780.

[ADDITION TO MR HAMMOND'S PAPER "NOTE ON AN EXCEPTIONAL CASE IN WHICH THE FUNDAMENTAL POSTULATE OF PROFESSOR SYLVESTER'S THEORY OF TAMISAGE FAILS."]

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THE extreme importance of Mr Hammond's result, as regards the entire subject of Covariants, leads me to reproduce his investigation in the notation of my Memoirs on Quantics, and with a somewhat different arrangement of the formulæ. For the binary seventhic

$$(a, b, c, d, e, f, g, h)(x, y)^7,$$

the four composite seminvariants of the deg-weight 5 . 11 (sources of covariants of the deg-order 5 . 13) are

I.		II.		
1 . 7	4 . 6	2 . 10	3 . 3	Deg-order.
1 . 0	4 . 11	2 . 2	3 . 9	Deg-weight.
a + 1	a^2eh + 1	ac + 1	$a\,ch$ + 2	
	fg − 1	b^2 − 1	dg − 7	
	$a\,bdh$ − 4		ef + 5	
	beg − 2		a^0b^2h − 2	
	bf^2 + 6		bcg + 7	
	c^2h + 3		bdf + 22	
	cdg − 2		be^2 − 25	
	cef − 6		c^2f − 27	
	d^2f + 10		cde + 45	
	de^2 − 5		d^3 − 20	
	a^0b^2ch 0			
	b^2dg + 20			
	b^2ef + 57			
	bc^2g − 15			
	$bcdf$ − 24			
	bce^2 − 30			
	bd^2e − 10			
	c^3f + 27			
	c^2de − 45			
	cd^3 + 20			

III.			IV.		
2 . 6	3 . 7	... ...	2 . 2	3 . 11	Deg-order.
2 . 4	3 . 7	... ...	2 . 6	3 . 5	Deg-weight.
$ae + 1$	$a^2h + 1$		$ag + 1$	$a^2f + 1$	
$bd - 4$	$a\,bg - 7$		$bf - 6$	$a\,be - 5$	
$c^2 + 3$	$cf + 9$		$ce + 15$	$cd + 2$	
	$de - 5$		$d^2 - 10$	$a^0b^2d + 8$	
	$a^0b^2f + 12$			$bc^2 - 6$	
	$bce - 30$				
	$bd^2 + 20$				

and it is here at once obvious that there exists a syzygy of the form I. = III. – IV.; in fact, if in III. and IV. we write $a = 0$, then the values are each

$$= -2b\,(4bd - 3c^2)\,(6bf - 15ce + 10d^2);$$

hence III. – IV. must divide by a, the quotient being a seminvariant of the deg-weight 4 . 11, which can only be a numerical multiple of the second factor of I., and is in fact = this second factor, that is, we have the syzygy I. = III. – IV.

Working out the values of the four products, and joining to them the expression for the irreducible seminvariant of the same deg-weight 5 . 11 (O, x^8 of my tables [774] for the binary sextic), we have the table:

5 . 10	5 . 11	O	I.	III.	IV.	II.
a^3dh	a^3eh		+ 1	+ 1		
eg	fg		– 1		+ 1	
f	a^2bdh		– 4	– 4		
a^2bch	beg		– 2	– 7	– 5	
bdg	bf^2		+ 6		– 6	
bef	c^2h		+ 3	+ 3		+ 2
c^2g	cdg		– 2		+ 2	– 7
cdf	cef	– 1	– 6	+ 9	+ 15	+ 5
ce^2	d^2f	+ 3	+10		– 10	
d^2e	de^2	– 2	– 5	– 5		
$a\,b^3h$	$a\,b^2ch$					– 4
b^2cg	b^2dg		+20	+ 28	+ 8	+ 7
b^2df	b^2ef	+ 1	+57	+ 12	– 45	– 5
b^2e^2	bc^2g		–15	– 21	– 6	+ 7
bc^2f	$bcdf$	–14	–24	– 36	– 12	+22
$bcde$	bce^2	+11	–30	– 30		–25
bd^3	bd^2e	+ 1	–10	+ 40	+ 50	
c^3e	c^3f	+ 9	+27	+ 27		–27
c^2d^2	c^2de	–14	–45	– 15	+ 30	+45
a^0b^4g	cd^3	+ 6	+20		– 20	–20
b^3cf	a^0b^4h					– 2
b^3de	b^3cg					– 7
b^2c^2e	b^3df	+ 8		– 48	– 48	–22
b^2cd^2	b^3e^2	– 9				+25
bc^3d	b^2c^2f	– 6		+ 36	+ 36	+27
c^5	b^2cde	+16		+120	+120	–45
	b^2d^3	– 8		– 80	– 80	+20
	bc^3e	– 3		– 90	– 90	
	bc^2d^2	+ 2		+ 60	+ 60	
	c^4d					

I have prefixed to the table the literal terms of the deg-weight 5 . 10; for the deg-weights 5 . 11 and 5 . 10, the numbers of terms are = 30 and 26 respectively; and it is the difference of these 30 – 26, = 4, which gives the number of asyzygetic seminvariants of the deg-weight 5 . 11.