64}$
	[cos - sin]	o	o	$+\frac{13}{160}$	$+\frac{1}{30}$	o	o	o	$+\frac{137}{5120}$
	[cos]	H	H	$-\frac{163}{80}$	$+\frac{173}{480}$	H	H	H	$-\frac{27783}{10240}$
	[sin]	H	H	$-\frac{339}{160}$	$+\frac{157}{480}$	H	H	H	$-\frac{28057}{10240}$
$\left(\frac{r}{a}\right)^4$	[cos + sin]	-5	$+\frac{21}{4}$	$+\frac{17}{48}$	$-\frac{71}{72}$	$+1$	-4	$+\frac{183}{64}$	$+\frac{65}{64}$
	[cos - sin]	o	o	$+\frac{2}{15}$	$+\frac{1}{720}$	o	o	o	$+\frac{37}{1024}$
	[cos]	H	H	$+\frac{30}{160}$	$-\frac{709}{1440}$	H	H	H	$+\frac{1077}{2048}$
	[sin]	H	H	$+\frac{53}{480}$	$-\frac{79}{160}$	H	H	H	$+\frac{1003}{2048}$

... Table $\left(\frac{r}{a}\right) \dots \left(\frac{r}{a}\right)^4 \cos \sin 3f \dots$

	$\cos \sin 3f$	$[]^4$				$[]^5$			
		e	e^3	e^5	e^7	e^2	e^4	e^6	
$\left(\frac{r}{a}\right)^1$	[cos + sin]	$+\frac{5}{2}$	$-\frac{63}{4}$	$+\frac{757}{24}$	$-\frac{1979}{72}$	$+\frac{37}{8}$	$-\frac{635}{24}$	$+\frac{54765}{1024}$	
	[cos - sin]	o	o	o	$-\frac{47}{5040}$	o	o	o	
	[cos]	H	H	H	$-\frac{138577}{10080}$	H	H	H	
	[sin]	H	H	H	$-\frac{15387}{1120}$	H	H	H	
$\left(\frac{r}{a}\right)^2$	[cos + sin]	$+2$	$-\frac{23}{2}$	$+\frac{65}{3}$	$-\frac{5317}{288}$	$+\frac{25}{8}$	$-\frac{50}{3}$	$+\frac{98875}{3072}$	
	[cos - sin]	o	o	o	$-\frac{17}{10080}$	o	o	o	
	[cos]	H	H	H	$-\frac{2908}{315}$	H	H	H	
	[sin]	H	H	H	$-\frac{31013}{3360}$	H	H	H	
$\left(\frac{r}{a}\right)^3$	[cos + sin]	$+\frac{3}{2}$	$-\frac{57}{8}$	$+\frac{11}{1}$	$-\frac{2905}{384}$	$+\frac{15}{8}$	$-\frac{135}{16}$	$+\frac{13975}{1024}$	
	[cos - sin]	o	o	o	$+\frac{57}{4480}$	o	o	o	
	[cos]	H	H	H	$-\frac{793}{210}$	H	H	H	
	[sin]	H	H	H	$-\frac{50923}{13440}$	H	H	H	
$\left(\frac{r}{a}\right)^4$	[cos + sin]	+ 1	- 3	$+\frac{37}{24}$	$+\frac{251}{144}$	$+\frac{7}{8}$	$-\frac{109}{48}$	$+\frac{535}{1024}$	
	[cos - sin]	o	o	o	$+\frac{79}{5040}$	o	o	o	
	[cos]	H	H	H	$+\frac{277}{315}$	H	H	H	
	[sin]	H	H	H	$+\frac{1451}{1680}$	H	H	H	

... Table $\left(\left(\frac{r}{a}\right) \dots \left(\frac{r}{a}\right)^4\right) \frac{\cos}{\sin} 3f \dots$

		$[]^6$			$[]^7$	
		e^3	e^5	e^7	e^4	e^6
$\left(\frac{r}{a}\right)^1$	[cos + sin]	$+\frac{61}{8}$	$-\frac{677}{16}$	$+\frac{56487}{640}$	$+\frac{4553}{384}$	$-\frac{84109}{1280}$
	[cos - sin]	o	o	o	o	o
	[cos]	H	H	H	H	H
	[sin]	H	H	H	H	H
$\left(\frac{r}{a}\right)^2$	[cos + sin]	$+\frac{9}{2}$	$-\frac{189}{8}$	$+\frac{3807}{80}$	$+\frac{2401}{384}$	$-\frac{127253}{3840}$
	[cos - sin]	o	o	o	o	o
	[cos]	H	H	H	H	H
	[sin]	H	H	H	H	H
$\left(\frac{r}{a}\right)^3$	[cos + sin]	$+\frac{9}{4}$	$-\frac{81}{8}$	$+\frac{2781}{160}$	$+\frac{343}{128}$	$-\frac{7889}{640}$
	[cos - sin]	o	o	o	o	o
	[cos]	H	H	H	H	H
	[sin]	H	H	H	H	H
$\left(\frac{r}{a}\right)^4$	[cos + sin]	$+\frac{3}{4}$	$-\frac{27}{16}$	$-\frac{81}{160}$	$+\frac{245}{384}$	$-\frac{147}{128}$
	[cos - sin]	o	o	o	o	o
	[cos]	H	H	H	H	H
	[sin]	H	H	H	H	H

... Table $\left(\frac{r}{a}\right) \dots \left(\frac{r}{a}\right)^4 \cos \sin 3f$.

$\cos \sin 3f$	$[]^8$		$[]^9$		$[]^{10}$	
	e^5	e^7	e^6		e^7	
$\left(\frac{r}{a}\right)^1$	$[\cos + \sin]$	$+\frac{8551}{480}$	$-\frac{288221}{2880}$	$+\frac{26809}{1024}$	$+\frac{305593}{8064}$	
	$[\cos - \sin]$	o	o	o	o	
	$[\cos]$	H	H	H	H	
	$[\sin]$	H	H	H	H	
$\left(\frac{r}{a}\right)^2$	$[\cos + \sin]$	$+\frac{128}{15}$	$-\frac{416}{9}$	$+\frac{59049}{5120}$	$+\frac{15625}{1008}$	
	$[\cos - \sin]$	o	o	o	o	
	$[\cos]$	H	H	H	H	
	$[\sin]$	H	H	H	H	
$\left(\frac{r}{a}\right)^3$	$[\cos + \sin]$	$+\frac{16}{5}$	$-\frac{76}{5}$	$+\frac{19683}{5120}$	$+\frac{3125}{672}$	
	$[\cos - \sin]$	o	o	o	o	
	$[\cos]$	H	H	H	H	
	$[\sin]$	H	H	H	H	
$\left(\frac{r}{a}\right)^4$	$[\cos + \sin]$	$+\frac{8}{15}$	$-\frac{26}{45}$	$+\frac{2187}{5120}$	$+\frac{625}{2016}$	
	$[\cos - \sin]$	o	o	o	o	
	$[\cos]$	H	H	H	H	
	$[\sin]$	H	H	H	H	

... Table $\left(\left(\frac{r}{a}\right)^{-1} \dots \left(\frac{r}{a}\right)^{-5} \right) \frac{\cos}{\sin} 3f \dots$

$\frac{\cos}{\sin} 3f$	[] ⁰			[] ¹		
	e^3	e^5	e^7	e^2	e^4	e^6
$\left(\frac{r}{a}\right)^{-1}$	[cos + sin]	$-\frac{1}{8}$	$-\frac{3}{3^2}$	$-\frac{9}{128}$	$+\frac{13}{8}$	$-\frac{41}{48}$
	[cos - sin]	"	"	"	o	$+\frac{1}{384}$
	[cos]	$-\frac{1}{8}$	$-\frac{3}{3^2}$	$-\frac{9}{128}$	H	$-\frac{109}{256}$
	[sin]	o	o	o	H	$-\frac{329}{768}$
$\left(\frac{r}{a}\right)^{-2}$	[cos + sin]	o	o	o	$+\frac{7}{8}$	$-\frac{5}{24}$
	[cos - sin]	"	"	"	o	$+\frac{5}{384}$
	[cos]	o	o	o	H	$-\frac{25}{256}$
	[sin]	o	o	o	H	$-\frac{85}{768}$
$\left(\frac{r}{a}\right)^{-3}$	[cos + sin]	o	o	o	$+\frac{3}{8}$	o
	[cos - sin]	"	"	"	o	$-\frac{1}{128}$
	[cos]	o	o	o	H	$-\frac{1}{256}$
	[sin]	o	o	o	H	$+\frac{1}{256}$
$\left(\frac{r}{a}\right)^{-4}$	[cos + sin]	o	o	o	$+\frac{1}{8}$	$+\frac{1}{48}$
	[cos - sin]	"	"	"	o	$+\frac{1}{384}$
	[cos]	o	o	o	H	$+\frac{3}{256}$
	[sin]	o	o	o	H	$+\frac{7}{768}$
$\left(\frac{r}{a}\right)^{-5}$	[cos + sin]	$+\frac{1}{8}$	$+\frac{7}{16}$	$+\frac{63}{64}$	$+\frac{1}{8}$	$+\frac{17}{48}$
	[cos - sin]	"	"	"	o	$+\frac{65}{384}$
	[cos]	$+\frac{1}{8}$	$+\frac{7}{16}$	$+\frac{63}{64}$	H	$+\frac{67}{256}$
	[sin]	o	o	o	H	$+\frac{71}{768}$

