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An experimental and theoretical study of the distortion of a travelling shock wave by wall effects(*)

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AN EXSPERIMENTAL and theoretical study of the distortion of a strong shock wave travelling along a solid boundary by a laminar boundary layer developing under the induced flow field was performed for the purpose of establishing the shock front slope near the wall. A simple continuum flow model is found to yield results which are in good agreement with experiment. It is shown that the deviation of the shock front from normal is quite small, which is in direct disagreement with the commonly held belief of a gradual decay of the shock wave to an acoustic pulse as the wall is approached.

W pracy przeprowadzono teoretyczne i doświadczalne badania nad odkształceniem silnej fali uderzeniowej, poruszającej się wzdłuż sztywnej ścianki, w wyniku działania laminarnej warstwy przyściennej powstającej w przepływie za falą. Celem ich było określenie kąta nachylenia fali w pobliżu ścianki. Stwierdzono, że prosty model teoretyczny, oparty na założeniu ośrodka ciągłego, daje wyniki dobrze zgadzające się z eksperymentem. Uzyskane wartości odchylenia fali uderzeniowej od normalnej do ścianki są niewielkie, co jest sprzeczne z powszechnym mniemaniem, że fala uderzeniowa w miarę zbliżania się do ścianki przechodzi w zaburzenie akustyczne.

В работе изложены теоретические и опытные исследования деформирования сильной ударной волны, движущейся вдоль жесткой стенки. Деформация фронта волны является результатом воздействия ламинарного пограничного слоя, возникающего за волной в изучаемом течении. Целью исследования являлось нахождение угла наклона фронта волны в окрестности стенки. Обнаружено, что хорошие результаты, согласующиеся с экспериментальными данными, дает простая теоретическая схема, основанная на континуальной модели течения. Полученные величины отклонения ударной волны от нормали к стенке имеют небольшие значения, что противоречит распространенному мнению о том, что ударная волна переходит в акустическое возмущение при приближении к стенке.

Notations

- C $\varrho\mu/(\varrho\mu)_2$,
- D half width of the channel,
- E energy parameter $= u_2^2/2h_2$,
- f stream function defined so that $f' = u/u_2$,
- g h/h_2,
- h specific enthalpy of gas,
- L characteristic dimension of the interaction,
- M rate of mass increase in the control volume, Eq. (1.1),
- Pr Prandtl number,
- q velocity in the leading edge region, Eq. (2.5),
- S transformed airfoil coordinate, Eq. (2.4),
- u velocity relative to the shock wave,
- U velocity far from the airfoil leading edge,

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x coordinate along the shock tube wall,
y coordinate normal to the shock tube wall,
Y transformed airfoil coordinate, Eq. (2.4),
δ^* displacement thickness, Eq. (2.2),
ε small parameter in thin airfoil theory, Eq. (2.4),
μ viscosity,
η $[\varrho_2 u_2/(2\xi)^{1/2}] \int\limits_0^y (\varrho/\varrho_2) dy'$,
ξ $\int\limits_0^x \varrho_2 u_2 \mu_2 dx$,
ψ stream function,
ϱ density,
θ deviation of the shock front from normal.

Subscripts 1 and 2 denote conditions ahead of and downstream of the shock wave, respectively, in a shock based coordinate system. Subscript e denotes edge of boundary layer velocity in a fixed coordinate system and subscript s denotes wall conditions.

1. Introduction

THE PROBLEM considered here has attracted considerable attention in the past — e.g., MIRELS [1], HARTUNIAN [2], DEBOER [3], AKAMATSU and URUSHIDANI [4] and several distinct methods of solution of different models of the flow have been exhibited. In general, the justifications for the adoption of a particular model have been rather scanty and this has given rise to certain misconceptions which have been propagated through the literature.

A common assumption is that of the decay of the shock wave to a Mach wave near the wall, since this automatically satisfies the condition of tangency at the wall. Our results indicate that this assumption is not true; the moving shock wave actually assumes an S-shape and intersects the wall at right angles. The difficulty of guessing the shape of the shock wave is avoided by SICHEL [5] who limits his attention to very weak shock waves. In our case, the shape of the shock wave is derived from the matching of the viscid and inviscid flows. In an essentially purely inviscid approach to the problem, DEBOER [3] assumed that the shape of the displacement thickness is known, and ignored the interaction of the boundary layer with the shock wave.

