# Certain solutions of the flow of a one-component three-phase mixture

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Two theoretical models of a one-component, three-phase mixture with solid and liquid phases dispersed in a volumetrically dominant gaseous phase are considered. The first model is in thermodynamic equilibrium, in the second one small perturbations of equilibrium state are admitted. Analising the properties of flows of the considered mixture some attention was paid to the vanishing velocity of propagation of acoustic perturbations with decreasing frequency, to the possibility of appearing of thin condensed layers called here "accumulation waves" and to the general properties of flows. It was underlined that the equilibrium model of the mixture in certain problems of flow may present unsatisfactory and even paradoxical results.

Rozpatrywane są dwa modele mieszaniny jednoskładnikowej z fazami stałą i ciekłą, rozproszonymi w dominującej objętościowo fazie gazowej. Pierwszy model znajduje się w izobarycznoizotermicznej równowadze termodynamicznej, w drugim uwzględniono niewielkie zaburzenia równowagi. Analizując własności przepływowe rozpatrywanej mieszaniny, zwrócono uwagę na znikanie prędkości rozchodzenia się zaburzeń akustycznych w miarę malenia częstości zaburzeń, na możliwość powstawania cienkich warstw z fazami skondensowanymi, nazwanych falami akumulacyjnymi oraz na ogólne własności przepływów izobarycznych. Podkreślono, że w niektórych zagadnieniach przepływowych równowagowy model mieszaniny może dawać niezadowalający opis rzeczywistości a nawet czasami może prowadzić do paradoksalnych wyników.

Рассмотрены две модели однокомпонентной смеси с твердой и жидкой фазами распределенными в преобладающей объемом газовой фазе. Первая модель находится в изобарно-изотермическом термодинамическом равновесии, во второй модели учтены небольшие возмущения равновесия. Анализируя свойства течения рассматриваемой смеси обращено внимание на исчезновение скорости распространения акустических возмущений по мере уменьшения частоты возмущений, на возможность возникновения тонких слоев с конденсированными фазами, называемыми акумулятивными волнами, а также на общие свойства изобарических течений. Подчеркнуто, что в некоторых проблемах течения равновесная модель смеси может приводить к неудотворительному описанию действительности и даже иногда может вести к парадоксальным результатам.

### 1. Introduction

A ONE-COMPONENT, three-phase mixture has such a peculiar property that it might exist in equilibrium in isobaric and isothermic conditions only. Isobaric flows are in general difficult to realize and the equilibrium conditions of the mixture considered should also not in general be fulfilled. However, also of interest seems to be the study of the properties of idealized equilibrium flows, and we shall here accord it the principal attention. Hence we shall consider two models of a one-component three-phase mixture: the first, denoted by E, in idealized isobaric and isothermic equilibrium; and the second, denominated by P, more realistic, in perturbed state, but with only two fractions of identical small liquid or solid spheres dispersed in a continuous, volumetrically dominant gaseous phase The surface tension in both models will be disregarded.

	₹[°K]	$\bar{p}\left[\frac{N}{m^2}\right]$	$\frac{\varrho^{\circ}}{\varrho^{1,2}}$	$\frac{\overline{s}^0 - \overline{s}^2}{\overline{s}^1 - \overline{s}^2}$
H <sub>2</sub> O	273	611	~10-6	6.75
NH <sub>3</sub>	195	6070	~10-4	4.48
CO <sub>2</sub>	217	518000	~10 <sup>-2</sup>	1.75

The phases diagram for the thermodynamic equilibrium, without taking into account the surface tension, is qualitatively demonstrated in Fig. 1. The triple point in pressure, temperature p, T, space corresponds to the interior of the triangle 012 in specific entropy, specific volume space s,  $1/\varrho$ . The physical constants in triple point state for water, ammonia and carbon dioxide are given in Table 1. The gaseous, liquid and solid phases



are denoted by the upper indices 0, 1, 2, respectively, and the triple point equilibrium state is denoted by a bar.