... Table $\left(\left(\frac{r}{a} \right)^{-1} \dots \left(\frac{r}{a} \right)^{-5} \right) \cos \sin 3f \dots$

	$\cos \sin 3f$	$[]^2$				$[]^3$			
		e	e^3	e^5	e^7	e^0	e^2	e^4	e^6
$\left(\frac{r}{a} \right)^{-1}$	$[\cos + \sin]$	$-\frac{5}{2}$	$+\frac{13}{2}$	$-\frac{341}{96}$	$+\frac{187}{576}$	$+1$	-9	$+\frac{1143}{64}$	$-\frac{365}{32}$
	$[\cos - \sin]$	0	0	$+\frac{1}{120}$	$-\frac{1}{2880}$	0	0	0	$+\frac{9}{1024}$
	$[\cos]$	H	H	$-\frac{567}{320}$	$+\frac{467}{2880}$	H	H	H	$-\frac{11671}{2048}$
	$[\sin]$	H	H	$-\frac{1709}{960}$	$+\frac{13}{80}$	H	H	H	$-\frac{11689}{2048}$
$\left(\frac{r}{a} \right)^{-2}$	$[\cos + \sin]$	-2	$+\frac{9}{2}$	$-\frac{41}{24}$	$+\frac{43}{288}$	$+1$	$-\frac{17}{2}$	$+\frac{951}{64}$	$-\frac{973}{128}$
	$[\cos - \sin]$	0	0	$+\frac{1}{120}$	$+\frac{23}{1440}$	0	0	0	$+\frac{27}{5120}$
	$[\cos]$	H	H	$-\frac{17}{20}$	$+\frac{119}{1440}$	H	H	H	$-\frac{38893}{10240}$
	$[\sin]$	H	H	$-\frac{103}{120}$	$+\frac{1}{15}$	H	H	H	$-\frac{38947}{10240}$
$\left(\frac{r}{a} \right)^{-3}$	$[\cos + \sin]$	$-\frac{3}{2}$	$+\frac{21}{8}$	$-\frac{19}{32}$	$+\frac{23}{192}$	$+1$	$-\frac{15}{2}$	$+\frac{687}{64}$	$-\frac{535}{128}$
	$[\cos - \sin]$	0	0	$-\frac{1}{80}$	$-\frac{1}{64}$	0	0	0	$-\frac{81}{5120}$
	$[\cos]$	H	H	$-\frac{97}{320}$	$+\frac{5}{96}$	H	H	H	$-\frac{21481}{10240}$
	$[\sin]$	H	H	$-\frac{93}{320}$	$+\frac{13}{192}$	H	H	H	$-\frac{21319}{10240}$
$\left(\frac{r}{a} \right)^{-4}$	$[\cos + \sin]$	-1	$+\frac{5}{4}$	$-\frac{7}{48}$	$+\frac{23}{288}$	$+1$	-6	$+\frac{423}{64}$	$-\frac{125}{64}$
	$[\cos - \sin]$	0	0	$+\frac{1}{120}$	$+\frac{13}{1440}$	0	0	0	$+\frac{81}{5120}$
	$[\cos]$	H	H	$-\frac{11}{160}$	$+\frac{2}{45}$	H	H	H	$-\frac{9919}{10240}$
	$[\sin]$	H	H	$-\frac{37}{480}$	$+\frac{17}{480}$	H	H	H	$-\frac{10081}{10240}$
$\left(\frac{r}{a} \right)^{-5}$	$[\cos + \sin]$	$-\frac{1}{2}$	$+\frac{3}{4}$	$+\frac{61}{96}$	$+\frac{191}{144}$	$+1$	-4	$+\frac{255}{64}$	$+\frac{17}{64}$
	$[\cos - \sin]$	0	0	$+\frac{109}{480}$	$+\frac{31}{45}$	0	0	0	$+\frac{1553}{5120}$
	$[\cos]$	H	H	$+\frac{69}{160}$	$+\frac{1451}{1440}$	H	H	H	$+\frac{2913}{10240}$
	$[\sin]$	H	H	$+\frac{49}{240}$	$+\frac{51}{160}$	H	H	H	$-\frac{193}{10240}$

... Table $\left(\left(\frac{r}{a}\right)^{-1} \dots \left(\frac{r}{a}\right)^{-5}\right) \frac{\cos}{\sin} 3f \dots$

\cos $\sin 3f$		$\overbrace{[]^4}$				$\overbrace{[]^5}$		
		e	e^3	e^5	e^7	e^2	e^4	e^6
$\left(\frac{r}{a}\right)^{-1}$	[cos + sin]	+ $\frac{7}{2}$	- $\frac{179}{8}$	+ $\frac{2009}{48}$	- $\frac{8893}{288}$	+ $\frac{67}{8}$	- $\frac{2279}{48}$	+ $\frac{274345}{3072}$
	[cos - sin]	o	o	o	+ $\frac{11}{1260}$	o	o	o
	[cos]	H	H	H	- $\frac{311167}{20160}$	H	H	H
	[sin]	H	H	H	- $\frac{103781}{6720}$	H	H	H
$\left(\frac{r}{a}\right)^{-2}$	[cos + sin]	+ 4	- 24	+ $\frac{241}{6}$	- $\frac{455}{18}$	+ $\frac{85}{8}$	- $\frac{1355}{24}$	+ $\frac{98525}{1024}$
	[cos - sin]	o	o	o	+ $\frac{1}{315}$	o	o	o
	[cos]	H	H	H	- $\frac{15923}{1260}$	H	H	H
	[sin]	H	H	H	- $\frac{5309}{420}$	H	H	H
$\left(\frac{r}{a}\right)^{-3}$	[cos + sin]	+ $\frac{9}{2}$	- 24	+ $\frac{545}{16}$	- $\frac{71}{4}$	+ $\frac{105}{8}$	- $\frac{125}{2}$	+ $\frac{94825}{1024}$
	[cos - sin]	o	o	o	- $\frac{2}{105}$	o	o	o
	[cos]	H	H	H	- $\frac{7463}{840}$	H	H	H
	[sin]	H	H	H	- $\frac{7447}{840}$	H	H	H
$\left(\frac{r}{a}\right)^{-4}$	[cos + sin]	+ 5	- 22	+ $\frac{607}{24}$	- $\frac{98}{9}$	+ $\frac{127}{8}$	- $\frac{3065}{48}$	+ $\frac{243805}{3072}$
	[cos - sin]	o	o	o	+ $\frac{8}{315}$	o	o	o
	[cos]	H	H	H	- $\frac{1711}{315}$	H	H	H
	[sin]	H	H	H	- $\frac{191}{35}$	H	H	H
$\left(\frac{r}{a}\right)^{-5}$	[cos + sin]	+ $\frac{11}{2}$	- $\frac{141}{8}$	+ $\frac{415}{24}$	- $\frac{5021}{1152}$	+ $\frac{151}{8}$	- $\frac{2809}{48}$	+ $\frac{63343}{1024}$
	[cos - sin]	o	o	o	+ $\frac{16319}{40320}$	o	o	o
	[cos]	H	H	H	- $\frac{19927}{10080}$	H	H	H
	[sin]	H	H	H	- $\frac{32009}{13440}$	H	H	H

... Table $\left(\left(\frac{r}{a} \right)^{-1} \cdots \left(\frac{r}{a} \right)^{-5} \right) \cos \sin 3f \dots$

$\cos \sin 3f$	$[]^6$			$[]^7$	
	e^3	e^5	e^7	e^4	e^6
$\left(\frac{r}{a} \right)^{-1}$	$[\cos + \sin]$	$+ 17$	$- \frac{2949}{32}$	$+ \frac{57213}{320}$	$+ \frac{12085}{384}$
	$[\cos - \sin]$	\circ	\circ	\circ	\circ
	$[\cos]$	H	H	H	H
	$[\sin]$	H	H	H	H
$\left(\frac{r}{a} \right)^{-2}$	$[\cos + \sin]$	$+ \frac{47}{2}$	$- \frac{239}{2}$	$- \frac{33951}{160}$	$+ \frac{17969}{384}$
	$[\cos - \sin]$	\circ	\circ	\circ	\circ
	$[\cos]$	H	H	H	H
	$[\sin]$	H	H	H	H
$\left(\frac{r}{a} \right)^{-3}$	$[\cos + \sin]$	$+ \frac{251}{8}$	$- \frac{4611}{32}$	$+ \frac{36249}{160}$	$+ \frac{8547}{128}$
	$[\cos - \sin]$	\circ	\circ	\circ	\circ
	$[\cos]$	H	H	H	H
	$[\sin]$	H	H	H	H
$\left(\frac{r}{a} \right)^{-4}$	$[\cos + \sin]$	$+ \frac{163}{4}$	$- \frac{2577}{16}$	$+ \frac{1089}{5}$	$+ \frac{35413}{384}$
	$[\cos - \sin]$	\circ	\circ	\circ	\circ
	$[\cos]$	H	H	H	H
	$[\sin]$	H	H	H	H
$\left(\frac{r}{a} \right)^{-5}$	$[\cos + \sin]$	$+ \frac{207}{4}$	$- \frac{2625}{16}$	$+ \frac{30483}{160}$	$+ \frac{47621}{384}$
	$[\cos - \sin]$	\circ	\circ	\circ	\circ
	$[\cos]$	H	H	H	H
	$[\sin]$	H	H	H	H

