Archives of Mechanics • Archiwum Mechaniki Stosowanej • 25, 6, pp. 975–984, Warszawa 1973

On plane micropolar thermoelasticity in multiply-connected domains and its application

Y. TAKEUTI (HAMAMATSU)

APPLYING Nowacki's theory [1], the present paper is concerned with some consideration of plane micropolar thermoelasticity in finite multiply-connected domains.

Praca niniejsza przedstawia, w oparciu o teorię Nowackiego [1], pewne rozważanie dotyczące płaskich zagadnień mikropolarnej termosprężystości dla skończonych obszarów wielospójnych.

В статье изложены некоторые результаты, полученные на основе теории Новацкого [1], относящиеся к плоским задачам микрополярной термоупругости для конечных многосвязных областей.

1. Introduction

THE PAPER begins with a presentation of fundamental relations of plane micropolar thermoelasticity for finite multiply-connected domains. In the second part with a view to illustrating the foregoing treatment, we deal with the steady thermal stresses in a regular polygonal prism with a hole, within the framework of micropolar thermoelasticity. Numerical work is carried out for the distribution of thermal stresses and couple-stresses in a square prism with a central circular hole.

2. Analysis

2.1. Basic equations for plane micropolar thermoelasticity

The fundamental stress-strain relations in plane strain problems are:

$$
\begin{aligned}
\gamma_{11} &= u_{1,1} = \frac{1}{2\mu}\left\{\sigma_{11} - \frac{\lambda}{2(\lambda+\mu)}(\sigma_{11}+\sigma_{22})\right\} + \frac{1}{2(\lambda+\mu)}\nu_1\tau,\\
\gamma_{22} &= u_{2,2} = \frac{1}{2\mu}\left\{\sigma_{22} - \frac{\lambda}{2(\lambda+\mu)}(\sigma_{11}+\sigma_{22})\right\} + \frac{1}{2(\lambda+\mu)}\nu_1\tau,\\
\gamma_{12} &= u_{2,1} - \omega_3 = \frac{1}{4\mu}(\sigma_{12}+\sigma_{21}) + \frac{1}{4\alpha}(\sigma_{12}-\sigma_{21}),\\
\gamma_{21} &= u_{1,2} + \omega_3 = \frac{1}{4\mu}(\sigma_{21}+\sigma_{12}) + \frac{1}{4\alpha}(\sigma_{21}-\sigma_{12}),\\
\varkappa_{13} &= \omega_{3,1} = \frac{1}{4\gamma}(\mu_{13}+\mu_{31}) + \frac{1}{4\varepsilon}(\mu_{13}-\mu_{31}) = \frac{1}{\gamma+\varepsilon}\mu_{13} = \frac{1}{\gamma-\varepsilon}\mu_{31},\\
\varkappa_{23} &= \omega_{3,2} = \frac{1}{4\gamma}(\mu_{23}+\mu_{32}) + \frac{1}{4\varepsilon}(\mu_{23}-\mu_{32}) = \frac{1}{\gamma+\varepsilon}\mu_{23} = \frac{1}{\gamma-\varepsilon}\mu_{32},
\end{aligned}
\tag{2.1}
$$

where

γ_{ij} components of strain,
u_i components of displacement,
$\varkappa_{ij}$ components of curvature,
ω_i components of rotation,
σ_{ij} components of stress,
μ_{ij} components of couple-stress,
λ, μ Lame's constants,
τ temperature change,
$\alpha, \beta, \gamma, \varepsilon$ new material constants,
ν_1 material constant $= \alpha_t E/(1-2\nu)$,
ν Poisson's ratio,
α_t coefficient of thermal expansion,
$'i$ partial differentiation with respect to i.

