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Motion of a crack in antiplane state of strain of an elastic strip

M. MATCZYŃSKI (WARSZAWA*)

THE PAPER presents the solution of the quasi-static problem of an infinite elastic medium weakened by an infinite number of semi-infinite, rectilinear, parallel and equally spaced cracks which are subjected to identic loads satisfying the conditions of antiplane state of strain. The stress intensity factors at the crack tips are determined for arbitrary loading of the cracks which are assumed to propagate at a constant velocity. Several particular cases are discussed. The solutions are used to discuss the problem of an infinite elastic strip with stress-free edges of the crack and with prescribed displacements at the boundary surfaces.

W pracy przedstawiono rozwiązanie quasi-statycznego zagadnienia dla nieograniczonego ośrodka sprężystego, osłabionego nieskończoną liczbą półnieskończonych, prostoliniowych, równoległych i jednakowo od siebie odległych szczelin o brzegach obciążonych identycznie i w sposób zapewniający warunki antypłaskiego stanu odkształcenia. Wyznaczono współczynniki intensywności naprężenia w końcach szczelin przy dowolnym obciążeniu oraz przy założeniu, że szczeliny rozprzestrzeniają się ze stałą prędkością. Przedyskutowano szereg przypadków szczególnych. Wyprowadzone rozwiązania wykorzystano do dyskusji zagadnienia pasma sprężystego ze szczeliną o brzegach swobodnych; na powierzchniach pasma dane są wartości przemieszczenia.

В работе рассмотрена квазистатическая задача о неограниченной упругой среде, ослабленной бесконечным числом прямолинейных параллельных и одинаково отстоящих друг от друга трещин, края которых подвержены воздействию одинаковых нагрузок, удовлетворяющих условиям антиплоского деформированного состояния. Определен коэффициент интенсивности напряжений для произвольной нагрузки на краю трещины. Полученные результаты тщательно изучены для некоторых случаев нагрузок. В случаях, когда края трещин подверженны действию постоянной нагрузки на всей длине, решение полной краевой задачи дается в замкнутом виде. Это решение используется затем для решения квазистатической задачи о бесконечной полосе из упругого материала, ослабленной полубесконечной трещиной. Рассмотрен случай, когда края трещины свободны от нагрузок, а на поверхности полосы заданы постоянные перемещения.

1. General formulation

IT IS KNOWN that the vector of elastic displacement u in antiplane state of strain may be expressed, in a rectangular coordinate system (x_1, x_2, x_3), in the form

$$\mathbf{u} = [0, 0, w(x_1, x_2, t)].$$

The non-vanishing components of this state of strain are thus given by the following relations:

$$\varepsilon_{13} = \frac{1}{2}\frac{\partial w}{\partial x_1}, \qquad \varepsilon_{23} = \frac{1}{2}\frac{\partial w}{\partial x_2},$$
$$\sigma_{13} = \mu\frac{\partial w}{\partial x_1}, \qquad \sigma_{23} = \mu\frac{\partial w}{\partial x_2}, \tag{1.1}$$

Here μ is the shear modulus.

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In the case of vanishing body forces the equations of motion are reduced to the single equation

$$\nabla^2 w = \frac{1}{c_2^2}\frac{\partial^2 w}{\partial t^2}, \tag{1.2}$$

where $c^2 = \mu/\varrho$ is the square of velocity of propagation of transversal elastic waves. If the case considered is of such character that the fixed coordinate system (x_1, x_2, x_3) may be replaced by the convectional system (x, y, z),

$$x_1 = x+ct, \quad x_2 = y, \quad x_3 = z, \tag{1.3}$$

where c is the velocity of motion of the system (x, y, z), then the Eqs. of motion (1.2) take the form

$$\beta^2\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2} = 0, \tag{1.4}$$

Here $\beta^2 = 1-c^2/c_2^2$.

