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Yield surfaces of the M-63 brass prestrained by cyclic biaxial loading

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EXPERIMENTAL results for thirty thin-walled specimens of the M-63 brass are presented in a study of the effect of biaxial cyclic loading on the shape of the yield surfaces. The alternating biaxial loading was superimposed upon a fixed biaxial isotropic state of stress. These results show that tested material strained cyclically beyond the initial yield locus works under a fully elastic regime after a few cycles of the initial transitory period of loading.

Przedstawiono wyniki badań eksperymentalnych trzydziestu cienkościennych próbek rurkowych z mosiądzu M-63, przeprowadzonych w celu określenia wpływu dwuosiowego obciążenia cyklicznego na kształt powierzchni plastyczności. Dwuosiowe obciążenie cykliczne było nałożone na stały dwuosiowy izotropowy stan naprężenia. Uzyskane wyniki pokazują, że badany materiał obciążony cyklicznie poza początkową granicę plastyczności reaguje w sposób całkowicie sprężysty po kilku cyklach stanowiących początkowy przejściowy okres zmian obciążenia.

Представлены результаты экспериментальных исследований тридцати тонкостенных трубчатых образцов из латуни М-63, проведенных с целью определения влияния двухосевой циклической нагрузки на форму поверхности пластичности. Двухосевая циклическая нагрузка наложена на постоянное двухосевое изотропное напряженное состояние. Полученные результаты показывают, что исследуемый материал, нагруженный циклически выше начального предела пластичности, реагирует полностью упругим образом после нескольких циклов, составляющих начальный переходный интервал изменений нагрузки.

Introduction

THE BEHAVIOUR of metals subjected to simple cyclic loading alternating between prescribed limits beyond the initial yield locus has been studied in numerous experimental works. In most studies, the simple cyclic tension-compression loadings or alternating torsion were applied to tested specimens and results were represented in the form of stress-strain diagrams (see books [1, 2] and review articles [3, 4]). These works were intended to study the plastic behaviour of metals undergoing loadings of the low-cycle fatigue type. Among other phenomena it has been discovered that there exists a limiting steady state to which the cycles are tending during an initial transitory period.

Only a few studies, however, are devoted to the more fundamental analysis of the behaviour of metals undergoing cyclic loading. In the works by R. A. ARUTUNIAN [5] and A. I. TSHISTIAKOV [6], tubular specimens of carbon steel were tested in order to establish the shape of the yield surface after prestraining by reversible torsion. Thus also in these works the prestraining history involved the simplest possible cyclic loading. R. L. SIERAKOWSKI and A. PHILLIPS [7] used biaxial proportional paths of cyclic loading in a study of the effect of such loading on the yield surface of an aluminium alloy. In real structures, the metal is, however, in most cases exposed to much more complex types of cyclic loadings, involving superposition of fixed and alternating loads. This type of loading has been theoretically analysed by Z. MRÓZ [8] in an attempt to describe the complex behaviour of metals undergoing cyclic plastic deformation. It is evident, however, that the key rôle in the study of the behaviour of metals exposed to low-cycle fatigue must be played by experimental investigations.

In the experiments here outlined a study was made of the effect of the initial transitory period of the alternating biaxial loading superimposed upon a fixed load on the behaviour of the yield surface. To establish the cyclic loading programme the kinematic hardening rule [9] was chosen. This simple hardening rule based upon previous concepts of A. JU. ISHLINSKII [10] and W. PRAGER [11] cannot of course fully describe the complex behaviour of metals. In previous experimental works [12, 13], however, we have shown, that the kinematic hardening rule is able to predict qualitatively certain complex phenomena connected with plastic deformation of metals under biaxial loadings.