The stress components in the form of stress functions are given by [1]:

(2.2)
$$\sigma_{11} = \varphi_{,22}-\psi_{,12}, \quad \sigma_{21} = -\varphi_{,21}+\psi_{,11}, \quad \sigma_{12} = -(\varphi_{12}+\psi_{,22}),$$
$$\sigma_{22} = \varphi_{,11}+\psi_{,21}, \quad \mu_{13} = \psi_{,1}, \quad \mu_{23} = \psi_{,2}.$$

The fundamental differential equations for ϕ and ψ and the conjugate relations are:

(2.3)
$$\Delta\Delta\phi+k\Delta\tau = 0,$$

(2.4)
$$(\psi-A^2\Delta\psi)_{,1} = -2B^2\{(1-\nu)\Delta\phi+\alpha_t E\tau\}_{,2},$$
$$(\psi-A^2\Delta\psi)_{,2} = 2B^2\{(1-\nu)\Delta\phi+\alpha_t E\tau\}_{,1},$$

where

k material constant $= E\alpha_t/(1-\nu)$,
A^2 new material constant $= (\gamma+\varepsilon)(\mu+\alpha)/4\mu\alpha$,
B^2 new material constant $= (\gamma+\varepsilon)/4\mu$;

Eqs. (2.3), (2.4) may be reduced to

(2.5)
$$\Delta(\psi-A^2\Delta\psi) = 0.$$

The boundary conditions are given by

$$P_1 = \sigma_{11}n_1+\sigma_{21}n_2, \quad P_2 = \sigma_{12}n_1+\sigma_{22}n_2,$$
$$g_3 = \mu_{13}n_1+\mu_{23}n_2,$$

where

P_i components of surface traction,
g_3 component of surface moment,
n_i component of direction cosine of the normal to the surface.

Now, let us consider the general problem of micropolar thermoelasticity when the cross-section of the body is multiply-connected. Let S be a connected region bounded by $n+1$ non-intersecting contours $L_0, L_1, \ldots, L_n$ of which L_0 contains all the others as shown in Fig. 1. As shown in our previous paper [2], the boundary value of ϕ at a variable point P_i on the contour L_i becomes:

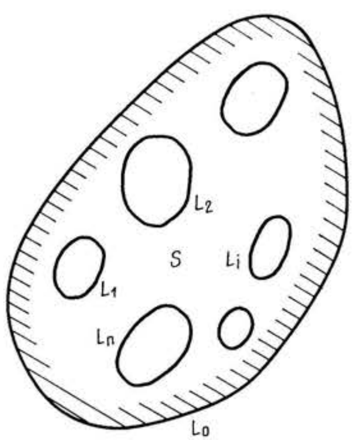

FIG. 1. Multiply-connected domain bounded by smooth non-intersecting contours.

$$(2.6)\qquad [\phi]_{P_i} = -\int_0^{P_i} dx_1 \int_{B_i}^{Q_i} P_2\, ds + \int_0^{P_i} dx_2 \int_{A_i}^{Q_i} P_1 ds + \int_0^{P_i} g_3\, ds + C_{1i}(x_1)_{P_i} + C_{2i}(x_1)_{P_i} + C_{3i}.$$

Moreover, the derivatives of ϕ and ψ on the contour become:

$$(2.7)\qquad \frac{\partial\phi}{\partial n} + \frac{\partial\psi}{\partial s} = -\int_{B_i}^{Q_i} P_2\, ds \cos(nx_1) + \int_{A_i}^{Q_i} P_1\, ds \cos(nx_2) + C_{2i}\cos(nx_1) + C_{1i}\cos(nx_2).$$

For a simply-connected domain, it is permissible to take these constants as zero. However, for multiply-connected regions, the constants C_{1i}, C_{2i} and C_{3i} generally assume different values on each boundary curve, and then additional boundary conditions are required to determine these constants. For this purpose, these constants must be so chosen that the displacement and the rotation may be single-valued. The condition which makes the change in rotation for an arbitrary path of integration (starting at a certain point and returning to the same point after including the inner boundary L_i) single-valued is

$$\oint_{L_i} d\omega_3 = \oint_{L_i} (\omega_{3,1}\, dx_1 + \omega_{3,2}\, dx_2) = \oint_{L_i} [(\gamma_{21,1} - \gamma_{11,2})\, dx_1 + (\gamma_{22,1} - \gamma_{12,2})\, dx_2.$$