The existing strong shock wave studies are marked by the employment of plausible, but physically unproved models of the flow. Most of the difficulties appear to stem from the failure to recognize the principal features of the flow — namely the existence of two velocity scales, i.e. the shock wave velocity which is equal to the velocity of the origin of the boundary layer and the induced velocity which determines the magnitude of the vorticity diffusing into the flow. The flow at the edge of the boundary layer lags behind the development of the boundary layer and thus a motion away from the wall is induced. The induced flow away from the wall is of second-order in the sense of asymptotic matching theory of viscid-inviscid flows, and thus the distortion of the shock wave must be of second order. Consequently, for all flows in which a distinct boundary layer can be recognized, the decay of a strong shock wave to an acoustic pulse near the wall cannot be accepted on mathematical grounds, since it would imply that second-order viscous effects are capable of causing first order inviscid flow perturbations.

At this point, it is convenient to illustrate the main features of the flow by considering the mass balance on a control volume bounded by a fixed plane normal to the duct axis, the plane of symmetry a distance D from the wall along which the shock wave is moving, and the shock front itself (Fig. 1). With subscripts 1 and 2 denoting the conditions ahead

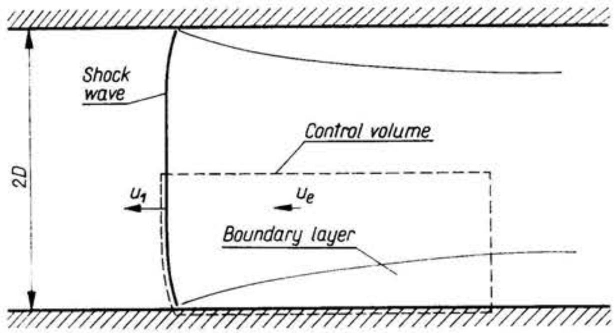

Fig. 1.

of and downstream of the shock wave respectively, and $u_e = u_1 - u_2$ being the induced velocity, the rate of change of mass in the control volume, $\dot{M}$, is

$$(1.1) \qquad \dot{M} = \int_0^D \varrho_1 u_1 \, dy + \int_0^D \varrho u dy = \varrho_1 u_1 D + \varrho_2 u_e \int_0^D [1 - (1 - \varrho u/\varrho_2 u_e)] dy$$
$$= \varrho_1 u_1 D + \varrho_2 u_e D(1 - \delta^*/D).$$

Here δ^* is the true boundary layer displacement thickness. One simple interpretation of the Eq. (1.1) is that the flow area of the duct is reduced by δ^*, which results in the displacement of the flow away from the wall. An alternative interpretation is that mass disappears from the control volume at the rate of $\varrho_2 u_e \delta^*$. The latter explanation leads directly to the appearance of the negative displacement thickness when the problem is analyzed in a coordinate system moving with the shock front.

2. Analysis

2.1. Boundary layer flow

We are considering a two-dimensional parallel wall duct in which flow is induced by a strong shock moving into a quiescent medium (Fig. 1). The induced inviscid flow is perturbed by the development of a laminar boundary layer which extends forward at the speed of the shock wave. The characteristic Reynolds number of the flow is taken to be sufficiently high to allow the use of the concept of a distinct boundary layer.

Only strong shock waves are considered here so that in the wall fixed coordinates the ratio of wall to total edge of boundary layer enthalpy will be small; the flow Mach number will also be small, so that following LEES [6] we can argue that the effects of the longitudinal pressure gradients will be negligible. Actually, in our analysis we do not need to employ this argument since in our use of thin airfoil theory we can take advantage of a fortuitous mathematical result that on an infinite parabolic cylinder the surface speed is equal to the free stream speed and no pressure gradient is induced. The displacement thickness on a flat plate grows parabolically and thus does not tend to induce a pressure gradient. In the shock front based coordinate system, we can therefore write the boundary layer equations in self-similar form:

$$(Cf'')' + ff'' = 0, \quad \left(\frac{C}{\mathrm{Pr}} g'\right)' + fg' = -2ECf''^2, \tag{2.1}$$

with the boundary conditions,

$$\begin{aligned} &\eta = 0, \quad f = 0, \quad f' = u_1/u_2, \quad g = g_s, \\ &\eta \to \infty, \quad f' \to 1, \quad g \to 1.0. \end{aligned}$$

