Archives of Mechanics • Archiwum Mechaniki Stosowanej • 25, 5, pp. 871–879, Warszawa 1973

A probabilistic interpretation of creep rupture curves

H. BROBERG (GOTHENBURG)

By considering ductile and brittle creep rupture as two independent mechanisms, the curved transition part of standard log-log creep rupture curves may be expressed in terms of the scatter present in both the basic mechanisms. The scatter also explains why phenomenological ductile creep rupture theory overestimates lifetime compared to rupture tests.

Na podstawie rozważań ciągliwego i kruchego zniszczenia pełzającego jako dwóch niezależnych mechanizmów zakrzywiona część przejściowa zwykłych logarytmicznych/krzywych zniszczenia pełzającego może być wyrażona za pomocą rozrzutu obecnego w obydwóch podstawowych mechanizmach. Rozrzut również wyjaśnia, dlaczego fenomenologiczna teoria ciągliwego zniszczenia pełzającego podaje poważnie zawyżone "czasy życia" w porównaniu z badaniami na zniszczenie.

Исследование хрупкого и пластического разрушений при ползучести, рассматриваемых в качестве независимых механизмов показывает, что можно описать искривленную часть логарифмических графиков разрушения при ползучести в терминах распределения свойств, присущего обоим основным механизмам. Разброс свойств объясняет также причины, по которым в рамках феноменологической теории пластического разрушения при ползучести получаются значительно завышенные величины "время жизни", по сравнению с результатами опытов на разрушение при ползучести.

1. Introduction

Creep rupture tests are normally done with bars subjected to a constant tensile force F at a constant temperature T. The time to rupture t_R is recorded at each combination of F and T. The results are often presented in diagrams, where $\log\sigma_0$ is plotted versus $\log t_R$. Here

$$\sigma_0 = F/A_0 \tag{1.1}$$

where A_0 denotes the initial cross sectional area of the loaded bar.

If several series of tests are run, each at a certain stress level σ_0, but all at the same temperature T, the results normally appear as shown in Fig. 1. The median points at the various stress levels have been connected by a "median creep rupture curve".

Such curves are found to have certain characteristic features, common to a large number of metals in very wide ranges of temperature:

1. at high stresses, the curves appear as straight lines with small slopes, often in the range — (0.1–0.3)
2. at low stresses, the curves appear as straight lines with larger slopes, often in the range — (0.2–0.5)

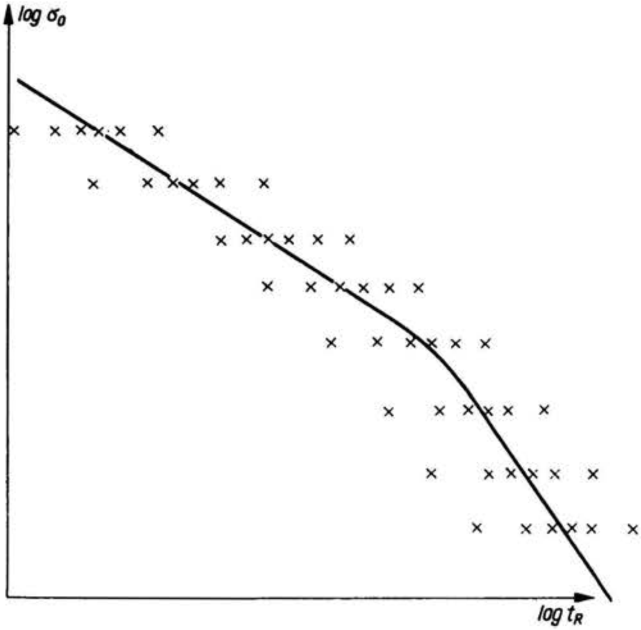

Fig. 1.

3. at intermediate stresses, the curves bend over, gradually, from the one type to the other
4. the scatter in rupture time is rather large and tends to be independent of the stress level

see e.g. NRIM, Elevated-Temperature properties at high-temperature materials, manufactured in Japan (1972).

It is the purpose of this paper to indicate a connection between the scatter and the gradual change in slope of such creep rupture curves.

2. Mechanistic interpretations of creep rupture curves

At high stresses, creep rupture is often preceded by large ductile deformation, whereas at low stresses creep, rupture takes place with very small previous deformation. From a phenomenological point of view, one may isolate these two types of creep rupture, viz. ductile and brittle, and analyse them, first, as two separate mechanisms. When these two limiting cases have been examined, one may consider their interaction.