2. Experimental programme

Since thin-walled tubular specimens loaded by internal pressure and axial tensile force were used, all results will be represented in the principal stress plane σ_z, σ_t, where σ_z is the axial stress and σ_t is the circumferential stress. The prestressing programmes are shown in Fig. 1. Six series of specimens, each of them containing five specimens, were investigated in order to establish the initial and subsequent yield surfaces in the first quadrant of the principal stress plane. Each of five specimens belonging to the same set was prestrained in the same manner. Next, after full unloading, the strain gauges were mounted on its surface and the yield locus of the prestrained specimens was investigated. The subsequent proportional loading paths were different for each specimen of one set. These loading paths are shown in Fig. 1 for series I as the radii $1, 2, \ldots, 5$ from the origin. For each specimen loaded along the prescribed loading path, the equivalent stress-equivalent strain diagram was plotted, from which were found stresses corresponding to plastic equivalent strains $\varepsilon^p_{eq} = 0.01$, 0.05, 0.2 and 0.5%. Moreover, the point of departure from the straight initial part of the diagram was determined. In this manner, the proportional limit was obtained. The experimental curves plotted through points corresponding to the same value of equivalent plastic strain are further marked by subscript notation — σ_{prop}, $\sigma_{0\cdot01}$, … …$\sigma_{0\cdot5}$, respectively.

Equivalent strains ε_{eq} were computed by summation of the small increments

$$\Delta\varepsilon_{eq} = \sqrt{\Delta\varepsilon_z^2 + \Delta\varepsilon_z\Delta\varepsilon_t + \Delta\varepsilon_t^2}\,,$$

where $\Delta\varepsilon_z$ and $\Delta\varepsilon_t$ are increments of axial and circumferential strains, respectively. Since changes of the wall thickness of specimens were not measured, the incompressibility of the material was assumed to establish the above formula.

Equivalent stresses σ_{eq} were obtained from the formula:

$$\sigma_{eq} = \sqrt{\frac{1}{6}\left((\sigma_z-\sigma_t)^2+(\sigma_t-\sigma_r)^2+(\sigma_r-\sigma_z)^2\right)},$$

where σ_r is the radial stress. If p is the internal pressure applied to the specimen, the radial stress has the value $\sigma_r = -p$ at the inner radius, and $\sigma_r = 0$ at the outer radius of the specimen. In calculations, the mean value $\sigma_r = -p/2$ has been assumed. Note that p is small as compared with σ_t, and, therefore, the further analysis may be carried out with

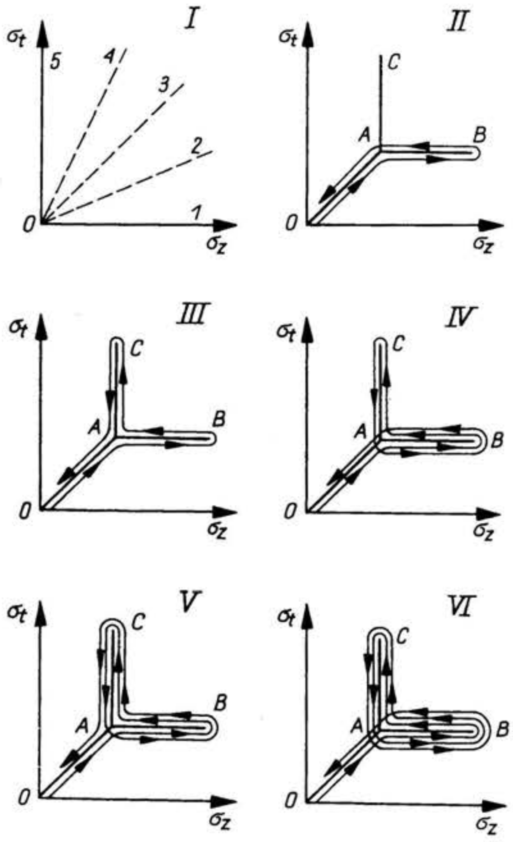

FIG. 1.

satisfactory accuracy in terms of plane stress theory, in spite of the existence of nonhomogeneously distributed radial stress.

The specimens of series I were not prestressed. Each of them was loaded along a prescribed radial path in order to find the conventional yield curves for the initial material. Specimens of series II were prestrained along the path OAB and then unloaded as shown in the figure. Prestraining programmes for the remaining series are shown in Fig. 1. The symmetric cyclic loading $ABACA$, where $AB = AC$, is superimposed upon the fixed loading represented by the vector OA. Series III is prestrained by one full cycle, series IV by one and a half, series V by two full cycles, and series VI by two and a half cycles. For each series, after unloading the conventional yield curves $\sigma_{prop}, \sigma_{0 \cdot 01}, \ldots, \sigma_{0,5}$ were found.