Using the (2.1) to introduce the stress-strain relation into the integrand, and expressing the strain in terms of the stress functions, we have

$$\oint_{L_i} d\omega_3 = \frac{\mu+\alpha}{4\mu\alpha} \oint_{L_i} \{(\Delta\psi)_{,1}\, dx_1 + (\Delta\psi)_{,2}\, dx_2\} + \frac{\lambda+2\mu}{4\mu(\lambda+\mu)} \oint_{L_i} \{-(\Delta\phi)_{,2}\, dx_1 + (\Delta\phi)_{,1}\, dx_2\} + \frac{\nu_1}{2(\lambda+\mu)} \oint_{L_i} (-\tau_{,2}\, dx_1 + \tau_{,1}\, dx_2).$$

Taking into account

$$\partial x_1/\partial s = -\partial x_2/\partial n, \qquad \partial x_2/\partial s = \partial x_1/\partial n,$$

the integral becomes:

$$\oint_{L_i} d\omega_3 = \frac{\mu+\alpha}{4\mu\alpha}\oint_{L_i}\frac{\partial}{\partial s}(\Delta\psi)ds+\frac{\lambda+2\mu}{4\mu(\lambda+\mu)}\oint_{L_i}\frac{\partial}{\partial n}(\Delta\phi)ds+\frac{\nu_1}{2(\lambda+\mu)}\oint_{L_i}\frac{\partial\tau}{\partial n}ds.$$

Then, from the condition of $\oint_{L_i} d\omega_3 = 0$, we obtain the next relation on each of the contours L_i

$$\oint_{L_i}\left[\frac{\partial}{\partial s}\left(\frac{\Delta\psi}{2}\right)+\left(\frac{B}{A}\right)^2\frac{\partial}{\partial n}\{(1-\nu)\Delta\phi+E\alpha_t\tau\}\right]ds = 0. \tag{2.8}$$

The condition for the single-valuedness of the displacement u_1 can be written as:

$$\oint_{L_i} du_1 = \oint_{L_i}(u_{1,1}\,dx_1+u_{1,2}\,dx_2)$$
$$= \oint_{L_i}\{d(x_1\gamma_{11})+d(x_2\gamma_{21})-d(x_2\omega_3)-x_1\,d\gamma_{11}-x_2\,d\gamma_{21}+x_2\,d\omega_3\}.$$

If the strain and rotation are single-valued, then the first three terms in the integrand must vanish:

$$\oint_{L_i} du_1 = \oint_{L_i} -[(x_1\gamma_{11,1}+x_2\gamma_{11,2})dx_1+\{x_1\gamma_{11,2}+x_2(\gamma_{21,2}+\gamma_{12,2})-x_2\gamma_{22,1}\}dx_2].$$

Applying the stress-strain relations and the stress-function relations, rearrangement of the integral leads to

$$\oint_{L_i} du_1 = -\frac{\lambda+2\mu}{4\mu(\lambda+\mu)}\oint_{L_i}\left(x_1\frac{\partial}{\partial s}-x_2\frac{\partial}{\partial n}\right)\Delta\phi\,ds$$
$$-\frac{\nu_1}{2(\lambda+\mu)}\oint_{L_i}\left(x_1\frac{\partial}{\partial s}-x_2\frac{\partial}{\partial n}\right)\tau ds+\frac{1}{2\mu}[x_1(\phi_{,11}+\psi_{,12})]_{A_i}^{A_i}$$
$$+\frac{1}{2\mu}[x_2(\phi_{,12}+\psi_{,22})]_{A_i}^{A_i}-\frac{1}{2\mu}\left[\oint_{L_i}\phi_{,11}\,dx_1+\oint_{L_i}\phi_{,12}\,dx_2\right]$$
$$-\frac{1}{2\mu}\left[\oint_{L_i}\psi_{,12}\,dx_1+\oint_{L_i}\psi_{,22}\,dx_2\right].$$