We shall review here two well established mechanistic theories of ductile and brittle creep rupture, respectively. The transition between the two will then be described in terms expressing their mechanical interaction. In the next section, an alternative approach will be presented, where the transition between ductile and brittle creep rupture is interpreted in probabilistic terms, as a conesquence of stochastic variation of material parameters.

2.1. Ductile creep rupture

The first known analysis is due to HOFF (1953). He assumed the material to be incompressible and to obey the Norton creep law

$$d\varepsilon/dt = K\sigma^n, \tag{2.1}$$

where K and n are temperature dependent material parameters. The analysis concerned a tensile bar subject to constant loading. The resulting differential equation predicts an instability at a finite time t_{RH} (H for HOFF), in the sence that $\sigma(t) \to \infty$ as $t \to t_{RH}$.

With the initial condition

$$\sigma(0) = \sigma_0, \tag{2.2}$$

where σ_0 is given by Eq. (1.1), follows

$$t_{RH} = 1/nK\sigma_0^n. \tag{2.3}$$

This relation represents a straight line with slope $-1/n$ in a $\log\sigma_0 - \log t_R$ diagram. This is in certain accord with experimental observations of creep rupture at high stresses.

However, the Hoff theory normally predicts longer lifetimes than are found in experiments, cf. Fig. 2, showing data obtained from STAL-LAWAL AB (1972). The reason for this discrepancy will be discussed later.

Alternatives to Hoff's analysis have been presented by ODQVIST (1962), RIMROTT and MUENSTERER (1964) and by others. These theories often give a fairly good description

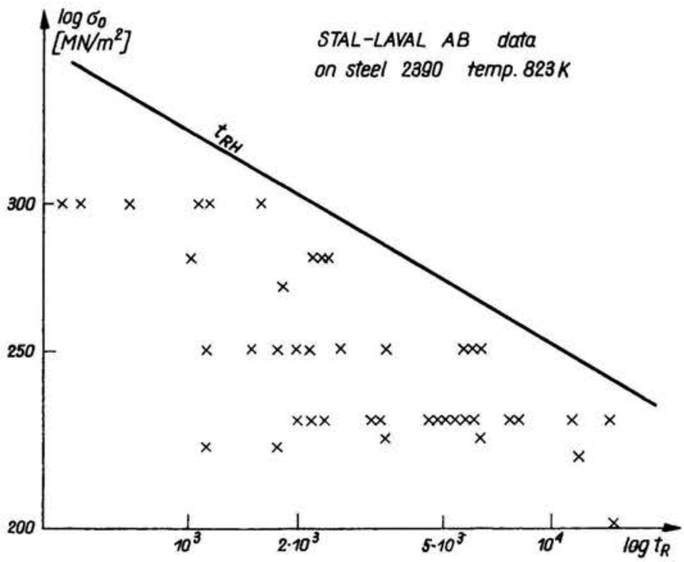

Fig. 2.

of the real rupture curves, in the ductile region, but they all seem to overestimate the lifetime and, of course, they cannot, and are not intended to, describe and explain the scatter present in creep rupture testing.

2.2. Brittle creep rupture

In brittle rupture, no equally simple mechanisms have yet been found. All analyses presented hereto are based on more free assumptions than were necessary for the ductile case. A well known theory by KACHANOV (1958) is based on a concept called "continuity" and denoted ψ. It describes the fraction of area still capable of transferring load. Hence, in the virgin state, $\psi = 1$ and at rupture, $\psi = 0$.

KACHANOV postulates the relation

$$d\psi/dt = -C(\sigma_0/\psi)^\nu, \tag{2.4}$$

where C and ν are temperature dependent material parameters. From this follows the time to brittle rupture

$$t_{RK} = 1/(1+\nu)C\sigma_0^\nu, \tag{2.5}$$

(subscript K for KACHANOV) which again is a straight line in the $\log\sigma_0 - \log t_R$ diagram. The two constants C and ν are normally chosen such that the straight line falls close to the median points, e.g. in the least square sense.