Figure 2 illustrates the theoretical analysis of the behaviour of the σ_{prop} curve for the material undergoing prestraining according to the programmes from Fig. 1. Our analysis will be limited here to the σ_{prop} curve, since our aim is to show that for certain types of complex cyclic loading the fully elastic response of the material can be reached after a few cycles. The ellipse I represents the initial σ_{prop} curve. The small circles represent

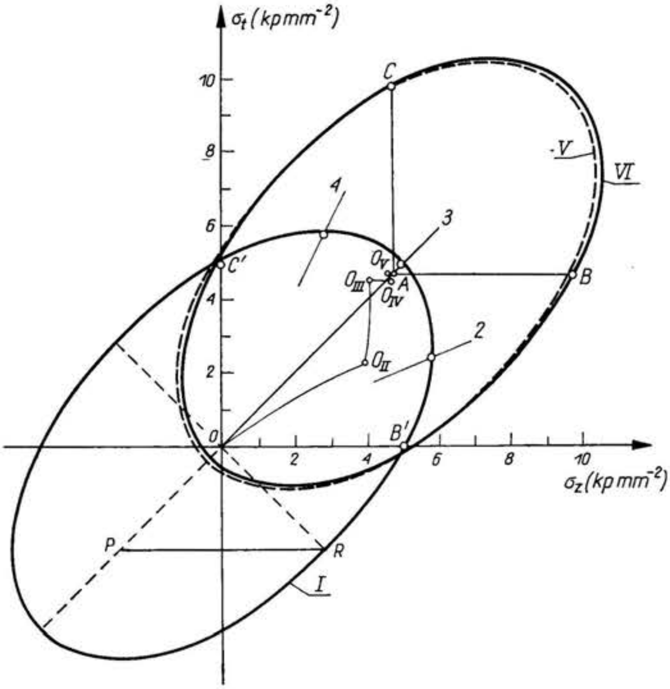

FIG. 2.

experimental points obtained for the unstressed material (series I). Through these points the theoretical Huber-Mises ellipse

$$\sigma_z^2 - \sigma_z\sigma_t + \sigma_t^2 = \sigma_0^2$$

was plotted with satisfactory accuracy, as shown in the figure.

Positions of the end points B and C of the cyclic part of prestressing programmes were so chosen that $AB = OB'$. If the kinematic hardening rule is assumed, the initial yield surface will be shifted as a rigid body in the $\sigma_z\sigma_t$ plane. For each of the prestraining programmes shown in Fig. 1, the corresponding position of the yield ellipse may be found according to Shield's and Ziegler's procedure (see [9]). Consecutive positions of the shifted yield ellipse for all prestraining programmes were obtained graphically using a small stress-increment technique. By way of example, the subsequent positions of the ellipse for the prestraining programmes V and VI are shown in Fig. 2. Central points of these ellipses are marked by O_{V} and O_{VI}. Central points for the remaining programmes II, III, IV are marked by O_{II}, O_{III} and O_{IV}, respectively. It is interesting to note that the point O_{VI} practically coincides with the point A. Thus the kinematic hardening rule predicts that after two and a half cycles further cycles of loading will in practice be accompanied by elastic strains only.

A similar conclusion is reached for smaller amplitudes of stress cycles — i.e., for $AB < OB'$ (the symmetry of stress cycles $AB = AC$ is preserved). Note that for larger symmetric stress cycles — i.e., for $AB > PR$ the kinematic workhardening rule predicts steady plastic cycling after an initial transitory period. According to this hardening rule, the fully elastic cycling is in such a case impossible.

In the present experiments, the tubular specimens were cyclically loaded along the prestraining path shown in Fig. 2. An experimental study for larger stress amplitudes will be published in a separate paper.