In the shock front coordinate system, the boundary conditions exhibit apparent slip at the inner boundary, but this is due to the apparent motion of the wall away from the shock wave, and no physical surface slip is implied. These equations are in standard form and may be solved easily using the simple, exact, semi-analytical technique developed in Ref. [7]. The crude series solution of these equations by AKAMATSU and URUSHIDANI [4], who used the degenerate gas properties $C = 1.0$ cannot be justified on either mathematical or physical grounds, since variation of physical properties is known to play a critical role in boundary layer flows (e.g. Ref. [8]). The Mirels' solution claimed to be "exact", suffers from precisely this defect, in addition to failing to match the viscous and inviscid flows. With a negligible pressure gradient the displacement thickness is given by:

$$\delta^* = \frac{(2\xi)^{\frac{1}{2}}}{\varrho_2 u_2} \int_0^\infty \left(\frac{\varrho_2}{\varrho} - f'\right) d\eta = \frac{(2\xi)^{\frac{1}{2}}}{\varrho_2 u_2} \Delta^* = 2^{\frac{1}{2}} x \Delta^* \mathrm{Re}_x^{-\frac{1}{2}} \tag{2.2}$$

with

$$\mathrm{Re}_x = \varrho_2 u_2 x/\mu_2 .$$

At the inner limit of the outer flow the velocity is directed along the tangent to the displacement thickness so that v, the y component of velocity is

$$v = \Delta^* u_2 (2\mathrm{Re}_x)^{-\frac{1}{2}} .$$

In the shock front based coordinate system, Δ^* appears to be negative so that v is directed towards the wall and, in terms of the asymptotic matching concepts, the outer flow at the inner boundary appears to flow along a parabola. In a fixed coordinate system, the streamlines again appear to be parabolic, but are directed towards the origin. The above observations are not valid at the origin itself, where in the simple formulation presented above the streamline slope and the normal velocity tends to infinity.

The above considerations bear directly upon the work of DEBOER [3], who calculated the curvature of the shock wave by solving Laplace's equation for the induced flow field with continuously curving boundaries formed by the shock wave and a laminar or turbulent boundary layer. While quite correct mathematically, DeBoer's solution is physically questionable since at the shock wave the slope of the boundary layer displacement thickness is infinite, and the tangent streamlines can be generated only by a shock wave lying parallel to the wall rather than by a weak wave demanded by the solution. Conversely, if DeBoer's discussion of the corrections for the region near the foot of the shock wave is taken, then his solution leads to a mathematical problem since discontinuities appear in his boundary conditions. In De Boer's analysis a connection between the induced flow field solution with the flow ahead of the shock front through the Rankine-Hugoniot and kinematic conditions does not appear. This difficulty is avoided here by the matching of the flow field generated by a perturbed shock wave with the flow field due to the boundary layer displacement thickness effects.

The physical and mathematical difficulties in the region at the foot of the shock wave are avoided by making use of the well known thin airfoil theory result that the flow in the leading edge region is independent of the details of the leading edge shape, to at least the second order of magnitude. Thus, while acknowledging the existence of a complex viscid-inviscid interaction region, we do not inquire into the details of its structure but simply deal with the local external flow due to the effective displacement thickness of the interaction. Here the implicit assumption of a relatively small interaction region is employed, so that the Reynolds number must be high enough to give a fairly distinct shock wave whose structure does not have to be examined. When the local leading edge region is connected with the downstream flow over the parabolic displacement thickness, the whole perturbation of the induced flow field by the boundary layer may be described in terms of thin airfoil theory.