In this paper we shall apply the complex integral Fourier transform defined by the following relations:

$$\begin{aligned} F(\alpha, y) &= \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} f(x, y)e^{i\alpha x}dx, \\ f(x, y) &= \frac{1}{\sqrt{2\pi}}\int_{-\infty+ic}^{\infty+ic} F(\alpha, y)e^{-i\alpha x}d\alpha, \end{aligned} \tag{1.5}$$

where the transform parameter α is a complex variable and the path of integration in Eq. $(1.5)_2$ lies within the strip $\alpha_1 < \operatorname{Im}\alpha < \alpha_2$ which represents the region of regularity of $F(\alpha, y)$.

From the theory of integral Fourier transforms it is known [1] that the function $F(\alpha, y)$ may also be represented in the form

$$F(\alpha, y) = F^-(\alpha, y)+F^+(\alpha, y), \tag{1.6}$$

where the functions

$$\begin{aligned} F^-(\alpha) &= \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{0} f(x, y)e^{i\alpha x}dx, \\ F^+(\alpha) &= \frac{1}{\sqrt{2\pi}}\int_{0}^{\infty} f(x, y)e^{i\alpha x}dx, \end{aligned} \tag{1.7}$$

are analytic in the lower $\operatorname{Im}\alpha < \alpha_2$ and upper $\operatorname{Im}\alpha > \alpha_1$ half-planes of the complex variable α, respectively.

Applying the integral Fourier transform (1.5) to the Eqs. (1.1), (1.4) we obtain

$$\Sigma_{xz}(\alpha, y) = -i\alpha\mu W(\alpha, y), \quad \Sigma_{yz}(\alpha, y) = \mu \frac{dW(\alpha, y)}{dy},$$
(1.8)
$$\frac{d^2 W(\alpha, y)}{dy^2} - \alpha^2 \beta^2 W(\alpha, y) = 0.$$

Solution of the Eq. $(1.8)_3$ yields then the Fourier transforms of the displacement w and stresses σ_{xz}, σ_{yz},

$$W(\alpha, y) = A(\alpha)\operatorname{sh}\alpha\beta y + B(\alpha)\operatorname{ch}\alpha\beta y,$$
(1.9)
$$\Sigma_{xz}(\alpha, y) = -i\alpha\mu[A(\alpha)\operatorname{sh}\alpha\beta y + B(\alpha)\operatorname{ch}\alpha\beta y],$$
$$\Sigma_{yz}(\alpha, y) = \mu\alpha\beta[A(\alpha)\operatorname{ch}\alpha\beta y + B(\alpha)\operatorname{sh}\alpha\beta y].$$

The unknown functions $A(\alpha)$, $B(\alpha)$ are to be determined from the boundary conditions of the particular problem considered.

2. Infinite medium with cracks

Let us consider the infinite elastic medium weakened by an infinite number of semi-infinite, rectilinear, parallel and uniformly spaced cracks (Fig. 1). The edges of the cracks are assumed to be loaded by identic forces; the cracks and their loads propagate at a constant velocity $c < c_2$ along the x_1-axis of the fixed rectangular coordinate system (x_1, x_2, x_3).

Owing to the symmetry of the problem it may be reduced to the problem of an infinite elastic strip of thickness $2h$ weakened in the middle plane $x_2 = 0$ by a semi-infinite crack

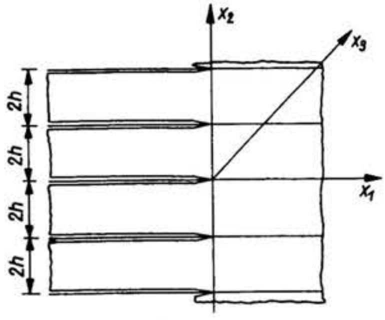

FIG. 1.

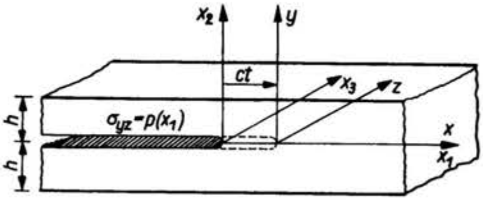

FIG. 2.

$x_1 < 0$ (Fig. 2). The surfaces $x_2 = \pm h$ are rigidly clamped while the surfaces of the crack are subject to the action of forces $\sigma_{23} = p(x_1)$. Both the loads and the crack itself propagate at a constant velocity $c < c_2$ along the x_1-axis.

