779.

[NOTE ON MR JEFFERY'S PAPER "ON CERTAIN QUARTIC CURVES WHICH HAVE A CUSP AT INFINITY, WHEREAT THE LINE AT INFINITY IS A TANGENT."]

[From the *Proceedings of the London Mathematical Society*, vol. XIV. (1883), p. 85.]

THE assumed form $\kappa\alpha^3\beta = u_2$, or, as this is afterwards written,

$$2\kappa x^3 y = ax^2 + 2bxy + cy^2 + 2ex + 2dy + \lambda,$$

is, I think, introduced without a proper explanation. Say, the form is $x^3y = z^2(*\between x, y, z)^2$, it ought to be shown how for a cuspidal quartic we arrive at this form; viz. taking the cusp to be at the point $(x=0,\ z=0)$, $z=0$ for the tangent at the cusp, and $x=0$ an arbitrary line through the cusp; then the line $z=0$ besides intersects the curve in a single point, and, if $y=0$ is taken as the tangent at that point, the equation of the curve must, it can be seen, be of the form

$$(x^3 + \theta x^2 z)\, y = z^2 (a,\ b,\ c,\ f,\ g,\ h\between x,\ y,\ z)^2.$$

The conic $(a, b, c, f, g, h\between x, y, z)^2 = 0$ touches the quartic at each of the two intersections of the quartic with the arbitrary line $x=0$; and we cannot, so long as the line remains arbitrary, find a conic which shall osculate the quartic at the two points in question; but, for the particular line $x + \frac{1}{3}\theta z = 0$, there exists such a conic, viz. writing x instead of $x + \frac{1}{3}\theta z$, the form is $x^3y = z^2(a', b', c', f', g', h'\between x, y, z)^2$, and the new conic $(a', \ldots\between x, y, z)^2 = 0$ has the property in question. This is the adopted form, and it thus appears that in it the line $x=0$ is a determinate line, viz. the line passing through the cusp and the two points of osculation of the osculating conic. It thus appears that in the assumed form the lines $x=0$, $y=0$, $z=0$ are determinate lines.