If the stress is single-valued, then the third and fourth terms in the right-hand side of the equation must vanish. Moreover, the fifth and sixth terms may be written as:

$$\oint_{L_i} d(\phi_{,1}+\psi_{,2}) = [\phi_{,1}+\psi_{,2}]_{A_i}^{A_i} = F_2,$$

where F_2 is the resultant force in x_2-direction. On account of the equilibrium of the force on all the boundaries, F_2 must be zero. Hence we finally obtain the following condition for the single-valuedness of u_1:

$$\oint_{L_i}\left(x_1\frac{\partial}{\partial s}-x_2\frac{\partial}{\partial n}\right)\Delta\phi\,ds+\frac{E\overline{\alpha}}{1-\nu}\oint_{L_i}\left(x_1\frac{\partial}{\partial s}-x_2\frac{\partial}{\partial n}\right)\tau\,ds = 0. \tag{2.9}$$

Similar reasoning leads to the third condition for the single-valuedness of u_2:

$$\oint_{L_i}\left(x_2\frac{\partial}{\partial s}+x_1\frac{\partial}{\partial n}\right)\Delta\phi\,ds+\frac{E\alpha_t}{1-\nu}\oint_{L_i}\left(x_2\frac{\partial}{\partial s}+x_1\frac{\partial}{\partial n}\right)\tau\,ds = 0. \tag{2.10}$$

It is seen that the last two conditions (2.9) and (2.10) have the same forms as in classical thermoelasticity. From the above reasoning, it follows that the Eqs. (2.8)–(2.10) become the additional boundary conditions for the multiply-connected domains in micropolar plane thermoelasticity. Therefore, the values of constants $C_{mi} (m = 1, 2, 3)$ in the Eq. (2.6) are so determined as to satisfy $3i$ integral relations of the Eqs. (2.8)–(2.10).

For the plane polar coordinates (r, θ), the Eqs. (2.4) become:

$$\frac{\partial}{\partial r}(\psi - A^2 \Delta\psi) = -2B^2 \frac{1}{r}\frac{\partial}{\partial\theta}\{(1-\nu)\Delta\phi + E\alpha_t \tau\},$$

(2.11)

$$\frac{1}{r}\frac{\partial}{\partial\theta}(\psi - A^2\Delta\psi) = 2B^2\frac{\partial}{\partial r}\{(1-\nu)\Delta\phi + E\alpha_t\tau\},$$

where

$$\Delta = \partial^2/\partial r^2 + r^{-1}\cdot\partial/\partial r + r^{-2}\cdot\partial^2/\partial\theta^2.$$

Let a be the radius of an arbitrary hole in the multiply-connected domains, the non-dimensional coordinate of r being defined as

$$r_0 = r/a. \tag{2.12}$$

Taking these dimensionless polar coordinates, the general solution of steady heat conduction with no heat source becomes:

$$\tau = A_0^* + B_0^* \ln r_0 + \sum_{n=1}^{\infty}\{(A_n^* r_0^{-n} + B_n^* r_0^n)\cos n\theta + (C_n^* r_0^{-n} + D_n^* r_0^n)\sin n\theta\}. \tag{2.13}$$

In this case, the Eq. (2.3) naturally reduces to the well known biharmonic equation:

$$\Delta\Delta\phi = 0. \tag{2.14}$$

The general solution of the Eq. (2.14) is

$$\begin{aligned}\phi = {} & A_0 + B_0\ln r_0 + C_0 r_0^2 + D_0 r_0^2 \ln r_0 + (A_1 r_0^{-1} + B_1 r_0 + C_1 r_0 \ln r_0 + D_1 r_0^3)\cos\theta \\ & + (L_1 r_0^{-1} + M_1 r_0 + N_1 r_0 \ln r_0 + O_1 r_0^3)\sin\theta + \sum_{1=n}^{\infty}\{(A_n r_0^{-n} + B_n r_0^n + C_n r_0^{2-n} + D_n r_0^{2+n})\cos n\theta \\ & + (L_n r_0^{-n} + M_n r_0^n + N_n r_0^{2-n} + O_n r_0^{2+n})\sin n\theta\}.\end{aligned} \tag{2.15}$$