The latter problem, the symmetry conditions being used again, is reduced to the problem of an infinite elastic strip with discontinuous boundary conditions which, by means of the convectional reference frame (x, y, z) defined by Eqs. (1.3), is now expressed in the form

$$\begin{aligned} w(x,y) &= 0 \quad &\text{for} \quad |x| < \infty,\ y = h, \\ w(x,y) &= 0 \quad &\text{for} \quad x > 0,\ y = 0, \\ \sigma_{yz}(x,y) &= p(x) \quad &\text{for} \quad x < 0,\ y = 0. \end{aligned}$$

Application of the Fourier transforms (1.5) and the relations (1.6), (1.7) yields the following Wiener-Hopf equation

$$W^-(\alpha) = -\frac{1}{\mu\beta}\frac{\operatorname{tg}\alpha\beta h}{\alpha}[\Sigma_{yz}^+(\alpha)+P(\alpha)], \tag{2.1}$$

where

$$P(\alpha) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{0} p(x)e^{i\alpha x}dx. \tag{2.2}$$

The region of existence of that equation is the region of regularity of the functions appearing in Eq. (2.1), i.e. the strip $-\pi/2\beta h < -\varepsilon < \operatorname{Im}\alpha < 0$. Equation (2.1) will be solved by means of the method of factorization.

To this end, let us represent the function

$$H(\alpha) = \frac{\operatorname{tg}\alpha\beta h}{\alpha} \tag{2.3}$$

in the form

$$H(\alpha) = \frac{\beta h}{\pi}H^-(\alpha)H^+(\alpha), \tag{2.4}$$

where

$$H^-(\alpha) = \frac{\Gamma\left(\frac{1}{2}-\frac{i\alpha\beta h}{\pi}\right)}{\Gamma\left(1-\frac{i\alpha\beta h}{\pi}\right)}, \quad H^-(\alpha) = H^+(-\alpha), \tag{2.5}$$

the functions $H^{\pm}(\alpha)$ being regular and non-zero in the respective halfplanes $\operatorname{Im}\alpha > -\pi/2\beta h$ and $\operatorname{Im}\alpha < \pi/2\beta h$. Using Eqs. (2.4) and (2.5), the Eq. (2.1) takes now the form

$$W^-(\alpha) = -\frac{h}{\pi\mu}H^-(\alpha)H^+(\alpha)[\Sigma_{yz}^+(\alpha)+P(\alpha)].$$

Applying the procedure used in [2] this equation may be written as

$$-\frac{\pi\mu}{h}\frac{W^-(\alpha)}{H^-(\alpha)} = H^+(\alpha)\Sigma_{yz}^+(\alpha)+E(\alpha), \tag{2.6}$$

where

$$E(\alpha) = H^+(\alpha)P(\alpha). \tag{2.7}$$

If the function $E(\alpha)$ is regular at least in the strip of existence of Eq. (2.1), it may be represented in the form

$$E(\alpha) = E^{+}(\alpha) - E^{-}(\alpha), \tag{2.8}$$

where

$$\begin{aligned} E^{-}(\alpha) &= \frac{1}{2\pi i} \int_{-\infty - i\delta_1}^{\infty - i\delta_1} \frac{E(\zeta)}{\zeta - \alpha} d\zeta, \\ E^{+}(\alpha) &= \frac{1}{2\pi i} \int_{-\infty - i\delta_2}^{\infty - i\delta_2} \frac{E(\zeta)}{\zeta - \alpha} d\zeta. \end{aligned} \tag{2.9}$$

Here $0 < \delta_1 < \delta_2 < \pi/2\beta h$, and the functions $E^{\pm}(\alpha)$ are regular in the respective half-planes $\operatorname{Im}\alpha > -\pi/2\beta h$ and $\operatorname{Im}\alpha < 0$. Using now the relation (2.8), we transform the Eq. (2.6),

$$-\frac{\pi\mu}{h} \frac{W^{-}(\alpha)}{H^{-}(\alpha)} + E^{-}(\alpha) = H^{+}(\alpha)\Sigma_{yz}^{+}(\alpha) + E^{+}(\alpha).$$