It should be emphasized that the density ratios of the gaseous to the condensed phases are very small and, consequently, in the models considered of a mixture with volumetrically dominant gaseous phase, we will disergard the specific volume and the total volume of condensed phases

(1.1) 
$$\frac{\varrho^0}{\varrho^{1,2}} \ll 1, \qquad \frac{\xi^{1,2}}{\xi^0} \cdot \frac{\varrho^0}{\varrho^{1,2}} \ll 1.$$

By  $\xi^0$ ,  $\xi^1$ ,  $\xi^2$  are represented the ratios of mass of each phase to the total mass of the mixture. In this case, the triangle 012 considered in Fig. 1 may be reduced to its simpler form in Fig. 2 and in the three-phase region of this triangle the proximal vicinity of the *s*-axis should be excluded.

Special attention will be paid to the mixture which may be isentropically compressed to a negligibly small volume of condensed phases only (the region below the  $s = \bar{s}^1$  line in Fig. 2).

For the equilibrium model E of the triple point mixture, we shall assume that all phases move with the same velocity **u** and we shall choose the mass ratios  $\xi^1$ ,  $\xi^2$  as two parameters of the thermodynamic state of the mixture. The values of the pressure and the temperature should be constant  $p = \bar{p}$ ,  $T = \bar{T}$ , while the density  $\varrho = \bar{\varrho}$  and the specific



entropy  $s = \bar{s}$  may change with changing of  $\xi^1$ ,  $\xi^2$ . The equations of flow of this model may be presented in the form [1]:

(1.2) 
$$\begin{aligned} \frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla) \mathbf{u} &= \mathbf{f}, \\ \frac{\partial \xi^n}{\partial t} + \mathbf{u} \cdot \nabla \xi^n &= -a^n (1 - \xi^1 - \xi^2) \nabla \cdot \mathbf{u}, \quad n = 1, 2, \end{aligned}$$

where f is the body force and  $a^1$ ,  $a^2$  are the ratios of the specific entropies, equivalent to the ratios of the latent heats:

(1.3) 
$$a^{1} = 1 - a^{2} = \frac{\bar{s}^{0} - \bar{s}^{2}}{\bar{s}^{1} - \bar{s}^{2}}.$$

The density  $\rho$  and the specific entropy s of the mixture are determined by the formulae:

(1.4) 
$$\frac{1}{\varrho} = \frac{\xi^0}{\varrho^0}, \quad \xi^0 = 1 - \xi^1 - \xi^2,$$
$$s = \xi^0 s^0 + \xi^1 s^1 + \xi^2 s^2.$$

In the model P of a three-phase mixture in the state of perturbed equilibrium, the velocities  $\mathbf{u}^n$  and the temperatures  $T^n$  of all phases (n = 0, 1, 2) may be different. Also the pressure p may be different from  $\bar{p}$ . Choosing in a manner similar to the model E the mean velocity

(1.5) 
$$\mathbf{u} = \xi^0 \mathbf{u}^0 + \xi^1 \mathbf{u}^1 + \xi^2 \mathbf{u}^2$$

and the mass ratios  $\xi^1$ ,  $\xi^2$  as the main parameters of flow, we shall additionally introduce here the parameters of perturbation

(1.6) 
$$\mathbf{w}^n = \mathbf{u}^n - \mathbf{u}, \quad \Delta T^n = T^n - \overline{T}, \quad \Delta p = p - \overline{p},$$

and consider only the linear isotropic approximation in respect to these last parameters. The existence of the viscous stress tensor and the heat flux in the gaseous phase will be disregarded, all dissipative effects being attributed to the interaction phenomena between condensed particles and the gaseous phase (exchange of mass, energy and momentum). The previously derived equations [1] for the model P are:

$$\begin{aligned} \frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla)\mathbf{u} &= \mathbf{f} - \xi^0 R \overline{T} \nabla \frac{\Delta p}{\overline{p}}, \\ \frac{\partial \xi^n}{\partial t} + \mathbf{u} \cdot \nabla \xi^n &= -\xi^0 \nabla \frac{\xi^n \mathbf{w}^n}{\xi^0} + \xi^n X^n, \\ (1.7) \quad \left(\frac{\partial}{\partial t} + \mathbf{u} \cdot \nabla\right) \left(\frac{\Delta T^0}{\overline{T}} - \frac{\Delta p}{\overline{p}}\right) &= (\nabla \cdot \mathbf{u}) \left(\frac{\Delta T^0}{\overline{T}} - \frac{\Delta p}{\overline{p}}\right) + \nabla (\mathbf{u} + \mathbf{w}^0) + \frac{\xi^1 X^1 + \xi^2 X^2}{\xi^0}, \\ \left(\frac{\partial}{\partial t} + \mathbf{u} \cdot \nabla\right) \left(C^0 \frac{\Delta T^0}{\overline{T}} - \frac{\Delta p}{\overline{p}}\right) &= \frac{\xi^1 (Q^1 X^1 - Z^1) + \xi^2 (Q^2 X^2 - Z^2)}{\xi^0}, \\ \frac{\partial}{\partial t} \frac{\Delta T^n}{\overline{T}} + \mathbf{u} \cdot \nabla \frac{\Delta T^n}{\overline{T}} &= \frac{1}{C^n} Z^n, \\ \frac{\partial \mathbf{w}^n}{\partial t} + (\mathbf{u} \cdot \nabla) \mathbf{w}^n + (\mathbf{w}^n \cdot \nabla) \mathbf{u} &= \mathbf{Y}^n + \xi^0 R \overline{T} \nabla \frac{\Delta p}{\overline{p}}, \\ X^n &= \mathcal{X}^n \frac{\Delta T^n - \Delta T^0}{\overline{T}} + \mathcal{L}^n \left(\frac{\Delta p}{\overline{p}} - Q^n \frac{\Delta T^n}{\overline{T}}\right), \\ (1.7) \quad Z^n &= -\mathcal{M}^n \frac{\Delta T^n - \Delta T^0}{\overline{T}} + \mathcal{N}^n \left(\frac{\Delta p}{\overline{p}} - Q^n \frac{\Delta T^n}{\overline{T}}\right), \\ \mathbf{Y}^n &= \mathcal{A}^n (\mathbf{w}^0 - \mathbf{w}^n), \quad \mathbf{w}^0 &= -(\xi^1 \mathbf{w}^1 + \xi^2 \mathbf{w}^2)/\xi^0, \\ n &= 1, 2, \end{aligned}$$

where R is the gas constant,  $C^0 = c_p^0/R$ ,  $C^{1,2} = c^{1,2}/R$  are non-dimensional specific heats,  $Q^{1,2} = (\bar{s}^0 - \bar{s}^{1,2})/R$  non-dimensional latent heats of evaporation, and  $\mathscr{A}^{1,2}$ ,  $\mathscr{K}^{1,2}$ ,  $\mathscr{L}^{12}$ ,  $\mathscr{M}^{1,2}$ ,  $\mathscr{N}^{1,2}$  are Onsager constants for determining the fluxes of mass  $X^{1,2}$ , energy  $Z^{1,2}$ , and momentum  $Y^{1,2}$  from the gaseous phase to the condensed phases.

The equations of flow for both models of a one-component three-phase mixture having been obtained, we shall now pay attention to some of their particular solutions and to the compatibility conditions on such discontinuity surfaces as may possibly occur.