3. Specimens and equipment

The thin walled tubular specimens were made of a drawn tube of 30mm inner diameter and 1 mm wall thickness. The material was M-63 brass containing 37% of zinc. All specimens were annealed before testing. The tubes were selected in order to obtain the most uniform distribution possible of the wall thickness and diameter. The largest deviation of the wall thickness from the mean value did not exceed 4%, while the deviation of the diameter was found not to exceed 0,3%.

The testing device used in the present investigation has two independent pressure installations. One of them gives the loading of the specimen by axial force and the other by internal pressure. A detailed description of this device was given in TURSKI's work [14]. The deformations have been measured by means of four ordinary resistance strain gauges with 15mm gauge length, applied on the surface of each specimen, two in axial and two in circumferential direction at diametrically opposite positions at mid-length of the specimen. In order to eliminate possible slight deviations from symmetry, the strains were taken as the mean value of readings of the two gauges oriented in the same manner. The measuring bridge used in the present work has the scale division corresponding to $\varepsilon = 5\cdot 10^{-6}$.

4. Experimental results

Although the experimental conventional yield curves σ_{prop}, $\sigma_{0\cdot 01}$, $\sigma_{0\cdot 05}$, $\sigma_{0\cdot 2}$ and $\sigma_{0\cdot 5}$ corresponding to different values of plastic equivalent strains have been established for each set of specimens, we shall discuss here in greater detail the behaviour of the σ_{prop} curves only in order to show that after a few cycles the fully elastic cycling is reached. Much more complete experimental results for this type of cycling are given in the unpublished thesis [15] by the first-named of the present authors ([1]).

In the following figures, the experimental σ_{prop} curves are compared with the respective theoretical σ_{prop} curves for each particular prestraining programme. The theoretical curves were obtained on the basis of the kinematic hardening rule. In each figure are presented the curves for the two consecutive series of specimens. Thus we are able to analyse directly the entire transistory period of cycling where the material is exposed to plastic deformations.

([1]) The experimental investigation outlined here has been performed while the first-named author was on leave from Belgrade University, Metallurgical Division in Bor, for a stay of one year at the Institute of Fundamental Technological Research in Warsaw.

In Fig. 3 are presented results for programmes I (continuos lines) and II (dashed lines). The results for programme I were previously used to establish the initial yield curve for the unstressed material shown in Fig. 2. The σ_{prop} curve for the specimens prestressed according to programme II shows a strong effect of the prestraining on its shape.

In Fig. 4 are compared σ_{prop} curves for programmes II (continuous lines) and III (dashed lines). Figure 5 shows experimental and theoretical curves for the programmes III and IV.

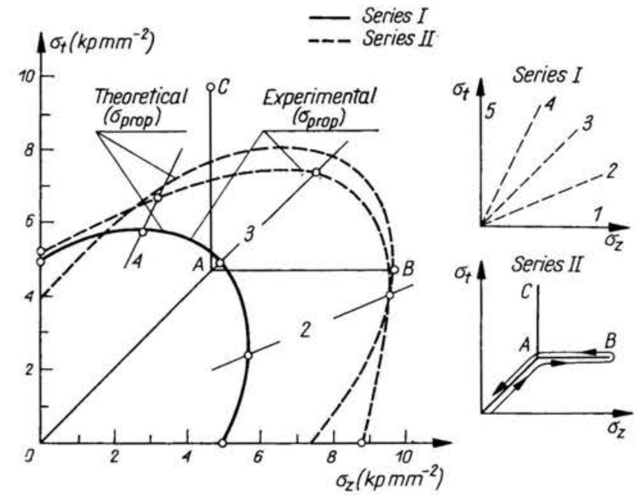

FIG. 3.

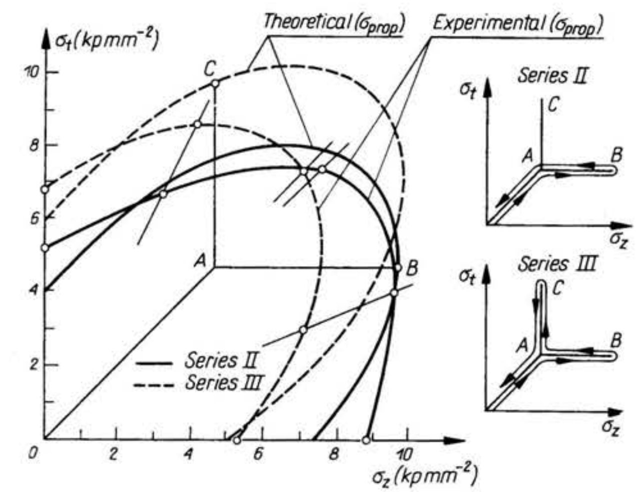

FIG. 4.