2.3. Interaction theories

The theories described above can now be coupled in different ways to smooth the passage between them. One can take into account large deformations in the continuity

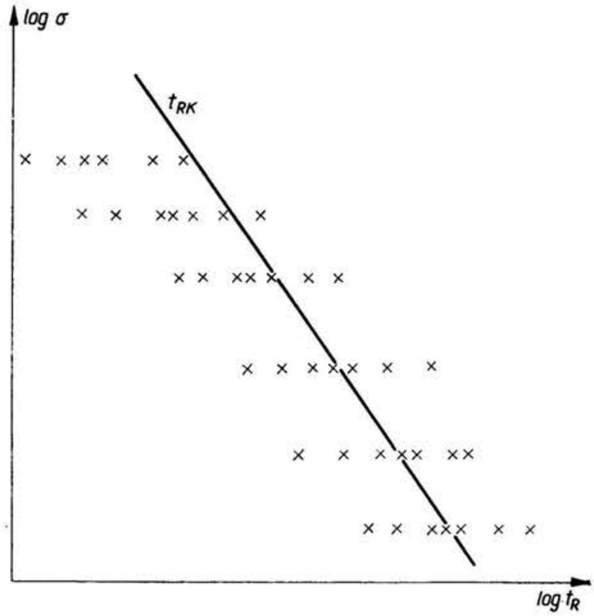

Fig. 3.

equation (a) or the continuity in the constitutive equation (b), or both (c). For Norton creep and Kachanov deterioration, the governing equations will then be

$$d\psi/dt = -C(\sigma/\psi)^\nu, \qquad d\varepsilon/dt = K(\sigma)^n, \tag{a}$$

(b) $$d\psi/dt = -C(\sigma_0/\psi)^{\nu}, \quad d\varepsilon/dt = K(\sigma/\psi)^{n},$$

(c) $$d\psi/dt = -C(\sigma/\psi)^{\nu}, \quad d\varepsilon/dt = K(\sigma/\psi)^{n}.$$

Equations (c) were introduced already by KACHANOV (1958). All three possibilities are discussed in the textbook by RABOTNOV (1966, English edition 1969). These coupled theories all smooth the passage between the ductile and brittle region.

3. Probabilistic interpretation of creep rupture curves

Two different mechanisms of creep rupture were considered in the previous section, viz. ductile and brittle. The material parameters involved were assumed to be constants. Various purely mechanical interaction mechanisms were considered in order to describe the transition between the two basic types of rupture.

Here, an alternative approach will be taken, based on the assumption that the material properties are not constant but vary from one point to another within the material. See GAROFALO (1965), p. 110.

3.1. Basic assumptions

For simplification, the following two assumptions will be made:

1. the mechanical parameters governing these two mechanisms are stochastic processes in one dimension, viz. along the bar length;
2. the two mechanisms of creep rupture, viz. ductile and brittle, are acting independently.

Both these assumptions may be relaxed without altering the principal conclusions.

3.2. Probabilistic analyses

1. For convenience, the Hoff theory of ductile rupture and the Kachanov theory of brittle rupture will be retained. The stochastic variation is assumed to be present only in the parameters K and C, whereas the exponents n and ν are assumed to be constants. This is in agreement with experimental observation. The constants K and n are determined from a $\log\dot{\varepsilon}-\log\sigma$ diagram obtained in standard creep tests. A variation in K corresponds to translation in such a plot (see Fig. 4a). A variation in n corresponds to slope change in the $\log\dot{\varepsilon}-\log\sigma$ plot (see Fig. 4b). It then follows that a variation in n leads to a pole at a certain stress σ_p, where no scatter can exist. No such stress σ_p has been observed in tests and if it exists, σ_p must be at a very low stress level which means very small variation in n (see Fig. 4c). Here, all variation is attributed to K and C for reasons stated above.

Denoting the axial coordinate by x ,the Norton creep law (2.1) then takes the form