... Table $\left(\left(\frac{r}{a} \right)^{-1} \dots \left(\frac{r}{a} \right)^{-5} \right) \cos \sin 3f \dots$

$\cos \sin 3f$	$[]^8$		$[]^9$	$[]^{10}$
	e^5	e^7	e^6	e^7
$\left(\frac{r}{a} \right)^{-1}$	[cos + sin]	$+ \frac{26371}{480}$	$- \frac{1710983}{5760}$	$+ \frac{471527}{5120}$
	[cos - sin]	o	o	o
	[cos]	H	H	H
	[sin]	H	H	H
$\left(\frac{r}{a} \right)^{-2}$	[cos + sin]	$+ \frac{2611}{30}$	$- \frac{39893}{90}$	$+ \frac{790053}{5120}$
	[cos - sin]	o	o	o
	[cos]	H	H	H
	[sin]	H	H	H
$\left(\frac{r}{a} \right)^{-3}$	[cos + sin]	$+ \frac{10531}{80}$	$- \frac{7363}{12}$	$+ \frac{1258449}{5120}$
	[cos - sin]	o	o	o
	[cos]	H	H	H
	[sin]	H	H	H
$\left(\frac{r}{a} \right)^{-4}$	[cos + sin]	$+ \frac{23029}{120}$	$- \frac{35614}{45}$	$+ \frac{385095}{1024}$
	[cos - sin]	o	o	o
	[cos]	H	H	H
	[sin]	H	H	H
$\left(\frac{r}{a} \right)^{-5}$	[cos + sin]	$+ \frac{65153}{240}$	$- \frac{2725199}{2880}$	$+ \frac{2850411}{5120}$
	[cos - sin]	o	o	o
	[cos]	H	H	H
	[sin]	H	H	H

...Table $\left(\frac{r}{a}\right)^1 \dots \left(\frac{r}{a}\right)^4 \frac{\cos}{\sin} 4f \dots$

$\cos \sin 4f$	$\left[\begin{array}{c} \\ \end{array} \right]^0$		$\left[\begin{array}{c} \\ \end{array} \right]^1$			$\left[\begin{array}{c} \\ \end{array} \right]^2$		
	e^4	e^6	e^3	e^5	e^7	e^2	e^4	e^6
$\left(\frac{r}{a}\right)^1$	[cos + sin]	$+\frac{15}{16}$	$+\frac{9}{32}$	$-\frac{229}{48}$	$+\frac{2735}{768}$	$+\frac{2719}{30720}$	$+\frac{29}{4}$	$-\frac{101}{6}$
	[cos - sin]	„	„	o	$+\frac{151}{3840}$	$+\frac{1423}{92160}$	o	$+\frac{37}{2880}$
	[cos]	$+\frac{15}{16}$	$+\frac{9}{32}$	H	$+\frac{6913}{3840}$	$+\frac{479}{9216}$	H	$+\frac{15341}{2880}$
	[sin]	o	o	H	$+\frac{1127}{640}$	$+\frac{3367}{92160}$	H	$+\frac{1913}{360}$
$\left(\frac{r}{a}\right)^2$	[cos + sin]	$+\frac{35}{16}$	$+\frac{7}{32}$	$-\frac{179}{24}$	$+\frac{2303}{384}$	$-\frac{1031}{3072}$	$+\frac{37}{4}$	$-\frac{83}{4}$
	[cos - sin]	„	„	o	$-\frac{17}{1920}$	$-\frac{1193}{15360}$	o	$+\frac{5}{576}$
	[cos]	$+\frac{35}{16}$	$+\frac{7}{32}$	H	$+\frac{5749}{1920}$	$-\frac{529}{2560}$	H	$+\frac{3961}{576}$
	[sin]	o	o	H	$+\frac{961}{320}$	$-\frac{1981}{15360}$	H	$+\frac{989}{144}$
$\left(\frac{r}{a}\right)^3$	[cos + sin]	$+\frac{35}{8}$	o	$-\frac{529}{48}$	$+\frac{6385}{768}$	$-\frac{20161}{30720}$	$+\frac{23}{2}$	$-\frac{139}{6}$
	[cos - sin]	„	„	o	$-\frac{1159}{3840}$	$-\frac{25297}{92160}$	o	$-\frac{23}{720}$
	[cos]	$+\frac{35}{8}$	o	H	$+\frac{15383}{3840}$	$-\frac{4289}{9216}$	H	$+\frac{10297}{1440}$
	[sin]	o	o	H	$+\frac{2757}{640}$	$-\frac{17593}{92160}$	H	$+\frac{10343}{1440}$
$\left(\frac{r}{a}\right)^4$	[cos + sin]	$+\frac{63}{8}$	o	$-\frac{187}{12}$	$+\frac{1769}{192}$	$-\frac{311}{7680}$	$+\frac{14}{1}$	$-\frac{137}{6}$
	[cos - sin]	„	„	o	$-\frac{1091}{960}$	$-\frac{13571}{23040}$	o	$-\frac{71}{720}$
	[cos]	$+\frac{63}{8}$	o	H	$+\frac{3877}{960}$	$-\frac{1813}{5760}$	H	$+\frac{953}{180}$
	[sin]	o	o	H	$+\frac{207}{40}$	$+\frac{6319}{23040}$	H	$+\frac{3883}{720}$

... Table $\left(\left(\frac{r}{a}\right)^1 \dots \left(\frac{r}{a}\right)^4\right) \cos \sin 4f \dots$

	$\cos \sin 4f$	[] ³				[] ⁴			
		e	e^3	e^5	e^7	e^0	e^2	e^4	e^6
$\left(\frac{r}{a}\right)^1$	[cos + sin]	$-\frac{9}{2}$	$+\frac{399}{16}$	$-\frac{5415}{128}$	$+\frac{58251}{2048}$	$+1$	$-\frac{31}{2}$	$+\frac{239}{4}$	$-\frac{6719}{72}$
	[cos - sin]	o	o	o	$+\frac{411}{71680}$	o	o	o	o
	[cos]	H	H	H	$+\frac{509799}{35840}$	H	H	H	H
	[sin]	H	H	H	$+\frac{1019187}{71680}$	H	H	H	H
$\left(\frac{r}{a}\right)^2$	[cos + sin]	-5	$+\frac{209}{8}$	$-\frac{2783}{64}$	$+\frac{31065}{1024}$	$+1$	$-\frac{29}{2}$	$+\frac{215}{4}$	$-\frac{5993}{72}$
	[cos - sin]	o	o	o	$+\frac{209}{35840}$	o	o	o	o
	[cos]	H	H	H	$+\frac{271871}{17920}$	H	H	H	H
	[sin]	H	H	H	$+\frac{543533}{35840}$	H	H	H	H
$\left(\frac{r}{a}\right)^3$	[cos + sin]	$-\frac{11}{2}$	$+\frac{411}{16}$	$-\frac{5025}{128}$	$+\frac{53459}{2048}$	$+1$	-13	$+\frac{353}{8}$	$-\frac{1151}{18}$
	[cos - sin]	o	o	o	$-\frac{541}{71680}$	o	o	o	o
	[cos]	H	H	H	$+\frac{467631}{35840}$	H	H	H	H
	[sin]	H	H	H	$+\frac{935803}{71680}$	H	H	H	H
$\left(\frac{r}{a}\right)^4$	[cos + sin]	-6	$+\frac{93}{4}$	$-\frac{933}{32}$	$+\frac{7929}{512}$	$+1$	-11	$+\frac{253}{8}$	$-\frac{346}{9}$
	[cos - sin]	o	o	o	$-\frac{459}{17920}$	o	o	o	o
	[cos]	H	H	H	$+\frac{4329}{560}$	H	H	H	H
	[sin]	H	H	H	$+\frac{138987}{17920}$	H	H	H	H

... Table $\left(\left(\frac{r}{a}\right)^1 \dots \left(\frac{r}{a}\right)^4 \right) \cos 4f \dots$

		$[]^5$				$[]^6$		
		e	e^3	e^5	e^7	e^2	e^4	e^6
$\left(\frac{r}{a}\right)^1$	[cos + sin]	$+\frac{7}{2}$	$-\frac{585}{16}$	$+\frac{47435}{384}$	$-\frac{3517045}{18432}$	$+\frac{33}{4}$	$-\frac{147}{2}$	$+\frac{15075}{64}$
	[cos - sin]	o	o	o	o	o	o	o
	[cos]	H	H	H	H	H	H	H
	[sin]	H	H	H	H	H	H	H
$\left(\frac{r}{a}\right)^2$	[cos + sin]	$+3$	$-\frac{239}{8}$	$+\frac{18875}{192}$	$-\frac{155175}{1024}$	$+\frac{25}{4}$	$-\frac{215}{4}$	$+\frac{10831}{64}$
	[cos - sin]	o	o	o	o	o	o	o
	[cos]	H	H	H	H	H	H	H
	[sin]	H	H	H	H	H	H	H
$\left(\frac{r}{a}\right)^3$	[cos + sin]	$+\frac{5}{2}$	$-\frac{365}{16}$	$+\frac{26845}{384}$	$-\frac{1884125}{18432}$	$+\frac{9}{2}$	-36	$+\frac{855}{8}$
	[cos - sin]	o	o	o	o	o	o	o
	[cos]	H	H	H	H	H	H	H
	[sin]	H	H	H	H	H	H	H
$\left(\frac{r}{a}\right)^4$	[cos + sin]	$+2$	$-\frac{63}{4}$	$+\frac{3989}{96}$	$-\frac{239275}{4608}$	$+3$	-21	$+\frac{435}{8}$
	[cos - sin]	o	o	o	o	o	o	o
	[cos]	H	H	H	H	H	H	H
	[sin]	H	H	H	H	H	H	H