In both figures a remarkable difference between corresponding pairs of experimental curves is clearly visible. Thus the initial stage of cyclic loading is accompanied by plastic deformations as predicted by the theoretical considerations based on the kinematic hardening rule.

Results for more advanced stages of cycling are shown in Figs. 6 and 7, where experimental and theoretical σ_{prop} curves are presented for the following pairs of prestraining programmes: IV–V and V–VI, respectively. It may be clearly seen that the more cycles of loading are included in the prestraining history, the smaller is the difference between σ_{prop} curves resulting from the two consecutive programmes. The experimental curves for the program-

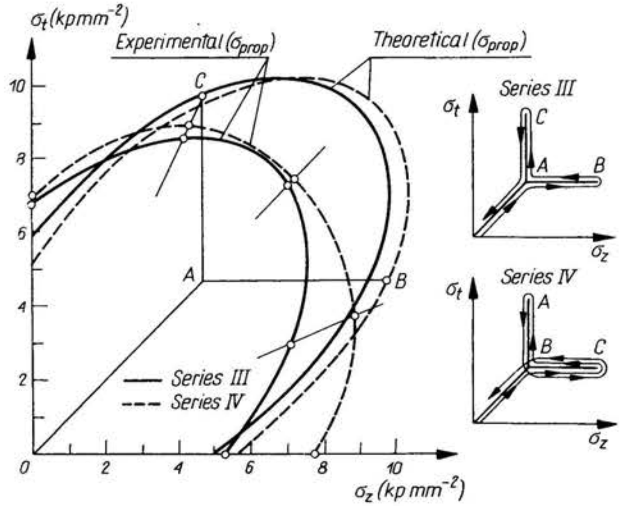

FIG. 5.

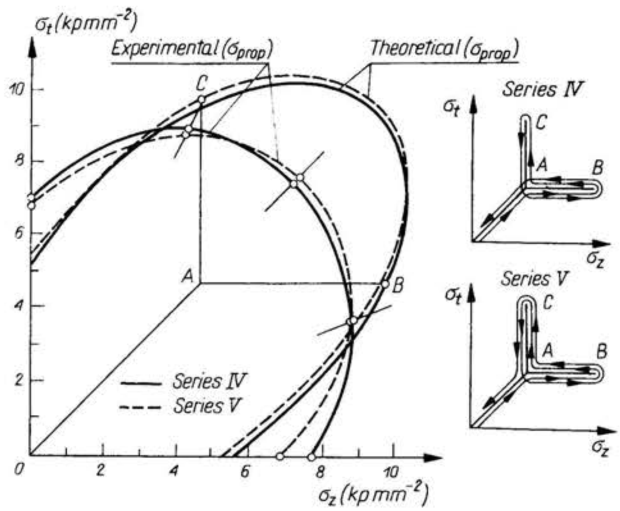

FIG. 6.

mes V (two full cycles) and VI (two and a half cycles) practically coincide. An analogous conclusion results from theoretical considerations.