#### 2. Acoustic perturbations

The disappearing of the speed sound is the main characteristic feature of the equilibrium model E considered. Since the isentrope  $s = \bar{s} = \text{const}$  is a horizontal straight line in the  $(p, 1/\varrho)$  space (Fig. 1), we immediately obtain:

(2.1) 
$$\bar{a} = \sqrt{\frac{\partial \bar{p}(\bar{\varrho}, \bar{s})}{\partial \bar{\varrho}}} = 0.$$

The isentropic speed of sound does not exist in the model P, where the dispersion and the attenuation of sound should be expected. Introducing into the equations of flow the harmonic plane acoustic perturbation proportional to  $\exp[-\mu x + i\omega(t-x/a)]$ , we found in [2,3] that for small  $\omega$  the phase velocity a and the damping coefficient  $\mu$  are proportional to  $\sqrt{\omega}$ :

$$(2.2) a \sim \sqrt{\omega}, \quad \mu \sim \sqrt{\omega}$$

Hence, the isentropic speed of sound  $\bar{a} = 0$  is also here equal to the limiting value of the phase velocity a for  $\omega \to 0$ . But quite unusually this limiting value is zero and unusually also the rate of changing of a and  $\mu$  (the derivative in respect to  $\omega$ ) is very large, being proportional to  $1/\sqrt{\omega}$ .

On the basis of further considerations it may be expected that the relations (2.2), derived for the model P only, should be more generally valid also for real one-component three-phase mixtures. But quantitatively, the region of their validity (small  $\omega$ ) is not very large and it may probably have little practical importance. However, it is difficult now to give a practical interpretation of the results obtained, because no research being made in acoustics of the one-component three-phase mixtures, either from the empirical or from the theoretical point of view, is known to the author.

#### 3. Accumulation waves

On the basis of Rankine-Hugoniot conditions it is easy to verify that the shock waves in the model E with no difference of pressure may not exist. At least on one side of the discontinuity surface the mixture must contain two phases only. Les us additionally consider now such triple point mixtures, with sufficiently small specific entropy, which may be compressed by a shock wave into mixtures composed of solid and liquid phases only (the region below  $s = \bar{s}^1$  line in Fig. 2). Since the volume of condensed phases is negligibly small (1.1) and, in consequence, the density ratio on both sides of the discontinuity surface should also be negligibly small  $\bar{\varrho}_{-}/\varrho_{0} \ll 1$ , thus we may in our case reduce the R.-H. conditions to the simpler approximative form:

$$\begin{aligned} \frac{\hat{u}_{0}}{\hat{u}_{-}} &= \frac{\bar{\varrho}_{-}}{\varrho_{0}} \ll 1, \quad \xi_{0}^{1} + \xi_{0}^{2} = 1, \\ (3.1) \qquad & \bar{p} + \bar{\varrho}_{-}(\hat{u}_{-})^{2} = p_{0}, \quad \bar{\varrho}_{-} = \bar{\varrho}^{0} / (1 - \xi_{-}^{1} - \xi_{-}^{2}), \\ & i_{-} + \frac{(\hat{u}_{-})^{2}}{2} = i_{0-}, \quad \frac{i_{0-} - i_{-}}{R\bar{T}} = Q^{1} (\xi_{-}^{1} - \xi_{0-}^{1}) + Q^{2} (\xi_{-}^{2} - \xi_{0-}^{2}). \end{aligned}$$

Lower indices denote the position towards the shock wave surface (Fig. 3). This system of equations may be solved and its solution for given difference of pressure  $\Delta p = p_0 - \bar{p}$  may be presented in the form:

2)  

$$\frac{(\hat{u}_{-})^{2}}{R\overline{T}} = \xi_{-}^{0} \frac{\Delta p}{\overline{p}}, \qquad \frac{G_{-}}{\overline{\varrho}^{0} \sqrt{R\overline{T}}} = \frac{1}{\sqrt{\xi_{-}^{0}}} \left| \sqrt{\frac{\Delta p}{\overline{p}}} \right|,$$

$$\xi_{0-}^{2} = 1 - \xi_{0-}^{1} = \left[ \xi_{-}^{2} \left( Q^{2} + \frac{1}{2} \frac{\Delta p}{\overline{p}} \right) - (1 - \xi_{-}^{1}) \left( Q^{1} + \frac{1}{2} \frac{\Delta p}{\overline{p}} \right) \right] / (Q^{2} - Q^{1}),$$

(3.2)

where the flux of mass

$$(3.3) G_{-} = \overline{\varrho}_{-} u_{-}$$

has been introduced. The region of validity of these results must be restricted to such mixtures as may totally be condensed by the shock wave.