... Table $\left(\left(\frac{r}{a}\right)^1 \dots \left(\frac{r}{a}\right)^4\right) \cos \sin 4f \dots$

$\cos \sin 4f$		$[]^7$			$[]^8$	
		e^3	e^5	e^7	e^4	e^6
$\left(\frac{r}{a}\right)^1$	[cos + sin]	$+\frac{787}{48}$	$-\frac{103937}{768}$	$+\frac{13103351}{30720}$	$+\frac{1423}{48}$	$-\frac{37724}{160}$
	[cos - sin]	o	o	o	o	o
	[cos]	H	H	H	H	H
	[sin]	H	H	H	H	H
$\left(\frac{r}{a}\right)^2$	[cos + sin]	$+\frac{269}{24}$	$-\frac{34601}{384}$	$+\frac{4312189}{15360}$	$+\frac{297}{16}$	$-\frac{13897}{96}$
	[cos - sin]	o	o	o	o	o
	[cos]	H	H	H	H	H
	[sin]	H	H	H	H	H
$\left(\frac{r}{a}\right)^3$	[cos + sin]	$+\frac{343}{48}$	$-\frac{41503}{768}$	$+\frac{4923079}{30720}$	$+\frac{32}{3}$	$-\frac{1184}{15}$
	[cos - sin]	o	o	o	o	o
	[cos]	H	H	H	H	H
	[sin]	H	H	H	H	H
$\left(\frac{r}{a}\right)^4$	[cos + sin]	$+\frac{49}{12}$	$-\frac{5243}{192}$	$+\frac{548261}{7680}$	$+\frac{16}{3}$	$-\frac{176}{5}$
	[cos - sin]	o	o	o	o	o
	[cos]	H	H	H	H	H
	[sin]	H	H	H	H	H

... Table $\left(\left(\frac{r}{a}\right)^1 \dots \left(\frac{r}{a}\right)^4\right) \frac{\cos}{\sin} 4f \dots$

$\cos \sin 4f$	$[]^9$		$[]^{10}$	$[]^{11}$	
	e^5	e^7	e^6	e^7	
$\left(\frac{r}{a}\right)^1$	[cos + sin]	$+\frac{64653}{1280}$	$-\frac{4051137}{10240}$	$+\frac{47603}{576}$	$+\frac{84728011}{645120}$
	[cos - sin]	o	o	o	o
	[cos]	H	H	H	H
	[sin]	H	H	H	H
$\left(\frac{r}{a}\right)^2$	[cos + sin]	$+\frac{18749}{640}$	$-\frac{1157699}{5120}$	$+\frac{25799}{576}$	$+\frac{1440343}{21504}$
	[cos - sin]	o	o	o	o
	[cos]	H	H	H	H
	[sin]	H	H	H	H
$\left(\frac{r}{a}\right)^3$	[cos + sin]	$+\frac{19683}{1280}$	$-\frac{1161297}{10240}$	$+\frac{3125}{144}$	$+\frac{19487171}{645120}$
	[cos - sin]	o	o	o	o
	[cos]	H	H	H	H
	[sin]	H	H	H	H
$\left(\frac{r}{a}\right)^4$	[cos + sin]	$+\frac{2187}{320}$	$-\frac{115911}{2560}$	$+\frac{625}{72}$	$+\frac{1771561}{161280}$
	[cos - sin]	o	o	o	o
	[cos]	H	H	H	H
	[sin]	H	H	H	H

Table $\left(\frac{r}{a}\right)^{-1} \dots \left(\frac{r}{a}\right)^{-5}$ $\cos \sin 4f \dots$

$\cos \sin 4f$	$[]^0$		$[]^1$			$[]^2$			
	e^4	e^6	e^3	e^5	e^7	e^2	e^4	e^6	
$\left(\frac{r}{a}\right)^{-1}$	[cos + sin]	$+\frac{1}{16}$	$+\frac{1}{16}$	$-\frac{73}{48}$	$+\frac{421}{768}$	$+\frac{4231}{30720}$	$+\frac{4}{4}$	$-\frac{33}{4}$	$+\frac{331}{96}$
	[cos - sin]	" "	"	\circ	$-\frac{19}{3840}$	$-\frac{1}{10240}$	\circ	\circ	$-\frac{7}{1440}$
	[cos]	$+\frac{1}{16}$	$+\frac{1}{16}$	H	$+\frac{1043}{3840}$	$+\frac{1057}{15360}$	H	H	$+\frac{2479}{1440}$
	[sin]	\circ	\circ	H	$+\frac{177}{640}$	$+\frac{2117}{30720}$	H	H	$+\frac{1243}{720}$
$\left(\frac{r}{a}\right)^{-2}$	[cos + sin]	\circ	\circ	$-\frac{17}{24}$	$+\frac{25}{384}$	$+\frac{47}{15360}$	$+\frac{11}{4}$	$-\frac{29}{6}$	$+\frac{21}{16}$
	[cos - sin]	" "	"	\circ	$-\frac{7}{1920}$	$-\frac{361}{46080}$	\circ	\circ	$-\frac{1}{1440}$
	[cos]	\circ	\circ	H	$+\frac{59}{1920}$	$-\frac{11}{4608}$	H	H	$+\frac{1889}{2886}$
	[sin]	\circ	\circ	H	$+\frac{11}{320}$	$+\frac{251}{46080}$	H	H	$+\frac{1891}{2880}$
$\left(\frac{r}{a}\right)^{-3}$	[cos + sin]	\circ	\circ	$-\frac{13}{48}$	$-\frac{29}{768}$	$-\frac{197}{6144}$	$+\frac{7}{4}$	$-\frac{29}{12}$	$+\frac{1}{3}$
	[cos - sin]	" "	"	\circ	$+\frac{11}{3840}$	$+\frac{407}{92160}$	\circ	\circ	$+\frac{1}{288}$
	[cos]	\circ	\circ	H	$-\frac{67}{3840}$	$-\frac{637}{46080}$	H	H	$+\frac{97}{576}$
	[sin]	\circ	\circ	H	$-\frac{13}{640}$	$-\frac{1681}{92160}$	H	H	$+\frac{95}{576}$
$\left(\frac{r}{a}\right)^{-4}$	[cos + sin]	\circ	\circ	$-\frac{1}{12}$	$-\frac{5}{192}$	$-\frac{149}{7680}$	$+\frac{1}{1}$	$-\frac{1}{1}$	$+\frac{1}{24}$
	[cos - sin]	" "	"	\circ	$-\frac{1}{960}$	$-\frac{11}{7680}$	\circ	\circ	$-\frac{1}{360}$
	[cos]	\circ	\circ	H	$-\frac{13}{960}$	$-\frac{1}{96}$	H	H	$+\frac{7}{360}$
	[sin]	\circ	\circ	H	$-\frac{1}{80}$	$-\frac{23}{2560}$	H	H	$+\frac{1}{45}$
$\left(\frac{r}{a}\right)^{-5}$	[cos + sin]	\circ	\circ	$-\frac{1}{48}$	$-\frac{7}{768}$	$-\frac{209}{30720}$	$+\frac{1}{2}$	$-\frac{1}{3}$	\circ
	[cos - sin]	" "	"	\circ	$+\frac{1}{3840}$	$+\frac{31}{92160}$	\circ	\circ	$+\frac{1}{720}$
	[cos]	\circ	\circ	H	$-\frac{17}{3840}$	$-\frac{149}{46080}$	H	H	$+\frac{1}{1440}$
	[sin]	\circ	\circ	H	$-\frac{3}{640}$	$-\frac{329}{92160}$	H	H	$-\frac{1}{1440}$

... Table $\left(\left(\frac{r}{a}\right)^{-1} \dots \left(\frac{r}{a}\right)^{-5} \right) \cos 4f \dots$