The interaction region is analysed using the approximation methods of thin airfoil theory. It should be realized that this theory was developed for convex shapes, while in our case the apparent region is concave. The similarity of the two situations is obvious from the reversibility and equivalence of purely inviscid flows. Thus we can use the results of Van Dyke's analysis [9] which showed that any leading edge shape of a thin airfoil may be approximated by a parabola. We are thus relieved of the problem of specifying exactly the effective displacement thickness in the interaction region, where neither the physics nor the mathematics of the problem are well understood. Also, we thus have a smooth junction with the downstream boundary layer displacement thickness which is known to vary parabolically with the streamwise direction. We follow the notation of VAN DYKE [9] and write for the effective displacement thickness:

$$Y = -(2S)^{\frac{1}{2}}, \tag{2.3}$$

with

$$Y = y/\varepsilon^2 L, \quad S = x/\varepsilon^2 L, \tag{2.4}$$

where ε is a small parameter of the problem. The velocity in the leading edge region of radius ε is given by:

$$q = U(1+\varepsilon), \tag{2.5}$$

with U being the unperturbed free stream velocity. In our case, we obviously have the small parameter from the form of the relation for δ^*, viz.([1]):

$$\varepsilon = -\Delta^*(\varrho_2 u_2 L/\mu_2)^{-\frac{1}{2}}, \tag{2.6}$$

with L being the characteristic dimension to be determined from the matching of the leading edge and the downstream solutions.

2.2. Induced flow

We must now estimate the distortion of the shock wave necessary to produce a flow acceleration of ε required by the Eq. (2.5). The inviscid flow downstream of the shock wave is described by:

$$\begin{aligned} (\varrho u)_x + (\varrho v)_y = 0, \qquad \varrho(uu_x + vu_y) + p_x = 0, \\ \varrho(uv_x + vv_y) + p_y = 0, \quad u(p/\varrho^\gamma)_x + v(p/\varrho^\gamma)_y = 0. \end{aligned} \tag{2.7}$$

In the von Mises coordinates, which use the stream function as the normal coordinate, we have:

$$\frac{\partial y}{\partial x} = v/u, \quad \frac{\partial y}{\partial \psi} = \frac{1}{\varrho u}, \quad \frac{\partial v}{\partial x} + \frac{\partial p}{\partial \psi} = 0,$$

$$u^2 + v^2 + \frac{2\gamma}{\gamma - 1}\frac{p}{\varrho} = 1, \quad p/\varrho^\gamma = f(\psi).$$

Normalization of the above equations by the upstream values of density and velocity does not change anything; and from now on the equations will be considered to be normalized even though old symbols will be retained. We begin the analysis by assuming

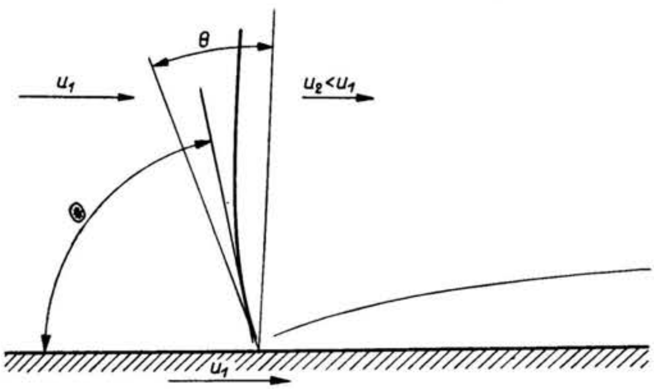

Fig. 2.

that a strong normal shock wave is perturbed smoothly in such a way that the leading terms for the shock wave angle (Fig. 2) may be taken to be:

$$\sin^2 \oplus = 1 - \theta^2 + f(x, \psi)$$

([1]) Substituting (2.4) into (2.3) and taking $y = \delta^*$, we obtain:

$$\varepsilon = \frac{\delta^*}{\sqrt{2XL}},$$

which, after using (2.2) gives (2.6).

The Rankine-Hugoniot relations are:

$$(2.8)\qquad \begin{aligned} &p = p_2/\varrho_1 u_1^2 = \frac{2}{\gamma+1}(1-\theta^2), \qquad \varrho = \varrho_2/\varrho_1 = (\gamma+1)/\gamma-1),\\ &u = u_2/u_1 = (\gamma-1)/(\gamma+1)+2\theta^2/(\gamma+1), \qquad v = v_2/u_1 = -2\theta/(\gamma+1). \end{aligned}$$

The solution is in the form:

$$u = u_0+\theta^2 u_{\rm II}, \quad v = \theta v_{\rm I}, \quad \varrho = \varrho_0+\theta^2\varrho_{\rm II}, \quad p = p_0+\theta^2 p_{\rm II}, \quad y = y_0+\theta y_{\rm I}+\theta^2 y_{\rm II}.$$

