55.

ON THE PREROGATIVE OF A TERNARY DENOMINATIONAL SYSTEM OF COINAGE.

[*Johns Hopkins University Circulars*, I. (1881), p. 132.]

PROFESSOR SYLVESTER drew attention to the fact that a system of coinage in which each coin is three times the value of the one below it would possess a superiority *above every other* in so far as it would admit of all payments up to any assigned limit, being effected with the smallest possible number of pieces, this advantage increasing with the size of the limit. Thus suppose the limit of 10 dollars to be selected, two persons each possessed of 7 coins of the respective value of 1, 3, 9, 27, 81, 243, 729 cents could pay each other by interchange of their coins any sum from 1 cent up to this limit. The full amount so capable of being paid being of course, $\frac{3^7-1}{2}$ cents, that is, \$10.93. Whereas with 7 coins doubling at each step, the extreme limit would be \$1.27.

Again if each coin were quadruple the value of its antecedent, the extreme limit attainable with 8 coins would be only $2(1+4+16+64)$, or \$1.70. Or, if each coin were five times the value of its antecedent, the sum of the geometrical progression $1+5+25+125+625$ being 781, 10 coins at least would be required to be possessed by each of two persons to enable one of them to pay the other any amount from 1 cent up to \$7.81, whereas as previously shown, 7 would be more than sufficient to allow of this being done on the ternary scale.

Thus the absolutely perfect system of coinage, so far as this depends on the smallness of the number of coins necessary to be used, is that which proceeds in a geometrical progression according to the ternary scale.

The following problem in arithmetic is suggested by the preceding considerations.

What is the condition that the sums and differences of the integers $a_1, a_2, a_3, \ldots a_n$, *not subject to any defined law of progression, may comprise between them all the numbers from* 1 *up to* $a_1+a_2+a_3 \ldots + a_n$?