Furthermore, the general solution of the Eq. (2.5) in plane polar form is

$$\begin{aligned}\psi = {} & R_0 + S_0 \ln r_0 + U_0 I_0(ar_0/A) + V_0 K_0(ar_0/A) \\ & + \sum_{n=1}^{\infty} = [\{R_n r_0^{-n} + S_n r_0^n + U_n I_n(ar_0/A) + V_n K_n(ar_0/A)\}\cos n\theta \\ & + W_n r_0^{-n} + X_n r_0^n + Y_n I_n(ar_0/A) + Z_n K_n(ar_0/A)\}\sin n\theta],\end{aligned} \tag{2.16}$$

where I_n and K_n are the modified Bessel functions. Substituting now the Eqs. (2.13), (2.15), (2.16) into Eqs. (2.8)–(2.10), we next obtain the relations between the unknown coefficients in the functions τ, ϕ and ψ.

$$S_0 = W_1 = R_1 = 0, \quad a^2 E\alpha_t C_1^* + 2(1-\nu)N_1 = 0, \quad a^2 E\alpha_t A_1^* + 2(1-\nu)C_1 = 0,$$

$$\left(\frac{B}{A}\right)^2 \{2a^2 E\alpha_\tau D_1^* + 16(1-\nu)O_1\} + \left(\frac{a}{A}\right)^2 S_1 = 0,$$

$$\left(\frac{B}{A}\right)^2 \{2a^2 E\alpha_t B_1^* + 16(1-\nu)D_1\} - \left(\frac{a}{A}\right)^2 X_1 = 0,$$

$$\left(\frac{B}{A}\right)^2 \{2a^2 E\alpha_t C_n^* + 8(1-\nu)(1-n)N_n\} - \left(\frac{a}{A}\right)^2 R_n = 0,$$

(2.17)
$$\left(\frac{B}{A}\right)^2 \{2a^2 E\alpha_s D_n^* + 8(1-\nu)(1+n)O_n\} + \left(\frac{a}{A}\right)^2 S_n = 0,$$

$$\left(\frac{B}{A}\right)^2 \{2a^2 E\alpha_t A_n^* + 8(1-\nu)(1-n)C_n\} + \left(\frac{a}{A}\right)^2 W_n = 0,$$

$$\left(\frac{B}{A}\right)^2 \{2a^2 E\alpha_t B_n^* + 8(1-\nu)(1+n)D_n\} - \left(\frac{a}{A}\right)^2 X_n = 0.$$

2.2. Polygonal prism with a circular hole

As a practical example, we consider the problem, shown in Fig. 2, of the thermal stresses and couple stresses in a regular p-sided polygonal prism having a central circular

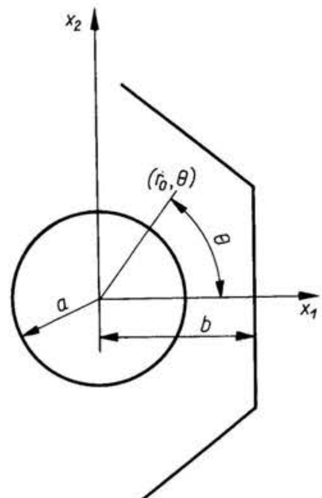

FIG. 2. Regular polygon with a circular hole.

hole under a steady temperature distribution with consideration of micropolar thermoelasticity. Let us assume that the inner and outer surfaces are at constant temperatures of T_0 and zero, respectively. Let a be the inner radius of the hole and b be the outer boundary of the prism. Now, we may show that the temperature and stress function must satisfy the boundary conditions. For this purpose, the numerical calculation required to obtain the unknown coefficients in τ, ϕ and ψ are enormous. Therefore, we use the point-matching technique to satisfy the boundary conditions at a selected finite set of outer boundary points of the polygonal region. If we replace $\sum_{n=1}^{\infty}$ in the Eqs. (2.13), (2.15) and (2.16) by $\sum_{n=1}^{N}$ approximately, we have to solve the equations of a finite number of unknowns. The solutions obtained satisfy almost exactly the prescribed boundary conditions in the interior of the body; and those on the outer boundary—approximately.

