172.

NOTE ON THE LOGIC OF CHARACTERISTICS.

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THE conditions in order that an equation of the sixth degree

$$(a, b, c, d, e, f, g), (x, y)^6 = 0$$

may have five of its roots equal are

$$\begin{aligned} A &= ae - 4bd + 3c^2 = 0, \\ B &= af - 3be + 2cd = 0, \\ C &= bf - 4ce + 3d^2 = 0, \\ D &= ag - 9ce + 8d^2 = 0, \\ E &= bg - 3cf + 2de = 0, \\ F &= cg - 4df + 3e^2 = 0, \end{aligned}$$

equivalent of course to four relations between the coefficients: among the connections of these equations are

$$fA - eB - bF + cE = 0,$$

$$(3e^2 - 2df) A - 2deB + ecD - AF - 2cdE + (3c^2 - 2bd) F = 0.$$

The system is one of the tenth order. To verify this, I write first

$$(A, B, C, F) = (A, B, C, F, cE) = (A, B, C, F, c) + (A, B, C, F, E),$$

i.e. the equations $A = 0$, $B = 0$, $C = 0$, $F = 0$ imply (by the first of the connectives) the additional equation $cE = 0$, viz. the system $A = 0$, $B = 0$, $C = 0$, $F = 0$, $cE = 0$, or what is the same thing, one of the systems $A = 0$, $B = 0$, $C = 0$, $F = 0$, $c = 0$ and $A = 0$, $B = 0$, $C = 0$, $F = 0$, $E = 0$.

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We have in like manner

$$(A,\ B,\ C,\ F,\ E) = (A,\ B,\ C,\ F,\ E,\ ceD) = (A,\ B,\ C,\ F,\ E,\ D)$$

since (A, B, C, F, E, c) and (A, B, C, F, E, e) respectively vanish as being each of them equivalent not to four but to five relations, and therefore as not adding to the order of the system.

Again,

$$\begin{aligned}(A,\ B,\ C,\ c) &= (A,\ B,\ C,\ c,\ bF)\\ &= (A,\ B,\ C,\ c,\ b) + (A,\ B,\ C,\ c,\ F)\\ &= (ae,\ af,\ d^2,\ b,\ c) + (A,\ B,\ C,\ c,\ F).\end{aligned}$$

But here

$$\begin{aligned}(ae,\ af,\ d^2,\ b,\ c) &= (a,\ af,\ d^2,\ b,\ c) + (e,\ af,\ d^2,\ b,\ c)\\ &= (a,\ af,\ d^2,\ b,\ c),\end{aligned}$$

(for (e, af, d^2, b, c) vanishes as being equivalent to five relations, and therefore as not adding to the order of the system),

$$\begin{aligned}&= (a,\ a,\ d^2,\ b,\ c) + (a,\ f,\ d^2,\ b,\ c)\\ &= (a,\ d^2,\ b,\ c),\end{aligned}$$

since, (a, f, d^2, b, c) vanishes for the above-mentioned reason.

Hence

$$(A,\ B,\ C,\ c) = (a,\ d^2,\ b,\ c) + (A,\ B,\ C,\ c,\ F).$$

We have consequently

$$\begin{aligned}(A,\ B,\ C,\ D,\ E,\ F) &= (A,\ B,\ C,\ E,\ F)\\ &= (A,\ B,\ C,\ F) - (A,\ B,\ C,\ F,\ c)\\ &= (A,\ B,\ C,\ F) - \{(A,\ B,\ C,\ c) - (a,\ b,\ c,\ d^2)\},\end{aligned}$$

which may be thus interpreted:—the system (A, B, C, c) contains the system (a, b, c, d^2), or what is the same thing, contains twice-over the system (a, b, c, d). Discarding this contained system, the remainder of the system (A, B, C, c) is contained in the system (A, B, C, F), and the residue of the last-mentioned system is the system (A, B, C, D, E, F), i.e. the system represented by the equations which express the equality of five roots of the given equation of the sixth degree.

It follows immediately that the order of the system (A, B, C, D, E, F) is $16-(8-2)=10$, i.e. that the system is, as above stated, one of the tenth order. The preceding is, I think, a good example of the kind of reasoning to be employed in what Mr Sylvester has most happily termed the Logic of Characteristics.

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