We may determine this region by means of the inequality

(3.4) 
$$\xi_{-}^{1} + \frac{Q^{2} + \frac{1}{2} \frac{\Delta p}{\bar{p}}}{Q^{1} + \frac{1}{2} \frac{\Delta p}{\bar{p}}} \xi_{-}^{2} > 1$$

obtained from the condition  $\xi_{0-}^2 > 0$ .

The main characteristic feature of the condensing shock wave considered is the cumulation of the flowing triple point mixture in a thin two-phase layer, close behind the surface of the shock wave. Since the volume of condensed phases should be negligibly

a two phase layer  

$$\begin{array}{c}
\underline{\hat{u}}_{-} & \underline{\hat{b}}_{-}^{2} \\
\underline{\hat{b}}_{-}^{1}, \underline{\hat{b}}_{-}^{2}, \overline{p} & \underline{\hat{b}}_{-}^{2} \\
\underline{\hat{b}}_{-}^{1}, \underline{\hat{b}}_{-}^{2}, \overline{p} \\
\underline{\hat{b}}_{-}^{1}, \underline{\hat{b}}_{-}^{1}, \underline{\hat{b}}_{-}^{2}, \overline{p} \\
\underline{\hat{b}}_{-}^{1}, \underline{\hat{b}}_{-}^{1}, \underline{\hat{b}}_{-}^{2}, \overline{p} \\
\underline{\hat{b}}_{-} & \underline{\hat{b}}_{-}^{1}, \underline{\hat{b}}_{-}^{2}, \overline{p} \\
\underline{\hat{b}}_{-} & \underline{\hat{b}}_{-}^{1}, \underline{\hat{b}}_{-}^{2}, \overline{p} \\
\underline{\hat{b}}_{-} & \underline{\hat{b}}_{-} & \underline{\hat{b}}_{-}^{1}, \underline{\hat{b}}_{-}^{2}, \overline{p} \\
\underline{\hat{b}}_{-} & \underline{\hat{b}}_{-}^{2}, \underline{\hat{b}}_$$

small, this thin layer is here considered as a discontinuity surface on which the mass of the mixture is totally condensed with the rate of cumulation  $G_{-}$ . Joining two condensing shock waves characterized by the same  $\Delta p$  and generated by two flows moving in opposite directions (Fig. 3), we may obtain a new wave, called here an accumulation wave. This wave might appear as a result of interaction of two intersecting streams of a triple point phase mixture. On both sides of an accumulation wave the pressure is the same and on its surface takes place the total condensation of the gaseous phase.

We shall now determine the parameters characterizing the plane accumulation wave generated by two unidimensional homogeneous flows of a triple point mixture (Fig. 3b).

Choosing a system of reference moving with the velocity U of the accumulation wave, we may reduce the problem to a superposition of the two condensating shock waves considered above with upstream velocities  $\hat{u}_{-} = (u_{-}-U)$  and  $u_{+} = -(\hat{u}_{+}-U)$ . Introducing these velocities into the first relation (3.2), we find