For all prestraining programmes, the difference between theoretical and experimental curves is so large that it is evident that the kinematic hardening rule departs qualitatively even very far from the real behaviour of metals undergoing plastic deformation. Similar results have also been obtained in previous works [12, 13, 16] for non-cyclic prestraining histories. One interesting feature, however, should be noted. In Fig. 7, the distance between theoretical curves for the two prestraining programmes is practically the same as the distance between experimental curves. This means that the kinematic hardening rule

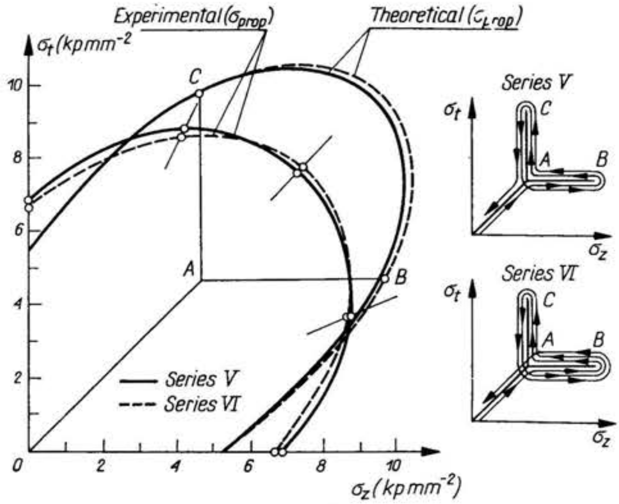

FIG. 7.

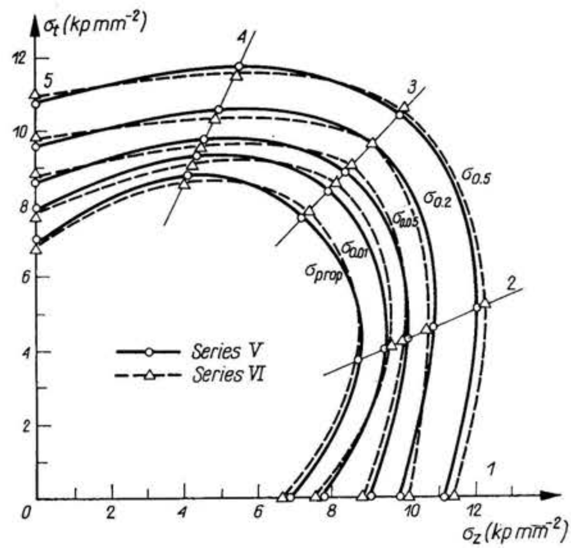

FIG. 8.

adequately predicts the number of stress cycles after which the material used here displays no further plastic deformations.

It is interesting to note, moreover, that also other conventional yield curves $\sigma_{0 \cdot 01}$, $\sigma_{0 \cdot 05}$, $\sigma_{0 \cdot 2}$ and $\sigma_{0 \cdot 5}$ practically coincide for the prestressing programmes V and VI as shown in Fig. 8.

5. Final remarks

The testing procedure described in Sec. 2 indicates that experimental curves obtained in this work do not represent the actual yield surfaces existing during cyclic loading. All specimens after cyclic prestressing were fully unloaded until the origin O along the path AO, and then the yield locus was investigated. Thus the curves plotted through experimental points represent the yield curves of the prestressed material and, therefore, provide no information as to the shape of actual yield surfaces. Even so, the present procedure is sufficient for analysis of the plastic behaviour of a material undergoing cyclic loading.

Results presented in this work indicate that M-63 brass, and probably numerous other metals strained cyclically beyond the initial yield locus, are capable of working under a fully elastic regime after a few cycles, provided the stress cycles are not too large. Such effect is similar to that known as the shake-down phenomenon in the theory of structures caused by redistribution of stresses during cyclic loading of a structure. The effect analysed in the present experiments can be attributed to the redistribution of micro-stresses after each cycle of loading. The micro-stresses at the microscopic level are in real polycrystalline metals distributed nonhomogeneously, even if at the macroscopic level the stress field existing in the material is homogeneous. The concept of micro-stresses has been found useful in attempts to describe complex strain-hardening phenomena of metals [17], and may be used also to introduce here the concept of the material shake-down.

The present experiments constitute the first step only towards the more fundamental experimental analysis of the behaviour of metals undergoing complex multi-axial cyclic loadings. First of all, it would be of interest to check whether for other cyclic loading programmes involving not only the first quadrant of the σ_z σ_t plane, a fully elastic regime can also be attained. Experiments for biaxial cyclic loading with large stress amplitudes would also be of significant practical value for a more profound understanding of the phenomena connected with the low-cycle fatigue of metals.

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