	$\cos \sin 4f$	[] ³				[] ⁴			
		e	e^3	e^5	e^7	e^0	e^2	e^4	e^6
$\left(\frac{r}{a}\right)^{-1}$	[cos + sin]	$-\frac{7}{2}$	$+\frac{307}{16}$	$-\frac{3717}{128}$	$+\frac{30571}{2048}$	$+1$	-16	$+\frac{239}{4}$	$-\frac{3019}{36}$
	[cos - sin]	o	o	o	$-\frac{261}{71680}$	o	o	o	o
	[cos]	H	H	H	$+\frac{267431}{35840}$	H	H	H	H
	[sin]	H	H	H	$+\frac{535123}{71680}$	H	H	H	H
$\left(\frac{r}{a}\right)^{-2}$	[cos + sin]	-3	$+\frac{123}{8}$	$-\frac{1305}{64}$	$+\frac{8547}{1024}$	$+1$	$-\frac{31}{2}$	$+\frac{433}{8}$	$-\frac{9793}{144}$
	[cos - sin]	o	o	o	$+\frac{27}{35840}$	o	o	o	o
	[cos]	H	H	H	$+\frac{74793}{17920}$	H	H	H	H
	[sin]	H	H	H	$+\frac{149559}{35840}$	H	H	H	H
$\left(\frac{r}{a}\right)^{-3}$	[cos + sin]	$-\frac{5}{2}$	$+\frac{183}{16}$	$-\frac{1611}{128}$	$+\frac{8035}{2048}$	$+1$	$-\frac{29}{2}$	$+\frac{365}{8}$	$-\frac{7111}{144}$
	[cos - sin]	o	o	o	$+\frac{243}{71680}$	o	o	o	o
	[cos]	H	H	H	$+\frac{70367}{35840}$	H	H	H	H
	[sin]	H	H	H	$+\frac{140491}{71680}$	H	H	H	H
$\left(\frac{r}{a}\right)^{-4}$	[cos + sin]	-2	$+\frac{31}{4}$	$-\frac{215}{32}$	$+\frac{819}{512}$	$+1$	-13	$+\frac{283}{8}$	$-\frac{1147}{36}$
	[cos - sin]	o	o	o	$-\frac{81}{17920}$	o	o	o	o
	[cos]	H	H	H	$+\frac{7146}{8960}$	H	H	H	H
	[sin]	H	H	H	$+\frac{14373}{17920}$	H	H	H	H
$\left(\frac{r}{a}\right)^{-5}$	[cos + sin]	$-\frac{3}{2}$	$+\frac{75}{16}$	$-\frac{393}{128}$	$+\frac{1251}{2048}$	$+1$	-11	$+\frac{199}{8}$	$-\frac{655}{36}$
	[cos - sin]	o	o	o	$+\frac{243}{71680}$	o	o	o	o
	[cos]	H	H	H	$+\frac{11007}{35840}$	H	H	H	H
	[sin]	H	H	H	$+\frac{21771}{71680}$	H	H	H	H

... Table $\left(\left(\frac{r}{a}\right)^{-1} \dots \left(\frac{r}{a}\right)^{-5}\right) \frac{\cos}{\sin} 4f \dots$

	$\cos \sin 4f$	[] ⁵				[] ⁶		
		e	e^3	e^5	e^7	e^2	e^4	e^6
$\left(\frac{r}{a}\right)^{-1}$	[cos + sin]	$+\frac{9}{2}$	$-\frac{757}{16}$	$+\frac{58801}{384}$	$-\frac{1316215}{6144}$	$+\frac{13}{1}$	$-\frac{459}{4}$	$+\frac{11205}{32}$
	[cos - sin]	o	o	o	o	o	o	o
	[cos]	H	H	H	H	H	H	H
	[sin]	H	H	H	H	H	H	H
$\left(\frac{r}{a}\right)^{-2}$	[cos + sin]	$+5$	$-\frac{405}{8}$	$+\frac{29485}{192}$	$-\frac{1800725}{9216}$	$+\frac{63}{4}$	$-\frac{267}{2}$	$+\frac{12255}{32}$
	[cos - sin]	o	o	o	o	o	o	o
	[cos]	H	H	H	H	H	H	H
	[sin]	H	H	H	H	H	H	H
$\left(\frac{r}{a}\right)^{-3}$	[cos + sin]	$+\frac{11}{2}$	$-\frac{833}{16}$	$+\frac{55135}{384}$	$-\frac{2975165}{18432}$	$+\frac{75}{4}$	$-\frac{595}{4}$	$+\frac{12513}{32}$
	[cos - sin]	o	o	o	o	o	o	o
	[cos]	H	H	H	H	H	H	H
	[sin]	H	H	H	H	H	H	H
$\left(\frac{r}{a}\right)^{-4}$	[cos + sin]	$+6$	$-\frac{205}{4}$	$+\frac{11935}{96}$	$-\frac{184955}{1536}$	$+\frac{22}{1}$	$-\frac{317}{2}$	$+\frac{5945}{16}$
	[cos - sin]	o	o	o	o	o	o	o
	[cos]	H	H	H	H	H	H	H
	[sin]	H	H	H	H	H	H	H
$\left(\frac{r}{a}\right)^{-5}$	[cos + sin]	$+\frac{13}{2}$	$-\frac{765}{16}$	$+\frac{37925}{384}$	$-\frac{1493005}{18432}$	$+\frac{51}{2}$	$-\frac{321}{2}$	$+\frac{2613}{8}$
	[cos - sin]	o	o	o	o	o	o	o
	[cos]	H	H	H	H	H	H	H
	[sin]	H	H	H	H	H	H	H

... Table $\left(\frac{r}{a}\right)^{-1} \dots \left(\frac{r}{a}\right)^{-5}$ $\frac{\cos}{\sin} 4f \dots$

		$[]^7$			$[]^8$	
		e^3	e^5	e^7	e^4	e^6
$\left(\frac{r}{a}\right)^{-1}$	[cos + sin]	$+\frac{1471}{48}$	$-\frac{190507}{768}$	$+\frac{22796399}{30720}$	$+\frac{1027}{16}$	$-\frac{119323}{240}$
	[cos - sin]	o	o	o	o	o
	[cos]	H	H	H	H	H
	[sin]	H	H	H	H	H
$\left(\frac{r}{a}\right)^{-2}$	[cos + sin]	$+\frac{959}{24}$	$-\frac{119119}{384}$	$+\frac{13442527}{15360}$	$+\frac{2141}{24}$	$-\frac{53003}{80}$
	[cos - sin]	o	o	o	o	o
	[cos]	H	H	H	H	H
	[sin]	H	H	H	H	H
$\left(\frac{r}{a}\right)^{-3}$	[cos + sin]	$+\frac{2443}{48}$	$-\frac{284557}{768}$	$+\frac{29629663}{30720}$	$+\frac{2893}{24}$	$-\frac{40387}{48}$
	[cos - sin]	o	o	o	o	o
	[cos]	H	H	H	H	H
	[sin]	H	H	H	H	H
$\left(\frac{r}{a}\right)^{-4}$	[cos + sin]	$+\frac{763}{12}$	$-\frac{81193}{192}$	$+\frac{7623959}{7680}$	$+\frac{1273}{8}$	$-\frac{61219}{60}$
	[cos - sin]	o	o	o	o	o
	[cos]	H	H	H	H	H
	[sin]	H	H	H	H	H
$\left(\frac{r}{a}\right)^{-5}$	[cos + sin]	$+\frac{3751}{48}$	$-\frac{352919}{768}$	$+\frac{29213303}{30720}$	$+\frac{4943}{24}$	$-\frac{23591}{20}$
	[cos - sin]	o	o	o	o	o
	[cos]	H	H	H	H	H
	[sin]	H	H	H	H	H

... Table $\left(\left(\frac{r}{a}\right)^{-1} \dots \left(\frac{r}{a}\right)^{-5}\right) \cos \sin 4f \dots$

$\cos \sin 4f$	$[]^9$		$[]^{10}$		$[]^{11}$	
	e^5	e^7	e^6	e^7	e^7	
$\left(\frac{r}{a}\right)^{-1}$	[cos + sin]	$+\frac{159263}{1280}$	$-\frac{9671209}{10240}$	$+\frac{65773}{288}$	$+\frac{86564393}{215040}$	
	[cos - sin]	o	o	o	o	
	[cos]	H	H	H	H	
	[sin]	H	H	H	H	
$\left(\frac{r}{a}\right)^{-2}$	[cos + sin]	$+\frac{117099}{640}$	$-\frac{6819921}{5120}$	$+\frac{25435}{72}$	$+\frac{210276803}{322560}$	
	[cos - sin]	o	o	o	o	
	[cos]	H	H	H	H	
	[sin]	H	H	H	H	
$\left(\frac{r}{a}\right)^{-3}$	[cos + sin]	$+\frac{333513}{1280}$	$-\frac{18298713}{10240}$	$+\frac{75947}{144}$	$+\frac{131087143}{129024}$	
	[cos - sin]	o	o	o	o	
	[cos]	H	H	H	H	
	[sin]	H	H	H	H	
$\left(\frac{r}{a}\right)^{-4}$	[cos + sin]	$+\frac{115617}{320}$	$-\frac{5856453}{2560}$	$+\frac{110099}{144}$	$+\frac{27485931}{17920}$	
	[cos - sin]	o	o	o	o	
	[cos]	H	H	H	H	
	[sin]	H	H	H	H	
$\left(\frac{r}{a}\right)^{-5}$	[cos + sin]	$+\frac{627003}{1280}$	$-\frac{28615329}{10240}$	$+\frac{155681}{144}$	$+\frac{1454039059}{645120}$	
	[cos - sin]	o	o	o	o	
	[cos]	H	H	H	H	
	[sin]	H	H	H	H	

Table $\left(\left(\frac{r}{a} \right)^{-1} \dots \left(\frac{r}{a} \right)^4 \right) \cos 5f \dots$