(3.5) 
$$\frac{\Delta p}{\bar{p}} = \frac{(u_{\mp} - U)^2}{\xi_{\mp}^0 R \bar{T}} = \frac{(u_{-} - u_{+})^2}{(\sqrt{\xi_{+}^0} + \sqrt{\xi_{-}^0})}, \quad U = \frac{u_{-} \sqrt{\xi_{+}^0} + u_{+} \sqrt{\xi_{-}^0}}{\sqrt{\xi_{+}^0} + \sqrt{\xi_{-}^0}},$$

and from the second relation (3.2) we find the total accumulation flow rate

(3.6) 
$$G = G_{-} + G_{+} = \bar{\varrho}^{\circ} \sqrt{R\bar{T}} \left( \frac{1}{\sqrt{\xi_{-}^{\circ}}} + \frac{1}{\sqrt{\xi_{+}^{\circ}}} \right) \sqrt{\frac{\Delta p}{\bar{p}}} = \bar{\varrho}^{\circ} \frac{u_{-} - u_{+}}{\sqrt{\xi_{-}^{\circ} \xi_{+}^{\circ}}}.$$

The diagrams of  $u_{+/}U$  versus  $u_{-/}U$  for  $\xi_{+}^0/\xi_{-}^0 = \text{const}$ ,  $u_{+/}u_{-} = \text{const}$  and V/U = const, where

(3.7) 
$$V = \sqrt{\xi_-^0 \xi_+^0} G/\overline{\varrho}^0 = \left(\sqrt{\xi_-^0} + \sqrt{\xi_+^0}\right) \sqrt{R\overline{T}} \cdot \sqrt{\Delta p/\overline{p}},$$

are presented in Fig. 4. For given velocities  $u_-$ ,  $u_+$ , and mass ratios  $\xi_-^1$ ,  $\xi_-^2$ ,  $\xi_+^1$ ,  $\xi_+^2$  on both sides of an accumulation wave we may find, from the diagrams in Fig. 4 or from the



formulae (3.5), (3.6), (3.2), the velocity and other quantities characterizing the accumulation wave. We shall explain this by a simple example.

Let us consider two triple point mixtures of water  $(R = 462 \text{ m}^2/\text{s}^2 \cdot \text{°K}, \overline{T} = 273 \text{ °K}, \overline{p} = 611 \text{ N/m}^2, Q^1 = 19.8, Q^2 - Q^1 = 2.9, \varrho^{1,2} \approx 10^3 \text{ kg/m}^3)$  with mass ratios

$$\xi_{-}^1 = 0.45, \quad \xi_{-}^2 = 0.5, \quad \xi_{+}^1 = 0, \quad \xi_{+}^2 = 0.9,$$

moving in opposite directions with velocities  $|u_{-}| = 10$  m/s and  $|u_{+}| = 2$  m/s. Thus we immediately obtain  $\bar{\varrho}^{0} = \bar{p}/R\bar{T} = 0.00485$  kg/m<sup>3</sup>,  $\xi_{-}^{o} = 0.05$ ,  $\xi_{+}^{o} = 0.1$  and for  $\xi_{+}^{o}/\xi_{-}^{o} = 2$ ,  $u_{+}/u_{-} = -0.2$ , we find from the diagram in Fig. 4 (the crossed point)  $u_{-}/U = 2$ , V/U = 2.4. We may verify also that, as assumed in (1.1), the volume ratios are very small  $\xi_{\pm}^{1,2} \bar{\varrho}^{0}/\xi^{0} \varrho^{1,2} \leq 10^{-4}$ . From the above data we obtain the velocities:

$$U = 5 \text{m/s}, \quad u_{-} = 10 \text{m/s}, \quad u_{+} = -2 \text{m/s}, \quad V = 12 \text{m/s};$$

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the flux of mass and the difference of pressure (3.7):

$$G = 0.82 \text{kg/m}^2 \text{s}, \quad \Delta p/\bar{p} = 0.0039;$$

and the mass ratios (3.2):

$$\xi_{0-}^2 = 1 - \xi_{0-}^1 = 0.16, \quad \xi_{0+}^2 = 1 - \xi_{0+}^1 = 0.22.$$

The thickening rate of the accumulation layer  $G/\varrho^{1,2} = 0.8 \cdot 10^{-3} \text{ m/s}$  is much less than the flow velocities.