Considering the symmetry of the body, the Eq. (2.13) becomes:

$$\tau = A_0^* + B_0^* \ln r_0 + \sum_{n=1}^{N} (A_{pn}^* r_0^{-pn} + B_{pn}^* r_0^{pn}) \cos np\theta. \tag{2.18}$$

Boundary conditions for temperature are:

$$\text{at} \quad r_0 = 1, \quad \tau = T_0, \tag{2.19}$$

$$\text{at} \quad x_1 = b, \quad \tau\left(\frac{1}{\cos \pi s/pN_s} \frac{b}{a} \frac{\pi s}{pN_s}\right) = 0, \quad s = 0, \ldots, N_s, \tag{2.20}$$

where N_s is a finite integer and represents a number of divisions of the angle π/p.

From the Eqs. (2.18) and (2.19), we have

$$A_0^* = T_0, \quad A_{pn}^* = -B_{pn}^*.$$

Then

$$\tau = T_0 + B_0^* \ln r_0 + \sum_{n=1}^{N} (r_0^{pn} - r_0^{-pn}) B_{pn}^* \cos np\theta. \tag{2.21}$$

Substituting the Eq. (2.21) into (2.20), we obtain the following (N_s+1) equations:

$$\ln\left(\frac{1}{\cos \pi s/pN_s}\right) \frac{B_0^*}{T_0} + \sum_{n=1}^{N} \left\{\left(\frac{1}{\cos \pi s/pN_s} \cdot \frac{b}{a}\right)^{np} - \left(\frac{1}{\cos \pi s/pN_s} \cdot \frac{b}{a}\right)^{-np}\right\} \times \cos \frac{\pi ns}{N} \cdot \frac{B_{np}^*}{T_0} = -1, \tag{2.22}$$

where $N < N_s$.

Using the method of least squares, we can determine $(N+1)$ unknown coefficients B_0 and B_{pn}^* in the function τ. Therefore, the temperature distribution in this problem may be entirely determined.

Now, we consider the stress problems. Because of the symmetrical arrangement, the stress functions become:

$$\phi = A_0 + B_0 \ln r_0 + C_0 r_0^2 + D_0 r_0^2 \ln r_0 + \sum_{n=1}^{N} (A_{pn} r_0^{-np} + B_{pn} r_0^{np} + C_{pn} r_0^{-np+2} + D_{pn} r_0^{np+2}) \cos np\theta, \tag{2.23}$$

$$(2.24)\qquad \psi = \sum_{n=1}^{N} \{W_{pn} r_0^{-np} + X_{pn} r_0^{np} + Y_{pn} I_{pn}(ar_0/A) + Z_{pn} K_{pn}(ar_0/A)\} \sin np\theta.$$

Substituting the Eqs. (2.23) and (2.24) into Eq. (2.2), the thermal stress components and the couple-stress components become:

$$(2.25)\qquad \sigma_{\theta\theta} a^2 = -r_0^{-2} B_0 + 2C_0 + (2\ln r_0 + 3) D_0$$
$$+ \sum_{n=1}^{N} [np(np+1) r_0^{-np-2} A_{np} + np(np-1) r_0^{np-2} B_{np} + (np-2)(np-1) r_0^{-np} C_{np}$$
$$+ (np+2)(np+1) r_0^{np} D_{np} - np(np+1) r_0^{-np-2} W_{np} + np(np-1) r_0^{np-2} X_{np}$$
$$+ \{np(a/Ar_0) I_{np-1}(ar_0/A) - np(np+1) r_0^{-2} I_{np}(ar_0/A)\} Y_{np}$$
$$- \{np(a/Ar_0) K_{np-1}(ar_0/A) + np(np+1) r_0^{-2} K_{np}(ar_0/A)\} Z_{np}] \cos np\theta;$$

$$(2.26)\qquad \mu_{rz} a = \sum_{n=1}^{N} [-npr_0^{-np-1} W_{np} + npr_0^{np-1} X_{np} + \{(a/A) I_{np-1}(ar_0/A)$$
$$- npr_0^{-1} I_{np}(ar_0/A)\} Y_{np} - \{(a/A) K_{np-1}(ar_0/A) + npr_0^{-1} K_{np}(ar_0/A)\} Z_{np}] \sin np\theta;$$

$$(2.27)\qquad \mu_{\theta z} a = \sum_{n=1}^{N} \{npr^{-np-1} W_{np} + npr_0^{np-1} X_{np} + npr_0^{-1} I_{np}(ar_0/A) Y_{np}$$
$$+ npr_0^{-1} K_{np}(ar_0/A) Z_{np}\} \cos np\theta.$$