(3.1) $$\partial\varepsilon/\partial t = K(x)\sigma^{n},$$

whereas the Kachanov deterioration law (2.4) takes the form

(3.2) $$\partial\psi/\partial t = -C(x)(\sigma_0/\psi)^{\nu}.$$

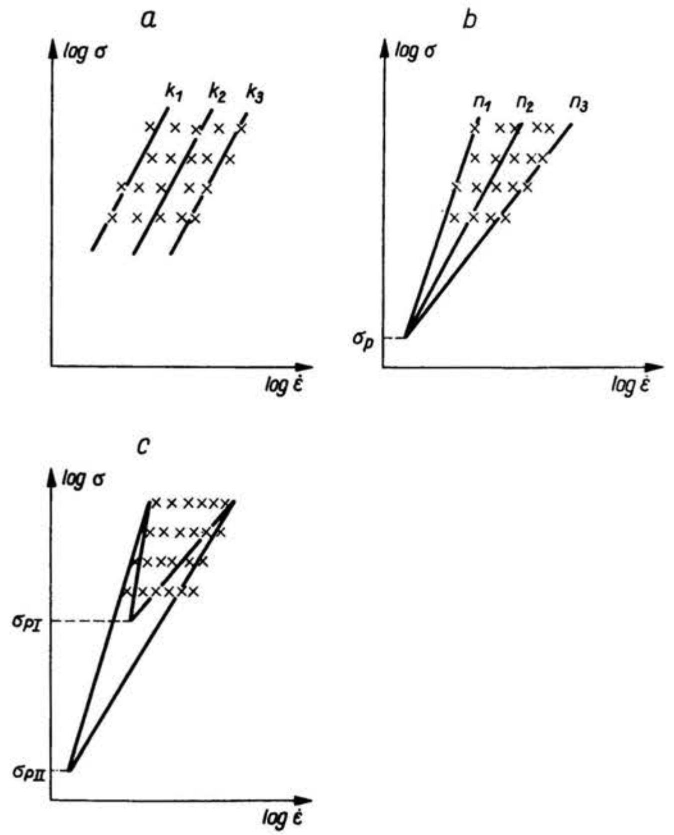

Fig. 4.

Ductile creep rupture will then occur at the cross-section where $\dot{\varepsilon}$ is largest, and since variation in n is neglected, this is equivalent to the cross-section, where K is largest.

According to Eq. (2.3),

$$t_{RH}^{(\mathrm{real})} = 1/nK_{\max}\sigma_0^n, \tag{3.3}$$

where $K_{\max}$ denotes extremum K within the loaded volume.

The experimental determination of the parameter K from ordinary creep tests is based on measuring the elongation rate $\dot{\Delta}$ over a gauge length L.

Hence, according to Eq. (3.1),

$$\dot{\Delta}L = (1/L)\int_0^L \dot{\varepsilon}\,dx = \sigma^n(1/L)\int_0^L K(x)dx = \sigma^n K_{\mathrm{ave}}. \tag{3.4}$$

Hence, according to Eq. (2.3), the calculated rupture time will be

$$t_{RH}^{(\mathrm{calc})} = 1/nK_{\mathrm{ave}}\sigma_0^n, \tag{3.5}$$

i.e. from Eq. (3.3),

$$t_{RH}^{(\mathrm{calc})} > t_{RH}^{(\mathrm{real})}. \tag{3.6}$$

The previously mentioned fact that the ductile rupture theories of HOFF and others all overestimate the lifetime, has hence been shown to be a direct consequence of the stochastic variation of the creep parameters.

For brittle rupture, the same analysis can be extracted. Equation (3.3) gives

$$t_{RB}^{(\text{real})} = 1/[(1+\nu)C_{\max}\sigma_0^{\nu}]. \tag{3.7}$$

The discrepancy between calculated and real creep rupture lifetime noted for the ductile case has no correspondence in the brittle case, since the parameters involved in the latter can only be determined from rupture tests. From Eq. (3.7) follows that $C_{\max}$ (strictly $\max[C(x)]$ in the interval $0 < x < L$) rather than C is determined.

2. Whereas the previous discussion referred to purely ductile or brittle creep rupture, their combined effect may also be described in statistical terms.

If two rupture processes are presented, one may use elementary probabilistic theory to get the resulting rupture times. Let A be the event of ductile rupture and B the event of brittle rupture. $A \cup B$ is then the event of rupture, ductile or brittle. Denoting by $P(A)$ and $P(B)$ the probability of ductile and brittle rupture respectively, we obtain

$$P(A \cup B) = P(A)+P(B)-P(A \cap B), \tag{3.8}$$

where, if A and B are independent,

$$P(A \cap B) = P(A)\cdot P(B). \tag{3.9}$$

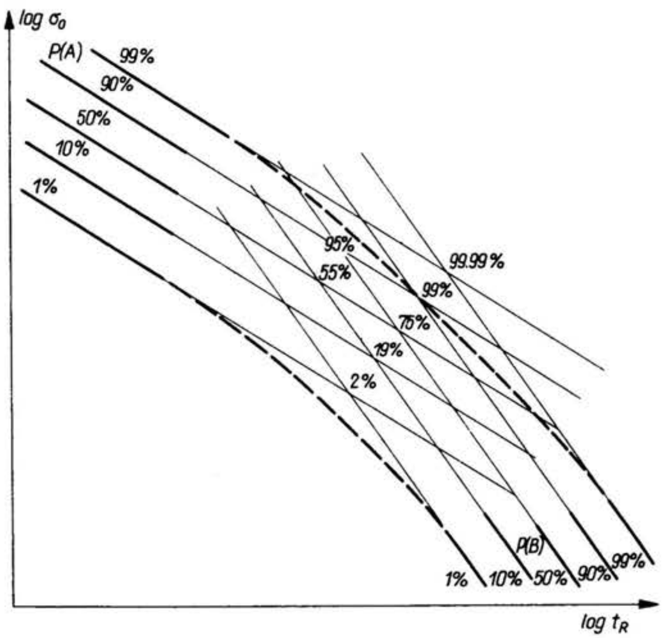

Fig. 5.