Both sides of this equation represent functions which are regular and non-zero in the respective halfplanes $\operatorname{Im}\alpha > -\pi/2\beta h$ and $\operatorname{Im}\alpha < 0$, and thus the Liouville theorem enables us to determine the solutions,

$$\begin{aligned} W^{-}(\alpha) &= \frac{h}{\pi\mu} H^{-}(\alpha) E^{-}(\alpha), \quad \text{reg. for} \quad \operatorname{Im}\alpha < 0, \\ \Sigma_{yz}^{+}(\alpha) &= -\frac{E^{+}(\alpha)}{H^{+}(\alpha)}, \quad \text{reg. for} \quad \operatorname{Im}\alpha > -\pi/2\beta h. \end{aligned} \tag{2.10}$$

From the point of view of the crack stability, the most interesting value is the stress intensity factor of σ_{yz} [3].

This factor as also the crack edge displacements will be determined by means of the Abel theorem concerning the Fourier transforms [4] which make it possible to determine the behaviour of the inverse Fourier transforms at $|x| \to \infty$ and $|x| \to 0$ from the behaviour of the corresponding transforms at the respective points $|\alpha| \to 0$ and $|\alpha| \to \infty$.

Using the relation (2.7) and the fact that $E^{\pm}(\alpha)$ defined by Eqs. (2.9) are assumed to be regular functions for $-\pi/2\beta h < \operatorname{Im}\alpha < 0$, they may be represented in the form

$$E^{\pm}(\alpha) = -\frac{1}{\alpha}\left[B - \frac{1}{2\pi i} \int_{-\infty - i\delta}^{\infty - i\delta} \frac{\zeta E(\zeta)}{\zeta - \alpha} d\zeta\right], \tag{2.11}$$

where

$$B = \frac{1}{2\pi i} \int_{-\infty - i\delta}^{\infty - i\delta} E(\zeta) d\zeta, \quad \delta_1 < \delta < \delta_2. \tag{2.12}$$

Consequently, basing on the properties of $E^+(\alpha)$ and using Eq. (2.11) as also the fact that

$$\frac{\Gamma(\alpha+\lambda)}{\Gamma(\alpha)} \sim \alpha^{\lambda}, \quad |\alpha| \to \infty,$$

it may be demonstrated that the functions $W^-(\alpha)$ and $\Sigma^+_{yz}(\alpha)$ for $|\alpha| \to \infty$ assume the form

$$W^-(\alpha) = -\frac{B}{\mu}\sqrt{\frac{h}{i\pi\beta}}\frac{1}{\alpha\sqrt{\alpha}}, \quad \Sigma^+_{yz}(\alpha) = B\sqrt{\frac{-i\beta h}{\pi}}\frac{1}{\sqrt{\alpha}}. \tag{2.14}$$

This, on the basis of the Abel theorem quoted before, yields the following relations:

$$\begin{aligned} w(x) &= \frac{2N}{\mu\beta}\sqrt{-x} \quad &&\text{for} \quad x \to (-0), \\ \sigma_{yz}(x) &= \frac{N}{\sqrt{x}} &&\text{for} \quad x \to (+0). \end{aligned} \tag{2.15}$$

Here N, the stress intensity factor, is equal to

$$N = -iB\sqrt{\frac{2\beta h}{\pi}}. \tag{2.16}$$

This result makes it possible to establish the exact value of the stress intensity factor for an arbitrary loading of the crack edges.