For the sake of brevity, the expressions for σ_{rr}, $\sigma_{r\theta}$ and $\sigma_{\theta r}$ are omitted here. Boundary conditions for the stress distribution are:

$$(2.28)\qquad \text{at} \quad r_0 = 1, \quad \sigma_{rr} = \sigma_{r\theta} = \mu_{rz} = 0;$$

$$(2.29)\qquad \text{at} \quad x_1 = b, \quad \sigma_{xx} = \sigma_{xy} = \mu_{xz} = 0.$$

Using the Eqs. (2.17) and (2.28), we can express the stress components by the terms with coefficients C_0, C_{np} and D_{np}. Then we use the point-matching technique to satisfy the outer boundary condition of the Eq. (2.27). Thus we can solve $3(N_s+1)-$ simultaneous equations for a selected finite set of the outer boundary points, and then the unknown coefficients of the stress functions are completely determined, and the problem is solved.

3. Numerical examples

The foregoing solutions will be illustrated numerically by the following data:

$$p = 4 \text{ (Square prism)}, N = 5, N_s = 9.$$

The variations in $\sigma_{\theta\theta}$ are shown in Figs. 3 and 4. Figures 5–8 illustrate the relation between $(\sigma_{\theta\theta})_{max}$, $(\mu_{rz})_{max}$, $(\mu_{\theta z})_{max}$ and b/a or B/A.

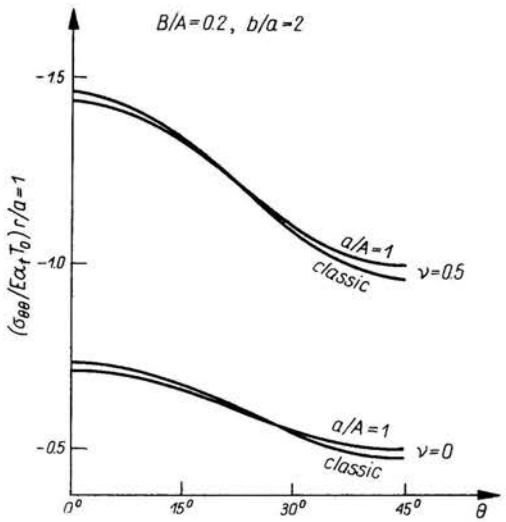

FIG. 3. Stress distribution of $\sigma_{\theta\theta}$ on the edge of the hole.

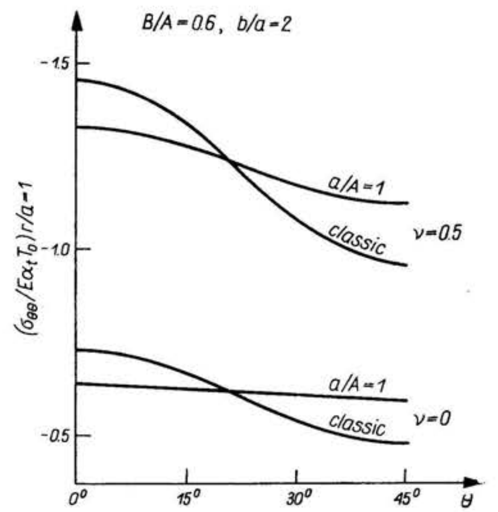

FIG. 4. Stress distribution of $\sigma_{\theta\theta}$ on the edge of the hole.

