

THERMAL STRESSES AND TWO TEMPERATURE HEAT TRANSFER

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1. Introduction

For description of thermal behaviour of metal films exposed to ultrashort laser pulses a two-temperature heat theory was introduced, cf. [1] - [4]. The theory involves the electron temperature T_e and the lattice temperature T_a . The energy transferred by electrons to lattice per unit volume of the crystal per unit time is proportional to the difference $T_e - T_a$. Mathematical properties of the theory were discussed in [5] and [6]. The problem has analogies in the diffusion theory (two-level diffusion of enzymes) and in viscous flow through non-homogeneous porous media with components of different porosity, [7].

2. Basic heat equations

Consider an elastic solid body B (a crystal, in particular a metal) irradiated by a laser pulse. Emerging of equilibrium between electrons and lattice in a crystal is realized by a relaxation process. During the relaxation processes crystal must be looked upon as a two-temperature system. The heat energy transferred by electrons to lattice per unit volume of the crystal per unit time is

$$(1) \quad \alpha(T_e - T_a) \quad \text{with} \quad \alpha \propto \frac{s^2}{\tau_e T_e}$$

where $T_e = T_e(x, t)$ is the electron temperature and $T_a = T_a(x, t)$ - the lattice temperature, both being functions of the position \mathbf{x} and time t . Moreover, s denotes the velocity of sound, and τ_e is the electron relaxation time, which can be regarded as a function of the electron temperature T_e . The expressions (1) are valid, when the temperatures T_e and T_a are much greater than Debye's temperature. Since, in this case $\tau_e \propto 1/T_e$, the heat transfer coefficient α is independent of the temperature.

Estimates based on the electrical conductivity of metals give values of α of the order 10^{10} J/(cm³ s K). If the specific heat of the lattice is c_a , the relaxation time for the phonon temperature is of the order of $c_a/\alpha \approx 10^{-10}$ s. For laser pulses of shorter duration, the violation of equilibrium between the electrons and lattice becomes important.

The heat energy balance equations for the metal absorbing the laser pulse has the form

$$(2) \quad \begin{aligned} c_e(T_e) \frac{\partial T_e}{\partial t} &= \lambda_e \Delta T_e - \alpha (T_e - T_a) + r(\mathbf{x}, t) \\ c_a(T_a) \frac{\partial T_a}{\partial t} &= \lambda_a \Delta T_a + \alpha (T_e - T_a) \end{aligned}$$

where λ denotes the heat conductivity coefficient, and the function $r(\mathbf{x}, t)$ describes the energy brought by the laser pulse. The symbol Δ denotes Laplacian.

3. Thermal deformations and stresses

We shall regard as the non-deformed state the state of the body before applying the heat source and consider deformations which accompany changes in the temperature of the body $T_a - T_0$. The stress tensor is, cf. [8].

$$(3) \quad \sigma_{ij} = 2\mu u_{ij} + \left(K - \frac{2}{3}\mu\right) u_{ll} \delta_{ij} - \gamma(T_a - T_0)\delta_{ij} \quad \text{with} \quad u_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i})$$

and $(u_i) = \mathbf{u}$ is the displacement in the elastic body. Moreover, K is the bulk modulus (modulus of hydrostatic compression) and μ is the shear modulus (modulus of rigidity). The coefficient $\gamma = 3K\alpha_T$, and α_T is the thermal expansion coefficient of the body. The equation of motion reads

$$(4) \quad \rho \frac{\partial^2 \mathbf{u}}{\partial t^2} = 2\mu \Delta \mathbf{u} + \left(K + \frac{1}{3}\mu\right) \nabla(\nabla \cdot \mathbf{u}) - \gamma \nabla T_a$$

and ∇ stands for the gradient operator.

In this study we solve a Danilovskaya problem for the new heat procedure, when the thermoelastic phenomenon is induced by a radiation pulse by two step action, $T_e \rightarrow T_a \rightarrow$ the stress, cf. [9, 10].

4. Applications

Description of laser - matter interaction may be useful in industrial applications, such as cutting, welding, material heat treatment.

In spectral analysis, arisen by the photothermal mechanism, the photoacoustic signal is useful in measuring the light absorption spectrum, particularly for transparent samples where the light absorption is very small.

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