3. Particular cases

3.1. Concentrated force

Let us consider the case when the edges $y = 0$ of the crack are loaded at $x = -l$ by a concentrated force of a constant intensity P. Then $p(x) = P\delta(x+l)$ and from Eq. (1.7) it is seen that

$$P(\alpha) = \frac{P}{\sqrt{2\pi}}e^{-i\alpha l}, \tag{3.1}$$

while the function $E(\alpha)$ described by Eq. (2.7) is a function regular within the strip $-\pi/2\beta h < \operatorname{Im}\alpha < 0$. Using the Eqs. (2.5), (2.7), (3.1), (2.12), we obtain the relation (2.16) in the form

$$N(P, \lambda) = -\frac{Pe^{\frac{-\pi\lambda}{2\beta}}}{\sqrt{\beta h}}\frac{1}{2\pi i}\int_{\varepsilon-i\infty}^{\varepsilon+i\infty}\frac{\Gamma(p)}{\Gamma\left(p+\frac{1}{2}\right)}e^{\frac{\pi\lambda}{\beta}p}\,dp,$$

where $\lambda = l/h$ and $0 < \varepsilon < 1/2$. The corresponding integration [5] yields now the stress intensity factor N in the case of concentrated loading of the crack edges,

$$N(P, \lambda) = -\frac{P}{\sqrt{\pi\beta h[\exp(\pi\lambda/\beta)-1]}} \tag{3.2}$$

with the notation $\lambda = l/h$. The stress intensity factor $N(P, \lambda)$ as a function of λ and of the crack propagation velocity is demonstrated graphically in Fig. 3.

It proves interesting to compare the conclusions following from Eq. (3.2) with the result obtained in [6], where a similar problem has been considered under the assumption of stress-free strip surfaces $x = \pm h$. It may be seen that with increasing crack propagation velocities the stress intensity factor (3.2) decreases, in contrast to the case considered in [6].

A similar effect is observed when the thickness of the strip decreases. In the case of a strip with stress-free edges, the stress intensity factor increases, and in the case considered

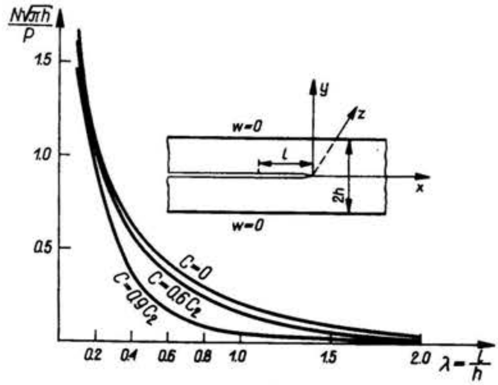

FIG. 3.

here — it decreases. Passing with the strip thickness to the limit $h \to \infty$ it is found, however, that in the both cases we obtain the same solution for a plane weakened by a crack loaded by concentrated forces P, the stress intensity factor being equal to

$$N(P, \lambda) = -\frac{P}{\pi\sqrt{l}} \quad \text{for} \quad h \to \infty.$$

3.2. Arbitrary loading

The considerations presented thus far may be generalized to the case of an arbitrary loading $p(x)$ of the crack edges. The formula (3.2) is then treated as a Green function what makes it possible to write the stress intensity factor N for an arbitrary distribution $p(x/h)$ in the form

$$N = -\sqrt{\frac{h}{\pi\beta}} \int_0^\infty \frac{p(\lambda)}{\sqrt{\exp(\pi\lambda/\beta)-1}}\, d\lambda.$$

If, for instance, the edges of the crack are subjected to the action of a constant load $p(x) = p_0$ along the interval $-l_1 < x < l_0$, then

$$N = -\frac{2p_0\sqrt{\beta h}}{\pi\sqrt{\pi}}\left[\operatorname{arctg}\sqrt{\exp(\pi\lambda_1/\beta)-1} - \operatorname{arctg}\sqrt{\exp(\pi\lambda_0/\beta)-1}\right], \tag{3.3}$$

where $\lambda_0 = l_0/h$, $\lambda_1 = l_1/h$. Figures 4 and 5 illustrate the behaviour of that function for various values of λ_1 and $\lambda_0 = k\lambda_1$ and various crack propagation velocities.

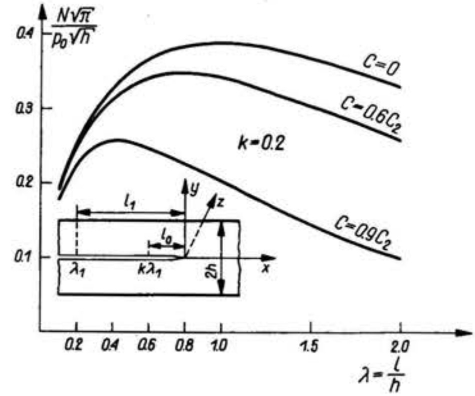

FIG. 4.