If, for example, $P(A) = 0.5$ and $P(B) = 0.8$, then $P(A \cup B) = 0.5+0.8-0.5\cdot 0.8 = 0.9$. This implies that if the probability of ductile rupture is 0.5 and the probability of brittle rupture is 0.8, the total probability of creep rupture is 0.9. The probability distribution functions $P(A)$ and $P(B)$ can be evaluated by extreme value statistics, if the probability distributions and autocorrelation functions of $K(x)$ and $C(x)$ are known. They can, of course, also be measured in rupture tests at stress levels, where they do not influence one another. Rupture tests have shown that the scatter is close to log-normal distributed, i.e. it is normal (Gaussian) distributed in a log time scale [see WALLES (1967)]. If $P(A)$ and $P(B)$ are both normal distributed in log time, one can show that $P(A \cup B)$ must also be normal-distributed.

This result implies that a curved transition region must exist in a creep rupture diagram, where ductile and brittle creep rupture lines of constant probability are drawn, cf. Fig. 5. A simple graphical construction is possible.

This curved transition has been derived here as a consequence simply of the scatter in the two phenomena of ductile and brittle rupture, assumed to be independent. This does not imply that rupture is caused entirely by one or the other mechanism.

In the curved region, the rupture must be of "mixed" kind, since both the processes leading to the two kinds of rupture must be activated. In any ruptured test specimen, one does always observe both deterioration and creep deformations.

It should be noted that no mechanical coupling is necessary here. The presence of such coupling in real situations contributes further to the smoothing of the transition. In such case, Eq. (3.9) must be replaced by another expression relation $P(A)$ and $P(B)$.

4. Conclusions

Two well known observations have been shown to be direct consequences of the inherent variation of the material parameters.

1. The time to ductile rupture, calculated from creep data determined in ordinary creep tests, overestimates the real lifetime.
2. The transition region in creep rupture diagrams is curved.

If no such variation were present, transition between the two rupture mechanisms would occur at one well defined stress level. This indicates that a complete theory must include both mechanical coupling and probabilistic aspects.

The probabilistic model also implies a size effect such that the creep rupture lifetime decreases with increasing size of the bar.

With the suggested approach to creep rupture analysis, one gains in simplicity, since the two rupture mechanisms may then be treated as independent. This is specially important in studying more complicated, statically indeterminate structures, where a coupled analysis becomes prohibitively involved.

References

1. NRIM, *Elevated temperature properties of high-temperature materials manufactured in Japan*, 1972.
2. N. J. HOFF, *Necking and rupture of rods subjected to constant tensile loads*, J. Appl. Mech., **20**, 105–108, 1953.
3. Private communication, STAL-LAVAL AB, Finspång, Sweden.

4. F. K. G. ODQVIST, *On theories of creep rupture*, Int. Symposium on second-order effects in elasticity, plasticity and fluid dynamics, Haifa 1962. Pergamon Press, 1964.
5. F. P. J. RIMROTT and H., MUENSTERER, *Creep rupture life prediction for a Prandtl material*, Conference on thermal loading and creep in structures and components, Paper 9, The Institution of Mechanical Engineers, London 1964.
6. L. M. KACHANOV, *On time to rupture under creep conditions* [in Russian], Izv. Ak. Nauk., **8**, 26–31, 1958.
7. YU. N. RABOTNOV, *Creep problems in structural members*, North-Holland Publishing Company, Amsterdam/London 1969, [English edition].
8. F. GAROFALO, *Foundamentals of creep and creep-rupture in metals*, The Macmillan Company, New York 1965.
9. K. F. A. WALLES, *Random and systematic factors in the scatter of creep data*, Aeronautical Research Council, current papers C.P. No. 935, Her Majesty's Stationary Office, London 1967.

DIVISION OF SOLID MECHANICS
CHALMERS UNIVERSITY OF TECHNOLOGY
GOTHENBURG, SWEDEN.

Received February 2, 1973.