$\cos \sin 5f$	$[]^0$		$[]^1$		$[]^2$			
	e^6	e^7	e^4	e^6	e^3	e^5	e^7	
$\left(\frac{r}{a} \right)^1$	[cos + sin]	$-\frac{21}{32}$	$-\frac{21}{64}$	$+\frac{1961}{384}$	$-\frac{4109}{1280}$	$-\frac{277}{24}$	$+\frac{575}{24}$	$-\frac{24781}{1920}$
	[cos - sin]	„	„	o	$-\frac{1091}{46080}$	o	o	$-\frac{209}{40320}$
	[cos]	$-\frac{21}{32}$	$-\frac{21}{64}$	H	$-\frac{29803}{18432}$	H	H	$-\frac{52061}{8064}$
	[sin]	o	o	H	$-\frac{146833}{92160}$	H	H	$-\frac{2032}{315}$
$\left(\frac{r}{a} \right)^2$	[cos + sin]	$-\frac{7}{4}$	$-\frac{7}{16}$	$+\frac{1131}{128}$	$-\frac{25429}{3840}$	$-\frac{97}{6}$	$+\frac{817}{24}$	$-\frac{2461}{120}$
	[cos - sin]	„	„	o	$-\frac{887}{46080}$	o	o	$-\frac{1}{105}$
	[cos]	$-\frac{7}{4}$	$-\frac{7}{16}$	H	$-\frac{61207}{18432}$	H	H	$-\frac{1149}{112}$
	[sin]	o	o	H	$-\frac{304261}{92160}$	H	H	$-\frac{17219}{1680}$
$\left(\frac{r}{a} \right)^3$	[cos + sin]	$-\frac{63}{16}$	$-\frac{21}{64}$	$+\frac{5509}{384}$	$-\frac{21763}{1920}$	$-\frac{263}{12}$	$+\frac{2135}{48}$	$-\frac{2641}{96}$
	[cos - sin]	„	„	o	$+\frac{6067}{46080}$	o	o	$+\frac{13}{2016}$
	[cos]	$-\frac{63}{16}$	$-\frac{21}{64}$	H	$-\frac{103249}{18432}$	H	H	$-\frac{6931}{504}$
	[sin]	o	o	H	$-\frac{528379}{92160}$	H	H	$-\frac{27737}{2016}$
$\left(\frac{r}{a} \right)^4$	[cos + sin]	$-\frac{63}{8}$	o	$+\frac{8501}{384}$	$-\frac{10543}{640}$	$-\frac{347}{12}$	$+\frac{2557}{48}$	$-\frac{3673}{120}$
	[cos - sin]	„	„	o	$+\frac{32251}{46080}$	o	o	$+\frac{281}{5040}$
	[cos]	$-\frac{63}{8}$	o	H	$-\frac{145369}{18432}$	H	H	$-\frac{30797}{2016}$
	[sin]	o	o	H	$-\frac{791347}{92160}$	H	H	$-\frac{154547}{10080}$

... Table $\left(\left(\frac{r}{a}\right)^1 \dots \left(\frac{r}{a}\right)^4\right) \frac{\cos}{\sin} 5f \dots$

$\cos \sin 5f$	$[]^3$			$[]^4$			
	e^2	e^4	e^6	e	e^3	e^5	e^7
$\left(\frac{r}{a}\right)^1$	[cos + sin]	$+\frac{93}{8}$	$-\frac{417}{8}$	$+\frac{77733}{1024}$	$-\frac{11}{2}$	$+\frac{207}{4}$	$-\frac{3715}{24}$
	[cos - sin]	o	o	o	o	o	o
	[cos]	H	H	H	H	H	H
	[sin]	H	H	H	H	H	H
$\left(\frac{r}{a}\right)^2$	[cos + sin]	$+\frac{113}{8}$	-62	$+\frac{94081}{1024}$	-6	$+\frac{109}{2}$	$-\frac{484}{3}$
	[cos - sin]	o	o	o	o	o	o
	[cos]	H	H	H	H	H	H
	[sin]	H	H	H	H	H	H
$\left(\frac{r}{a}\right)^3$	[cos + sin]	$+\frac{135}{8}$	$-\frac{1119}{16}$	$+\frac{103311}{1024}$	$-\frac{13}{2}$	$+\frac{443}{8}$	$-\frac{1885}{12}$
	[cos - sin]	o	o	o	o	o	o
	[cos]	H	H	H	H	H	H
	[sin]	H	H	H	H	H	H
$\left(\frac{r}{a}\right)^4$	[cos + sin]	$+\frac{159}{8}$	$-\frac{1191}{16}$	$+\frac{101103}{1024}$	-7	$+54$	$-\frac{3379}{24}$
	[cos - sin]	o	o	o	o	o	o
	[cos]	H	H	H	H	H	H
	[sin]	H	H	H	H	H	H

... Table $\left(\left(\frac{r}{a}\right)^1 \dots \left(\frac{r}{a}\right)^4\right) \cos 5f \dots$

		$[]^5$				$[]^6$			
		e^0	e^2	e^4	e^6	e	e^3	e^5	e^7
$\left(\frac{r}{a}\right)^1$	[cos + sin]	+ 1	- $\frac{49}{2}$	+ $\frac{9575}{64}$	- $\frac{441125}{1152}$	+ $\frac{9}{2}$	- $\frac{561}{8}$	+ $\frac{11415}{32}$	- $\frac{109167}{128}$
	[cos - sin]	o	o	o	o	o	o	o	o
	[cos]	H	H	H	H	H	H	H	H
	[sin]	H	H	H	H	H	H	H	H
$\left(\frac{r}{a}\right)^2$	[cos + sin]	+ 1	- $\frac{47}{2}$	+ $\frac{8975}{64}$	- $\frac{412175}{1152}$	+ 4	- $\frac{121}{2}$	+ $\frac{1213}{4}$	- $\frac{11619}{16}$
	[cos - sin]	o	o	o	o	o	o	o	o
	[cos]	H	H	H	H	H	H	H	H
	[sin]	H	H	H	H	H	H	H	H
$\left(\frac{r}{a}\right)^3$	[cos + sin]	+ 1	- 22	+ $\frac{7999}{64}$	- $\frac{177925}{576}$	+ $\frac{7}{2}$	- $\frac{201}{4}$	+ $\frac{7755}{32}$	- $\frac{9065}{16}$
	[cos - sin]	o	o	o	o	o	o	o	o
	[cos]	H	H	H	H	H	H	H	H
	[sin]	H	H	H	H	H	H	H	H
$\left(\frac{r}{a}\right)^4$	[cos + sin]	+ 1	- 20	+ $\frac{6695}{64}$	- $\frac{138775}{576}$	+ 3	- $\frac{159}{4}$	+ $\frac{2859}{16}$	- $\frac{12591}{32}$
	[cos - sin]	o	o	o	o	o	o	o	o
	[cos]	H	H	H	H	H	H	H	H
	[sin]	H	H	H	H	H	H	H	H

... Table $\left(\left(\frac{r}{a}\right)^1 \dots \left(\frac{r}{a}\right)^4 \right) \cos 5f \dots$

	$\cos 5f$ $\sin 5f$	[] ⁷			[] ⁸			
		e^2	e^4	e^6	e	e^3	e^5	e^7
$\left(\frac{r}{a}\right)^1$	[cos + sin]	+ $\frac{103}{8}$	- $\frac{3941}{24}$	+ $\frac{780003}{1024}$	+ $\frac{359}{12}$	- $\frac{32885}{96}$	+ $\frac{1453367}{960}$	
	[cos - sin]	o	o	o	o	o	o	o
	[cos]	H	H	H	H	H	H	H
	[sin]	H	H	H	H	H	H	H
$\left(\frac{r}{a}\right)^2$	[cos + sin]	+ $\frac{83}{8}$	- $\frac{259}{2}$	+ $\frac{1829093}{3072}$	+ $\frac{133}{6}$	- $\frac{2999}{12}$	+ $\frac{263873}{240}$	
	[cos - sin]	o	o	o	o	o	o	o
	[cos]	H	H	H	H	H	H	H
	[sin]	H	H	H	H	H	H	H
$\left(\frac{r}{a}\right)^3$	[cos + sin]	+ $\frac{65}{8}$	- $\frac{4667}{48}$	+ $\frac{1331743}{3072}$	+ $\frac{379}{24}$	- $\frac{8255}{48}$	+ $\frac{141719}{192}$	
	[cos - sin]	o	o	o	o	o	o	o
	[cos]	H	H	H	H	H	H	H
	[sin]	H	H	H	H	H	H	H
$\left(\frac{r}{a}\right)^4$	[cos + sin]	+ $\frac{49}{8}$	- $\frac{3283}{48}$	+ $\frac{293853}{1024}$	+ $\frac{32}{3}$	- $\frac{328}{3}$	+ $\frac{6674}{15}$	
	[cos - sin]	o	o	o	o	o	o	o
	[cos]	H	H	H	H	H	H	H
	[sin]	H	H	H	H	H	H	H

... Table $\left(\left(\frac{r}{a}\right)^1 \dots \left(\frac{r}{a}\right)^4\right) \cos \sin 5f$.