In the framework of the equilibrium model E of the triple point mixture, we obtained some information concerning the conditions of generating accumulation waves. To obtain more details concerning the inner structure and the real thickness of these waves, it would be necessary to take into account the very complex dissipative phenomena occurring in the flow but such is not the object of this paper.

#### 4. Isobaric flows of the model E

The triple point mixture may exist in equilibrium in isobaric conditions described by its equations of flow (1.2).

When the body force is equal to zero  $\mathbf{f} = 0$ , the solution of the first Eqs. (1.2) is very simple: each point of the mixture moves in the isobaric region with a constant velocity on its straight line trajectory. The field of flow in the triple point region would be composed of all these straight line trajectories, with accumulation waves eventually appearing. But outside the triple point region, two phase regions may exist and the unknown surfaces dividing these regions should be determined by solving a complex problem with given boundary conditions.

The existence of a body force  $\mathbf{f} \neq 0$  has an important influence on the problem under consideration because the isobaric conditions are here especially difficult to realize. For instance if  $\mathbf{u} = 0$ , the state of mechanical equilibrium of a triple point mixture may be realized on a surface  $p = \bar{p} = \text{const}$  only, and not in a three-dimensional space. Not only here but also in other flow problems it is doubtful whether the idealized model E may be used for a satisfactory description of the reality.

Returning to the first model of an isobaric flow with body force equal to zero  $\mathbf{f} = 0$ , we will present here, as examples, two exact solutions of the Eqs. (1.2), describing steady, cylindrical or spherical one-dimensional flows of the triple point mixture. Since, according to the first equation (1.2)  $\partial \mathbf{u}/\partial t + (\mathbf{u} \cdot \nabla)\mathbf{u} \equiv u du/dr = 0$ , u = const along straight stream lines, we obtain:

(4.1) 
$$\nabla \cdot \mathbf{u} \equiv \frac{d(r^k u)}{r^k dr} = k \frac{u}{r}, \quad k = \begin{cases} 1 \text{ for cylindrical case,} \\ 2 \text{ for spherical case,} \end{cases}$$

where r is the cylindrical or spherical coordinate (radius) and u is the radial velocity. Introducing (4.1) into the second Eqs. (1.2), we obtain two equations:

(4.2) 
$$\frac{d\xi^n}{dr} = -ka^n \frac{1}{r} (1-\xi^1-\xi^2),$$

which are fulfilled by the solution:

(4.3) 
$$\xi^{n} = a^{n} \frac{(r^{m})^{k} - (r)^{k}}{a^{1}(r^{2})^{k} + a^{2}(r^{1})^{k}}, \qquad \substack{m \neq n, \\ m, n = 1, 2. \end{cases}$$

The constants  $r^1 < r^2$  chosen determine the boundaries of the triple point region. On  $r = r^1$  or  $r = r^2$  the solid or liquid phases respectively vanish and the region of isobaric expansion (u > 0) or compression (u < 0) is determined by  $r^1 < r < r^2$ . We shall not discuss here the possibilities of physical realization of such flows.

## 5. Final remarks

For description of flow of many continuous media, two sorts of approximation models are, in general, applied. In the first approximation the dissipation of energy is disregarded (for instance the non-viscous gas, etc.) and in the second approximation the dissipation is taken into account to correct the non-dissipative solution.

For a one-component three-phase mixture, the flow of its non-dissipative model E must be isobaric, with all resulting consequences — often paradoxical (such as vanishing of the speed of sound, impossibility of realizing of straight line trajectories in the neighbourhood of curved walls, etc). Since the non-vanishing gradient of pressure exerts an important influence on flow phenomena, it may be expected that in non-isobaric flows the non-dissipative model might not give a satisfactory description of reality, and the dissipative effects, even in the first approximation, should be taken into account. However, even the simplest dissipative models (such as the model P) are sufficiently complicated to involve considerable difficulty in solving even comparably simple flow problems.

It seems that the peculiar properties of flows with slow velocities of one-component three-phase mixtures would be an interesting theme for experimental research also, and should yield information about their real properties.

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