Substitution of the above relations into the full problem with boundary conditions results in:

$$(2.9)\qquad \begin{aligned} u_0 &= (\gamma-1)/(\gamma+1), \quad p_0 = 2/(\gamma+1),\\ \varrho_0 &= (\gamma+1)/(\gamma-1), \quad y_0 = \psi, \end{aligned}$$

for the basic solution, and

$$(2.10)\qquad \begin{aligned} &u_{\rm II} = 2/(\gamma+1), \quad \varrho_{\rm II} = 0, \quad v_{\rm I} = -2/(\gamma+1),\\ &p_{\rm II} = -2/(\gamma+1), \quad y_{\rm I} = -2x/(\gamma-1), \quad y_{\rm II} = -2\psi/(\gamma-1) \end{aligned}$$

for the perturbations due to the distortion of the shock wave. We have the immediate result(²):

$$(2.11)\qquad \theta = \left(\frac{\gamma-1}{2}\varepsilon\right)^{\frac{1}{2}}.$$

2.3. Displacement thickness effects

As was pointed out above, the boundary layer equations can be solved easily and accurately using the method of Ref. [7] and a very simple computer program. Having solved the equations, Δ^* can be evaluated according the Eq. (2.2)

The final quantity still to be determined is the characteristic dimension of the problem, L. We know from thin airfoil theory that the region affected by the leading edge disturbance is of the order of ε. Since the only physical dimension associated with the problem is the duct half width, and a dimensionless radius ε is affected by viscous effects, we conclude that D characterized the physical magnitude of the problem and set $L = D$. Another argument in favor of the duct half width as the characteristic dimension is that mathematically our solution breaks down as $\varepsilon \to 1.0$, which means physically that the boundary layer fills the whole duct and our basic assumptions break down. The value of the shock wave inclination angle is then finally:

$$(2.12)\qquad \theta = \left(\frac{\gamma-1}{2}\right)^{\frac{1}{2}}|\Delta^*|^{\frac{1}{2}}\mathrm{Re}_D^{-\frac{1}{4}},$$

(²) The Eq. $(2.8)_3$ can be transformed to the form:

$$u = \frac{u_2}{u_1} = \frac{\gamma-1}{\gamma+1}\left(1+\frac{2\theta^2}{\gamma-1}\right).$$

Comparing it with (2.5), using also (2.9) and taking $u_0 = U$, we obtain:

$$\varepsilon = \frac{2\theta^2}{\gamma-1},$$

which gives immediately (2.11).

with

$$\mathrm{Re}_D = \varrho_2 u_2 D/\mu_2. \tag{2.13}$$

The region affected by the viscous effects is

$$\frac{y^*}{D} = |\Delta^*|\,\mathrm{Re}_D^{-\frac{1}{2}}. \tag{2.14}$$

It should be noted that the inclination of the shock wave from normal is a weaker function of the Reynolds number than the region influenced by the boundary layer.

The above relations show that the deviation of the shock wave from normal is quite small for any flow in which some sort of rational boundary layer exists. With the condition of the tangency of the flow at the wall and a small deviation from normal away from the wall, it is clear that the popular assumption of the decay of the shock front to an acoustic pulse near the wall cannot be accepted.

3. Experimental results(³)

The experiments were performed in a 120 mm diameter shock tube 10·m long. Air, at an initial pressure of 0.05 Tr and initial temperature about 293°K was used as the test gas in which shock waves at Mach 3 and Mach 6 were generated. The intent was to obtain Mach numbers sufficiently high to justify the strong shock assumptions without getting into the problems of real gas effects. Under these conditions, the mean free path ahead of the shock front was about 1 mm, the shock wave thickness was about 4 mm, and the contact surface was never less that 50 mm from the shock wave at the test station.