	$\cos \sin 5f$	$[]^9$		$[]^{10}$		$[]^{11}$		$[]^{12}$	
		e^4	e^6	e^5	e^7	e^6		e^7	
$\left(\frac{r}{a}\right)^1$	[cos + sin]	+ $\frac{7887}{128}$	- $\frac{848529}{1280}$	+ $\frac{11263}{96}$	- $\frac{1402147}{1152}$	+ $\frac{9742939}{46080}$		+ $\frac{409821}{1120}$	
	[cos - sin]	o	o	o	o	o		o	
	[cos]	H	H	H	H	H		H	
	[sin]	H	H	H	H	H		H	
$\left(\frac{r}{a}\right)^2$	[cos + sin]	+ $\frac{5431}{128}$	- $\frac{578003}{1280}$	+ $\frac{1817}{24}$	- $\frac{37429}{48}$	+ $\frac{5931103}{46080}$		+ $\frac{236641}{1120}$	
	[cos - sin]	o	o	o	o	o		o	
	[cos]	H	H	H	H	H		H	
	[sin]	H	H	H	H	H		H	
$\left(\frac{r}{a}\right)^3$	[cos + sin]	+ $\frac{3563}{128}$	- $\frac{184221}{640}$	+ $\frac{4433}{96}$	- $\frac{66859}{144}$	+ $\frac{3389017}{46080}$		+ $\frac{101971}{896}$	
	[cos - sin]	o	o	o	o	o		o	
	[cos]	H	H	H	H	H		H	
	[sin]	H	H	H	H	H		H	
$\left(\frac{r}{a}\right)^4$	[cos + sin]	+ $\frac{2187}{128}$	- $\frac{107163}{640}$	+ $\frac{625}{24}$	- $\frac{71875}{288}$	+ $\frac{1771561}{46080}$		+ $\frac{1944}{35}$	
	[cos - sin]	o	o	o	o	o		o	
	[cos]	H	H	H	H	H		H	
	[sin]	H	H	H	H	H		H	

Table $\left(\left(\frac{r}{a}\right)^{-1} \dots \left(\frac{r}{a}\right)^{-5}\right) \cos \sin 5f\dots$

\cos $\sin 5f$	$[]^0$		$[]^1$		$[]^2$		
	e^5	e^7	e^4	e^6	e^3	e^5	e^7
$\left(\frac{r}{a}\right)^{-1}$	[cos + sin]	$-\frac{1}{32}$	$-\frac{5}{128}$	$+\frac{167}{128}$	$-\frac{247}{960}$	$-\frac{31}{6}$	$+\frac{859}{96}$
	[cos - sin]	„	„	o	$+\frac{151}{46080}$	o	$+\frac{13}{6720}$
	[cos]	$-\frac{1}{32}$	$-\frac{5}{128}$	H	$-\frac{2341}{18432}$	H	$-\frac{1867}{1344}$
	[sin]	o	o	H	$-\frac{12007}{92160}$	H	$-\frac{779}{560}$
$\left(\frac{r}{a}\right)^{-2}$	[cos + sin]	o	o	$+\frac{209}{384}$	$+\frac{43}{1280}$	$-\frac{19}{6}$	$+\frac{55}{12}$
	[cos - sin]	„	„	o	$+\frac{37}{46080}$	o	$-\frac{1}{2016}$
	[cos]	o	o	H	$+\frac{317}{18432}$	H	$-\frac{809}{2016}$
	[sin]	o	o	H	$+\frac{1511}{92160}$	H	$-\frac{101}{252}$
$\left(\frac{r}{a}\right)^{-3}$	[cos + sin]	o	o	$+\frac{73}{384}$	$+\frac{211}{3840}$	$-\frac{43}{24}$	$+\frac{193}{96}$
	[cos - sin]	„	„	o	$-\frac{47}{46080}$	o	$-\frac{17}{20160}$
	[cos]	o	o	H	$+\frac{497}{18432}$	H	$-\frac{109}{2016}$
	[sin]	o	o	H	$+\frac{2579}{92160}$	H	$-\frac{1073}{20160}$
$\left(\frac{r}{a}\right)^{-4}$	[cos + sin]	o	o	$+\frac{7}{128}$	$+\frac{49}{1920}$	$-\frac{11}{12}$	$+\frac{35}{48}$
	[cos - sin]	„	„	o	$+\frac{19}{46080}$	o	$+\frac{1}{1120}$
	[cos]	o	o	H	$+\frac{239}{18432}$	H	$+\frac{1}{84}$
	[sin]	o	o	H	$+\frac{1157}{92160}$	H	$+\frac{37}{3360}$
$\left(\frac{r}{a}\right)^{-5}$	[cos + sin]	o	o	$+\frac{5}{384}$	$+\frac{1}{128}$	$-\frac{5}{12}$	$+\frac{5}{24}$
	[cos - sin]	„	„	o	$-\frac{1}{9216}$	o	$-\frac{1}{2016}$
	[cos]	o	o	H	$+\frac{71}{18432}$	H	$+\frac{5}{1008}$
	[sin]	o	o	H	$+\frac{73}{18432}$	H	$+\frac{11}{2016}$

...Table $\left(\left(\frac{r}{a}\right)^{-1} \dots \left(\frac{r}{a}\right)^{-5}\right)$ $\frac{\cos}{\sin} 5f\dots$

	$\cos 5f$ $\sin 5f$	[] ³			[] ⁴				
		e^2	e^4	e^6	e	e^3	e^5	e^7	
$\left(\frac{r}{a}\right)^{-1}$	[cos + sin]	$+\frac{59}{8}$	$-\frac{501}{16}$	$+\frac{39963}{1024}$	$-\frac{9}{2}$	$+\frac{337}{8}$	$-\frac{5683}{48}$	$+\frac{26119}{192}$	
	[cos - sin]	o	o	o	o	o	o	o	
	[cos]	H	H	H	H	H	H	H	
	[sin]	H	H	H	H	H	H	H	
$\left(\frac{r}{a}\right)^{-2}$	[cos + sin]	$+\frac{45}{8}$	$-\frac{177}{8}$	$+\frac{24501}{1024}$	-4	+ 36	$-\frac{565}{6}$	$+\frac{3475}{36}$	
	[cos - sin]	o	o	o	o	o	o	o	
	[cos]	H	H	H	H	H	H	H	
	[sin]	H	H	H	H	H	H	H	
$\left(\frac{r}{a}\right)^{-3}$	[cos + sin]	$+\frac{33}{8}$	$-\frac{29}{2}$	$+\frac{13281}{1024}$	$-\frac{7}{2}$	$+\frac{59}{2}$	$-\frac{3349}{48}$	$+\frac{17803}{288}$	
	[cos - sin]	o	o	o	o	o	o	o	
	[cos]	H	H	H	H	H	H	H	
	[sin]	H	H	H	H	H	H	H	
$\left(\frac{r}{a}\right)^{-4}$	[cos + sin]	$+\frac{23}{8}$	$-\frac{139}{16}$	$+\frac{6263}{1024}$	-3	+ 23	$-\frac{1145}{24}$	$+\frac{1703}{48}$	
	[cos - sin]	o	o	o	o	o	o	o	
	[cos]	H	H	H	H	H	H	H	
	[sin]	H	H	H	H	H	H	H	
$\left(\frac{r}{a}\right)^{-5}$	[cos + sin]	$+\frac{15}{8}$	$-\frac{75}{16}$	$+\frac{2535}{1024}$	$-\frac{5}{2}$	$+\frac{135}{8}$	$-\frac{715}{24}$	$+\frac{20945}{1152}$	
	[cos - sin]	o	o	o	o	o	o	o	
	[cos]	H	H	H	H	H	H	H	
	[sin]	H	H	H	H	H	H	H	

... Table $\left(\left(\frac{r}{a} \right)^{-1} \dots \left(\frac{r}{a} \right)^{-5} \right) \cos \sin 5f \dots$

	$\cos \sin 5f$	[] ⁵				[] ⁶			
		e^0	e^2	e^4	e^6	e	e^3	e^5	e^7
$\left(\frac{r}{a} \right)^{-1}$	[cos + sin]	+ 1	- 25	+ $\frac{9575}{64}$	- $\frac{103325}{288}$	+ $\frac{11}{2}$	- 86	+ $\frac{13593}{3^2}$	- $\frac{60877}{64}$
	[cos - sin]	o	o	o	o	o	o	o	o
	[cos]	H	H	H	H	H	H	H	H
	[sin]	H	H	H	H	H	H	H	H
$\left(\frac{r}{a} \right)^{-2}$	[cos + sin]	+ 1	- $\frac{49}{2}$	+ $\frac{8999}{64}$	- $\frac{363365}{1152}$	+ 6	- $\frac{183}{2}$	+ $\frac{3465}{8}$	- $\frac{29175}{3^2}$
	[cos - sin]	o	o	o	o	o	o	o	o
	[cos]	H	H	H	H	H	H	H	H
	[sin]	H	H	H	H	H	H	H	H
$\left(\frac{r}{a} \right)^{-3}$	[cos + sin]	+ 1	- $\frac{47}{2}$	+ $\frac{8095}{64}$	- $\frac{298655}{1152}$	+ $\frac{13}{2}$	- $\frac{759}{8}$	+ $\frac{13509}{3^2}$	- $\frac{26213}{3^2}$
	[cos - sin]	o	o	o	o	o	o	o	o
	[cos]	H	H	H	H	H	H	H	H
	[sin]	H	H	H	H	H	H	H	H
$\left(\frac{r}{a} \right)^{-4}$	[cos + sin]	+ 1	- 22	+ $\frac{6935}{64}$	- $\frac{114085}{576}$	+ 7	- $\frac{383}{4}$	+ $\frac{6275}{16}$	- $\frac{2751}{4}$
	[cos - sin]	o	o	o	o	o	o	o	o
	[cos]	H	H	H	H	H	H	H	H
	[sin]	H	H	H	H	H	H	H	H
$\left(\frac{r}{a} \right)^{-5}$	[cos + sin]	+ 1	- 20	+ $\frac{5615}{64}$	- $\frac{80455}{576}$	+ $\frac{15}{2}$	- $\frac{375}{4}$	+ $\frac{11055}{3^2}$	- $\frac{8595}{16}$
	[cos - sin]	o	o	o	o	o	o	o	o
	[cos]	H	H	H	H	H	H	H	H
	[sin]	H	H	H	H	H	H	H	H