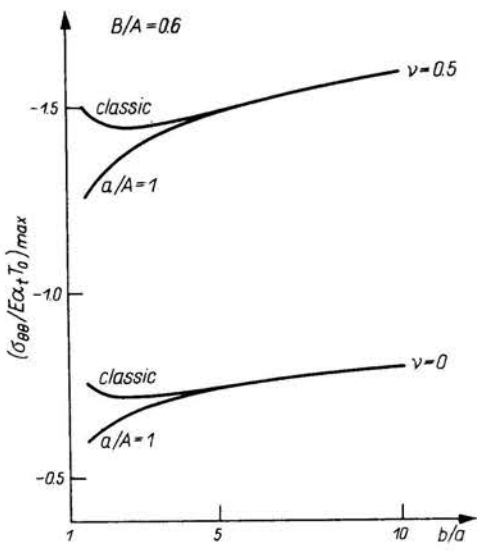

FIG. 5. Relation between $(\sigma_{\theta\theta})_{max}$ and b/a.

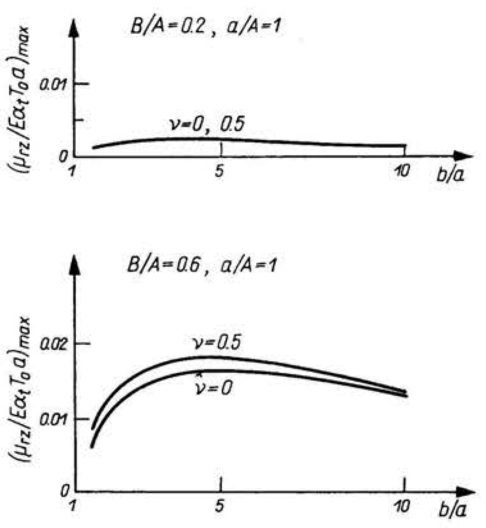

FIG. 6. Relation between $(\sigma_{rz})_{max}$ and b/a.

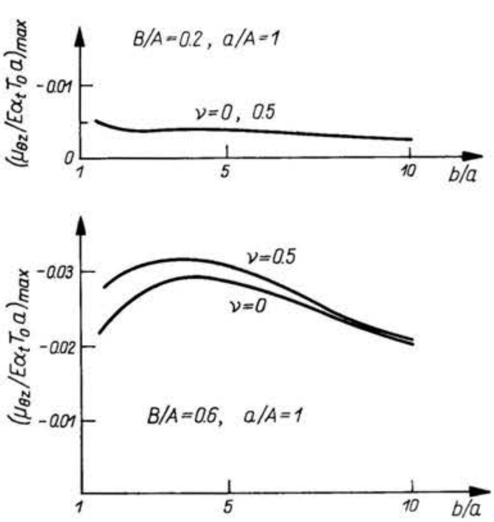

FIG. 7. Relation between $(\mu_{\theta z})_{max}$ and b/a.

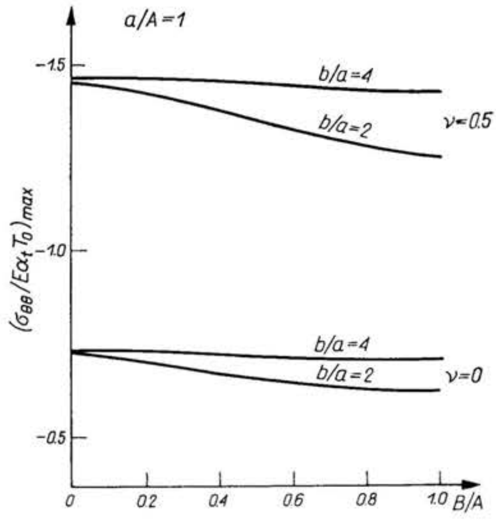

FIG. 8. Relation between $(\sigma_{\theta\theta})_{max}$ and A/B.

References

1. W. NOWACKI, *The plane problem of micropolar thermoelasticity*, Arch. Mech. Stos., **22**, 1, 3–26, 1970.
2. Y. TAKEUTI and K. YAMAZATO, *The effects of couple-stresses on thermal stress distributions in multiply-connected domains*, Z.A.M.M., **53**, 3, 155–166, 1973.

DEPARTMENT OF MECHANICAL ENGINEERING,
SHIZUOKA UNIVERSITY, HAMAMATSU JAPAN.

Received April 4, 1973.