The shape of the shock wave was studied in the vicinity of a glass plate placed in the shock tube (Fig. 3), using the electron beam attenuation technique, Refs. [12–16]. The

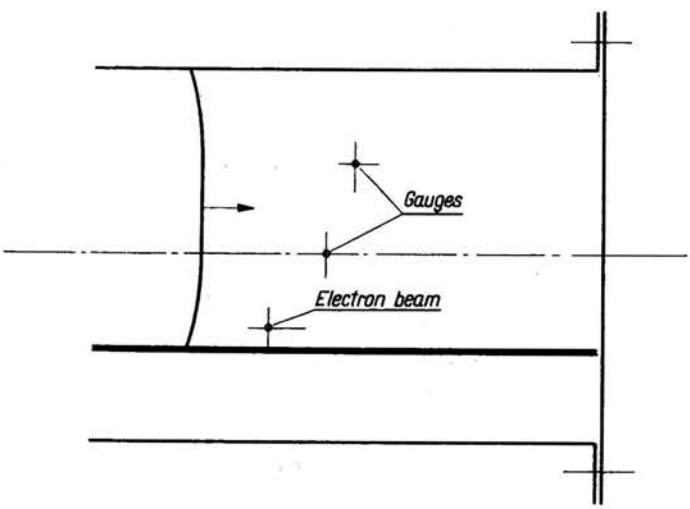

Fig. 3.

(³) Detailed description of the experiments may be found in Refs. [10 and 11].

electron beam, 0.5 mm dia, was generated with an ordinary TV type electron gun. Thin film heat transfer gages were placed in the shock tube to indicate the location of the shock wave and its inclination to the tube axis. The electron beam could be translated normal to the glass plate surface from a distance of 1.5 mm to a distance of 12 mm. The shape of the shock wave was determined in a series of measurements at various distances from the plate with identical initial shock tube conditions.

It is estimated that maximum error in any single measurement of the position of the shock is about 1 mm, which is less than 1/3 of the shock wave thickness.

4. Comparison of test data with theoretical predictions

A representative example of the comparison of calculated shape of the shock wave (solid line) with measured data (open circles) is shown in Fig. 4. Theoretical predictions

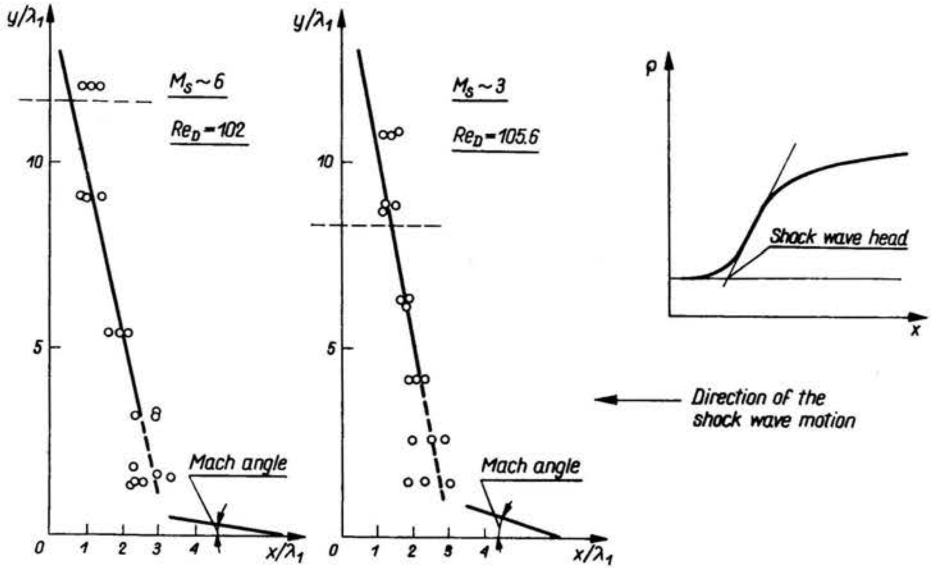

Fig. 4.

are seen to agree, as regards mean inclination of the shock wave, very closely with experiments. The agreement in the extent of the affected region (dashed lines) is not equally good.

5. Conclusions

The good agreement of the theoretical and test data indicates that a simple, but mathematically and physically consistent, continuum flow model is adequate for accurate estimates of the distortion of a travelling shock wave by viscous effects in the wall region.

It is also concluded that the commonly held belief that a shock wave decays to an acoustic wave as the wall is approached has neither theoretical nor experimental basis in fact. The results presented here may be used to estimate the actual transverse disturbances of the shock tube flow due to the distortion of the shock wave. The important fact is that these disturbances are always small.

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