... Table $\left(\left(\frac{r}{a} \right)^{-1} \dots \left(\frac{r}{a} \right)^{-5} \right) \cos \sin 5f \dots$

		$[]^7$			$[]^8$		
		e^2	e^4	e^6	e^3	e^5	e^7
$\left(\frac{r}{a} \right)^{-1}$	[cos + sin]	$+\frac{149}{8}$	$-\frac{3771}{16}$	$+\frac{3242339}{3072}$	$+\frac{1201}{24}$	$-\frac{54151}{96}$	$+\frac{4603927}{1920}$
	[cos - sin]	o	o	o	o	o	o
	[cos]	H	H	H	H	H	H
	[sin]	H	H	H	H	H	H
$\left(\frac{r}{a} \right)^{-2}$	[cos + sin]	$+\frac{175}{8}$	$-\frac{6461}{24}$	$+\frac{1183371}{1024}$	$+\frac{188}{3}$	$-\frac{4115}{6}$	$+\frac{16781}{6}$
	[cos - sin]	o	o	o	o	o	o
	[cos]	H	H	H	H	H	H
	[sin]	H	H	H	H	H	H
$\left(\frac{r}{a} \right)^{-3}$	[cos + sin]	$+\frac{203}{8}$	$-\frac{896}{3}$	$+\frac{3712093}{3072}$	$+\frac{463}{6}$	$-\frac{38771}{48}$	$+\frac{46693}{15}$
	[cos - sin]	o	o	o	o	o	o
	[cos]	H	H	H	H	H	H
	[sin]	H	H	H	H	H	H
$\left(\frac{r}{a} \right)^{-4}$	[cos + sin]	$+\frac{233}{8}$	$-\frac{5149}{16}$	$+\frac{3701999}{3072}$	$+\frac{281}{3}$	$-\frac{22135}{24}$	$+\frac{99181}{30}$
	[cos - sin]	o	o	o	o	o	o
	[cos]	H	H	H	H	H	H
	[sin]	H	H	H	H	H	H
$\left(\frac{r}{a} \right)^{-5}$	[cos + sin]	$+\frac{265}{8}$	$-\frac{16135}{48}$	$+\frac{1169805}{1024}$	$+\frac{2695}{24}$	$-\frac{24485}{24}$	$+\frac{160513}{48}$
	[cos - sin]	o	o	o	o	o	o
	[cos]	H	H	H	H	H	H
	[sin]	H	H	H	H	H	H

... Table $\left(\left(\frac{r}{a}\right)^{-1} \dots \left(\frac{r}{a}\right)^{-5}\right) \cos \sin 5f.$

$\cos \sin 5f$	$[]^9$		$[]^{10}$		$[]^{11}$		$[]^{12}$	
	e^4	e^6	e^5	e^7	e^6	e^7		
$\left(\frac{r}{a}\right)^{-1}$	[cos + sin]	$+\frac{14987}{128} - \frac{394607}{320}$		$+\frac{11989}{48} - \frac{161685}{64}$		$+\frac{22980061}{46080}$		$+\frac{2122251}{2240}$
	[cos - sin]	o	o	o	o	o		o
	[cos]	H	H	H	H	H		H
	[sin]	H	H	H	H	H		H
$\left(\frac{r}{a}\right)^{-2}$	[cos + sin]	$+\frac{19863}{128} - \frac{2029257}{1280}$		$+\frac{8359}{24} - \frac{983731}{288}$		$+\frac{33554587}{46080}$		$+\frac{40401}{28}$
	[cos - sin]	o	o	o	o	o		o
	[cos]	H	H	H	H	H		H
	[sin]	H	H	H	H	H		H
$\left(\frac{r}{a}\right)^{-3}$	[cos + sin]	$+\frac{25791}{128} - \frac{2521323}{1280}$		$+\frac{1423}{3} - \frac{1282973}{288}$		$+\frac{47722543}{46080}$		$+\frac{2391591}{1120}$
	[cos - sin]	o	o	o	o	o		o
	[cos]	H	H	H	H	H		H
	[sin]	H	H	H	H	H		H
$\left(\frac{r}{a}\right)^{-4}$	[cos + sin]	$+\frac{32907}{128} - \frac{1516377}{640}$		$+\frac{7597}{12} - \frac{67409}{12}$		$+\frac{66363649}{46080}$		$+\frac{1726833}{560}$
	[cos - sin]	o	o	o	o	o		o
	[cos]	H	H	H	H	H		H
	[sin]	H	H	H	H	H		H
$\left(\frac{r}{a}\right)^{-5}$	[cos + sin]	$+\frac{41355}{128} - \frac{353187}{128}$		$+\frac{79711}{96} - \frac{987035}{144}$		$+\frac{18100973}{9216}$		$+\frac{3906009}{896}$
	[cos - sin]	o	o	o	o	o		o
	[cos]	H	H	H	H	H		H
	[sin]	H	H	H	H	H		H

The foregoing tables are read as follows: for instance, in the Table $(x^0, x^1, x^2, x^3) \frac{\cos f}{\sin f}$, p. 376, *et seq.*, the third and fourth lines of the x^0 -compartment show that

$$\begin{aligned}\cos f = & -e \\ & + (\frac{1}{2} e^2 - \frac{9}{16} e^4 + \frac{25}{384} e^6) 2 \cos g \\ & + (-\frac{1}{2} e^3 - \frac{2}{3} e^5 + \frac{3}{16} e^7) 2 \cos 2g \\ & + \text{ &c.} \\ \sin f = & (\frac{1}{2} e^2 - \frac{7}{16} e^4 + \frac{17}{384} e^6) 2 \sin g \\ & + (-\frac{1}{2} e^3 - \frac{7}{12} e^5 + \frac{1}{6} e^7) 2 \sin 2g \\ & + \text{ &c.}\end{aligned}$$

and the first and second lines give the sum and difference respectively of the corresponding coefficients of the cosine and sine series.

Addition, 28th Dec. 1860.—The tables have been verified by me, on the proof-sheets; the x -tables, the cosine lines of the $\left(\frac{r}{a}\right)$ -tables, and a portion of the sine lines of the same tables, in the manner explained at the commencement of the memoir; but for the remainder of the sine lines it was found easier to employ the following mode of verification; viz. the equation $\frac{r}{a} = 1 + x$, gives

$$\left\{ \left(\frac{r}{a}\right)^4 - 4 \left(\frac{r}{a}\right)^3 + 6 \left(\frac{r}{a}\right)^2 - 4 \left(\frac{r}{a}\right) \right\} \sin jf = (-1 + x^4) \sin jf;$$

and as regards the terms up to e^7 , the limit of the tables,

$$\left\{ \left(\frac{r}{a}\right)^{-5} - 5 \left(\frac{r}{a}\right)^{-4} + 10 \left(\frac{r}{a}\right)^{-3} - 10 \left(\frac{r}{a}\right)^{-2} + 5 \left(\frac{r}{a}\right)^{-1} \right\} \sin jf = (1 - x^5 + 5x^6 - 15x^7) \sin jf,$$

which equations afford the verification referred to. In going over the earlier sheets I omitted to see that the fractions were in their least terms, and it may happen that, in some instances, they are not so. [A few reductions have been made, and I believe that the fractions are now all in their least terms.]

The expression for the true anomaly f itself has been repeatedly calculated to a much greater extent, in particular by Schubert (*Astronomie Théorique*, Pet. 1822), as far as e^{20} . The easiest way of obtaining it seems to be by means of the equation

$$\frac{df}{dg} = \left(\frac{r}{a}\right)^{-2} \sqrt{1 - e^2};$$

and, for the convenience of reference, I here give it as far as e^7 , viz.,

$$f = g + \Sigma [\sin]^i \sin ig,$$

where, as in the other sine series, i is to be taken as well negatively as positively, and $[\sin]^{-i} = -[\sin]^i$. Or, what is the same thing,

$$f = g + \Sigma [\sin]^i 2 \sin ig,$$

where i has only positive values. And the coefficients are

e	e^2	e^3	e^4	e^5	e^6	e^7
$[\sin]^1 =$	1	$-\frac{1}{8}$		$+\frac{5}{192}$		$+\frac{107}{9216}$
$[\sin]^2 =$	$+\frac{5}{8}$	$-\frac{11}{48}$		$+\frac{17}{384}$		
$[\sin]^3 =$		$+\frac{13}{24}$	$-\frac{43}{128}$		$+\frac{95}{1024}$	
$[\sin]^4 =$			$+\frac{103}{192}$	$-\frac{451}{960}$		
$[\sin]^5 =$				$+\frac{1097}{1920}$	$-\frac{5957}{9216}$	
$[\sin]^6 =$					$+\frac{1223}{1920}$	
$[\sin]^7 =$						$+\frac{47273}{64512}$

The expression for $\frac{r}{a}$, as far as e^{13} , is given in Schubert's work, above referred to; and that of $\log\left(\frac{r}{a}\right)$, as far as e^9 , was calculated by Oriani, see the Introduction to Delambre's *Tables du Soleil*, Paris, 1806.