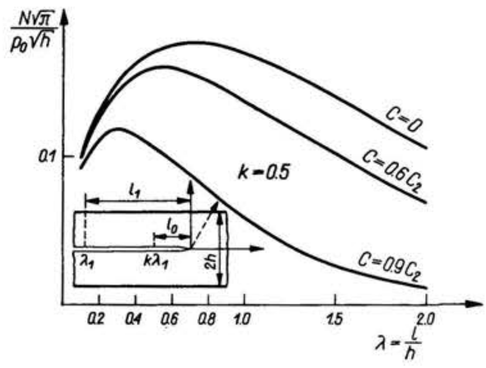

FIG. 5.

3.3. Constant load $p(x) = p$ for $x < 0$.

Passing to the limits with $\lambda_0 \to 0$ and $\lambda_1 \to \infty$, we obtain the stress intensity factor N in the case when the edges of the crack are loaded on their entire length by $p(x) = p_0$, and then

$$N = -p_0 \sqrt{\frac{\beta h}{\pi}}. \tag{3.4}$$

In such a case, the exact solution of the complete boundary value problem can be determined.

From the Eq. (22), we obtain

$$P(\alpha) = \frac{k}{\alpha}, \quad \text{where} \quad k = \frac{p_0}{i\sqrt{2\pi}}$$

while $E(\alpha)$ defined by Eq. (2.7) is a function regular within the strip $-\pi/2\beta h < \operatorname{Im}\alpha < 0$. By means of the relations (2.9), (2.11), we may therefore obtain the functions $E^{\pm}(\alpha)$,

$$E^-(\alpha) = -\frac{kH^+(0)}{\alpha} \qquad \text{reg. for} \quad \operatorname{Im}\alpha < 0,$$

$$E^+(\alpha) = \frac{k}{\alpha}[H^+(\alpha) - H^+(0)] \qquad \text{reg. for} \quad \operatorname{Im}\alpha > -\frac{\pi}{2\beta h}.$$

whence, using the relations (2.10), we determine

$$\begin{aligned} W^-(\alpha) &= -\frac{khH^+(0)}{\pi\mu}\frac{H^-(\alpha)}{\alpha} \qquad \text{reg. for} \quad \operatorname{Im}\alpha < 0, \\ \Sigma^+_{yz}(\alpha) &= \frac{k}{\alpha}\left[\frac{H^+(0)}{H^+(\alpha)} - 1\right] \qquad \text{reg. for} \quad \operatorname{Im}\alpha > -\frac{\pi}{2\beta h}, \end{aligned} \tag{3.5}$$

$H^{\pm}(\alpha)$ being defined by Eqs. (2.5).

Since the functions $W^{-}(\alpha)$ and $\Sigma_{yz}^{+}(\alpha)$ are regular functions in the respective lower ($\mathrm{Im}\,\alpha < 0$) and upper $\mathrm{Im}\,\alpha > -\pi/2\beta h$ halfplanes of the complex variable α, application of the Eqs. $(1.5)_2$ and (3.5) yields

$$w(x) = -\frac{p_0 h}{\mu\sqrt{\pi}}\frac{1}{2\pi i}\int\limits_{-\infty-i\delta}^{\infty-i\delta}\frac{H^{-}(\alpha)}{\alpha}e^{-i\alpha x}d\alpha \qquad \text{for} \quad x<0,$$

$$\sigma_{yz}(x) = \frac{p_0\sqrt{\pi}}{2\pi i}\int\limits_{-\infty+i\delta}^{\infty+i\delta}\frac{e^{-i\alpha x}}{\alpha H^{+}(\alpha)}d\alpha - \frac{p_0}{2\pi i}\int\limits_{-\infty+i\delta}^{\infty+i\delta}\frac{e^{-i\alpha x}}{\alpha}d\alpha \qquad \text{for} \quad x>0.$$

