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ON A LADY'S FAN, ON PARALLEL MOTION, AND ON AN ORTHOGONAL WEB OF JOINTED RODS.

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By means of Prof. Sylvester's Fan, it is possible to divide any angle into any assigned equal number of parts; and the trajectories of points taken in the several links connecting together the sticks of the fan have finite nodes, whose numbers are successively 1, 2^4, 3^4, 4^4, ...

Prof. Sylvester stated, in his second communication, That parallel motions exist at all is a paradox more wonderful than ever, now that his method gives the means of determining the conditions to be satisfied, and comparing their number with that of the disposable constants. The orders for 3, 5, 7, ... bars are 6, 20, 72, ... Formerly the existence of *one* was doubted; now a finite number for *every* order of linkwork is rendered highly probable. In particular, Prof. Sylvester showed how to determine whether *Parallel Motions* exist, and, if so, how to find them for any given number of bars and mode of colligation. He showed how to form a determinant involving only the lengths of the bars and other quantities which fix their direction; this determinant, if a parallel motion exists, must vanish identically for all values of the latter set of quantities. This is called the *Determinant of Parallel Motion*. The determinant is formed as a Jacobian of Equations, involving only linear functions of the lengths, and of a determinant corresponding to a set of equations of the same form as the above. Its evanescence gives a system of conditions to be satisfied, all expressed as rational functions of the lengths; and, by known algebraical methods, these enable us to find *necessary* relations of the lengths, if a Parallel Motion exists. It must then be ascertained whether these solutions are sufficient, and the problem is solved.

Prof. Sylvester's remarks on "An Orthogonal Web of Jointed Rods" were to the following effect: If two sets of joints be taken respectively

in two lines perpendicular to each other, either in a plane or in space, and a *linkage* be formed by connecting each point in one set with each point in the other by jointed rods, this constitutes an orthogonal web. It is *not* a fixture, and its motion is subject to this curious condition, that either each set of points must always continue to lie in the same right line, which may be called a neutral position, or else one set will lie in a right line, and the other in a plane at right angles to such line. Starting from the neutral position (a position of *double-lock*), the system may be said to be subject to an optional locking about one or the other of two perpendicular lines, and an unlocking about the others; but, when once put in motion, the system must again be brought into the same, or a new neutral position, before the one axis of lock can be got rid of, and another at right angles thereto substituted in its stead. If the whole motion be confined to a plane, the paradox consists in the link-combination forming one degree of liberty of deformation (*ἀλλοίωσις*, as distinguished by Plato from *κίνησις*), although a calculation of the amount of restraint by the general method applicable to such questions would seem to indicate that it ought to form an absolutely rigid system except in the case where there are only two joints in one at least of two sets. Taken in space, there is the further and more striking paradox, that the number of degrees of liberty of deformation, according to the choice made of one or the other of the two sets of points to be unlocked out of the rectilinear into the planar position, will be the *alternative of two numbers*, viz., the number of points in the one set or in the other set (which need not be the same), a kind of indeterminateness in the "Index of Freedom" without precedent in mathematical speculations. As lightning clears the air of impalpable noxious vapours, so an incisive paradox frees the human intelligence from the lethargic influence of latent and unsuspected assumptions. Paradox is the slayer of Prejudice.