Performing then the integration [5] we arrive at the conclusion that the displacement of the upper edge of the crack and the stresses σ_{yz} along the positive x-axis are expressed by the following formulae:

$$w(x) = -\frac{2p_0 h}{\pi\mu}\arccos[\exp(\pi x/2\beta h)] \qquad \text{for} \quad x<0,$$

$$\sigma_{yz}(x) = p_0\left[1-\frac{1}{\sqrt{1-\exp(-\pi x/\beta h)}}\right] \qquad \text{for} \quad x>0.$$

Passing to the limit with $|x|\to 0$ we conclude that the displacement w and stress σ_{yz} in the neighbourhood of the crack tip are described by the formulae established before, Eqs. (2.15), and the stress intensity factor N is given by the Eq. (3.4).

From the relation (3.6) it additionally results that in the case when $|x|\to\infty$,

$$w(x) = -\frac{p_0 h}{\mu} \qquad \text{for} \quad x\to-\infty,$$

$$\sigma_{yz}(x) = 0 \qquad \text{for} \quad x\to+\infty.$$

3.4. Crack with stress-free edges

To conclude our considerations it should be mentioned that the relations (3.6) obtained here may be used to solve the following problem. Let an infinite elastic strip weakend

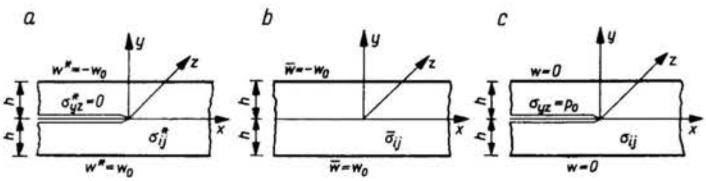

FIG. 6.

in its middle surface by a semi-infinite crack have prescribed displacements $w^*(x, \pm h) = \mp w_0 = \text{const}$ at the boundary surfaces $y = \pm h$; the edges of the crack are stress-free (Fig. 6a). By means of the superposition principle the solution may be represented as a sum of solutions for a continuous strip having prescribed displacement $\bar{w} = \mp w_0$ on the surfaces $y = \pm h$ (Fig. 6b) and for a strip with a crack acted on by a constant load $\sigma_{yz} = p_0$ (Fig. 6c).

The displacement w and stresses $\bar{\sigma}_{xz}, \bar{\sigma}_{yz}$ in the problem shown in Fig. 6b are calculated by means of the relations

$$\begin{gathered}\bar{w}(x, y) = -\varkappa y,\\ \bar{\sigma}_{xz}(x, y) = 0, \quad \bar{\sigma}_{yz}(x, y) = -p_0 = -\mu x,\end{gathered} \tag{3.7}$$

where $\varkappa = w_0/h$. Combining the corresponding solutions (3.6) and (3.7), the displacement w^* at the upper edge of the crack and the stress σ^*_{yz} along the positive x-axis assume, in the case described in Fig. 6a, the form

$$\begin{aligned} w^*(x) &= -\frac{2w_0}{\pi}\arccos[\exp(\pi x/2\beta h)] \quad &\text{for} \quad x < 0,\\ \sigma^*_{yz}(x) &= -\frac{\mu\varkappa}{\sqrt{1-\exp(-\pi x/\beta h)}} \quad &\text{for} \quad x > 0.\end{aligned} \tag{3.8}$$

Using the relations (3.4), $(3.7)_2$ or passing to the limit $x \to (+0)$ in Eq. $(3.8)_2$, we obtain the stress intensity factor N^*,

$$N^* = -\mu w_0 \sqrt{\beta/\pi h}.$$

From this formula it follows that — in contrast to the problem considered in Sec. 3.3 — the stress intensity factor N increases with decreasing values of the thickness of the strip.

In order to obtain the solutions of the corresponding static problems in all the formulae derived in this paper it should be assumed that $\beta = 1$.

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POLISH ACADEMY OF SCIENCES
INSTITUTE OF FUNDAMENTAL TECHNOLOGICAL RESEARCH
and
CHAIR OF MECHANICS A, MUNICH TECHNOLOGICAL UNIVERSITY.

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