ON FE MODELING OF A VIBRATING BODY
CONTROLED BY SLIDING SOME OF ITS COMPONENTS

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1. Extended Abstract

A FE procedure to simulate a multibody system consisting of a main body and sliding small components is proposed. The procedure is suitable for control purposes since it allows imposing explicitly any relative motion of each component as input to obtain the response of the main body as output. In particular, attenuation effects in a vibrating main body can be generated by precisely synchronizing the current body configuration with relative motion of one or more of its components. Such effects are possible to obtain because the dynamic interaction between the main body and the sliding component triggers the Coriolis type forces that are generally capable of producing either attenuation or amplification in different phases of a cycle of the vibrating main body.

By maximizing these forces during attenuation phase and minimizing during amplification phase the reduction of the system's vibration in each cycle can be achieved [1]. It should be noted that handling such a task by the 'standard' FE approach (as used, for example, in vehicle-bridge or vehicle-rail tract simulations and involving typically only constant not controlled relative velocities) would be challenging mainly due to complexity of the geometrical constraints to be imposed between the sliding components and the main body.

In our procedure each component is forced to slide with an assumed $s(t)$ along a prescribed path (AB in Figure 1a) modeled by the beam elements (that represents a guiding beam, which can be real or fictitious) attached at nodes (see Figure 1b) to the main body meshed by any elements appropriate to simulating its vibrations. This way the sliding component interacts with the main body via the guiding beam in which only one currently traversed element (referred to as the composite element) is affected by the relative motion.
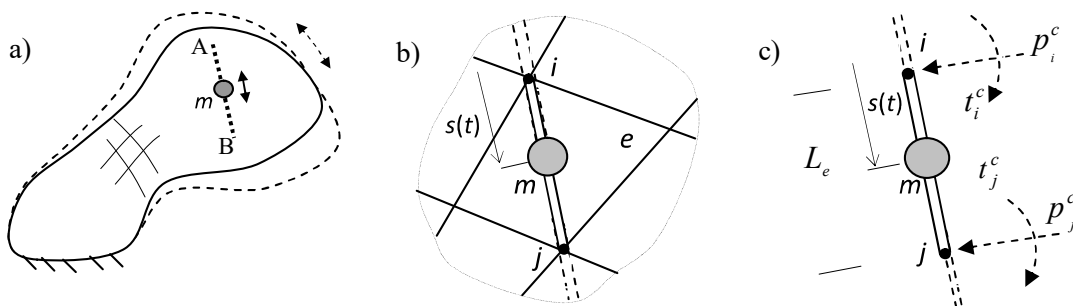

Figure 1: The vibrating system (a) the guiding beam (b), and the composite element (c)

This element has a time-dependent mass matrix, $\mathbf{M}_e(t)$ and, as it was shown in [2], the Coriolis forces generated in it can be explicitly identified in terms of the component's current position and velocity, which in turn permits relating directly these forces to assumed trajectories of the components and selecting the one resulting in 'best' attenuation. This is possible because the element equation for the composite element is derived in the following forms (in order to concentrate on the active attenuation any passive damping is omitted):

(1a,b) $\qquad \mathbf{M}_e(t)\ddot{\mathbf{u}}_e + \mathbf{C}_e\dot{\mathbf{u}}_e + \mathbf{K}_e\mathbf{u}_e = \mathbf{F}_e \qquad$ or $\qquad \mathbf{M}_e(t)\ddot{\mathbf{u}}_e + \mathbf{K}_e\mathbf{u}_e = \mathbf{F}_e - \mathbf{f}_e^c$

where $\mathbf{C}_e = \dfrac{d\mathbf{M}_e}{dt} = m\dot{s}\dfrac{\partial}{\partial s}(\mathbf{N}^T\mathbf{N})$ represent the Coriolis effects and can be treated as an 'instantaneous' damping matrix (where $\mathbf{N}=\mathbf{N}(s(t))$ are the values of the beam's shape functions matrix at the current mass location). The term $\mathbf{f}_e^c = \mathbf{C}_e\dot{\mathbf{u}}_e$ defines the 'Coriolis' force vector due to the relative motion (consisting of forces p^c and moments t^c as indicated in Figure 1c) that is assigned to the end nodes of the element. As can be seen the matrix $\mathbf{C}_e$ and vector $\mathbf{f}_e^e$ depend *explicitly* on the current relative velocity $\dot{s}$ of the mass and *implicitly* on its current position between the nodes of the element (the position is hidden in functions $\mathbf{N}$).

The first form, eq. (1a), indicates how the relative motion relates to periods of attenuation ($C_e^{ii} > 0$, if $\dot{s} > 0$) and to periods of amplification ($C_e^{ii} < 0$ if $\dot{s} < 0$), while the second form, eq. (1b), is more suitable for the practical simulation due to the fact that all the LHS terms can be routinely handled by typical FE software (in our case by ANSYS), while the values of $\mathbf{f}_e^e$ on the RHS can be easily calculated and added to forces at each time step of the integration procedure. The system's attenuation is achieved by controlling forces $\mathbf{f}_e^e$ (via properly choosing $s(t)$) that are to be maximized in the periods of attenuation and minimized in the periods of amplification.

The corresponding control schemes for the components' motion resulting in attenuation (active) of the main body will be presented. In fact these schemes generate 'inverse' self-excited vibrations that can be characterized by a positive damping coefficient. As illustration the test case of using relative motion of two small masses to attenuate vibrations of a frame is presented in Figure 2. For this particular case an effective active damping ratio of about 2.7% was obtained.
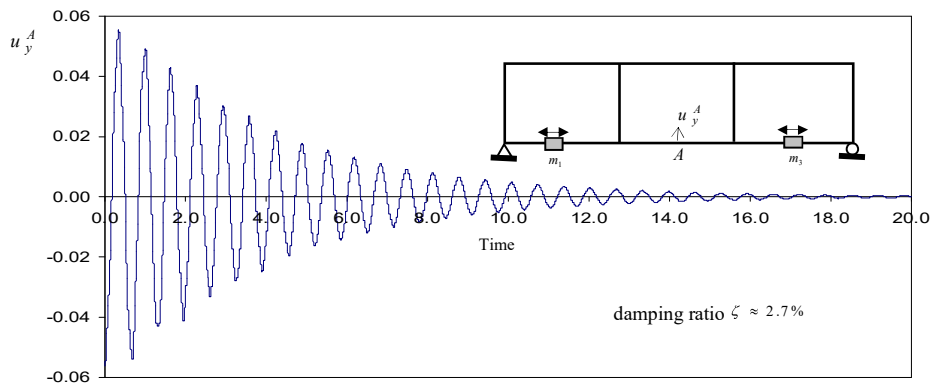

Figure 2: The response of vibrating frame controlled by motion of two masses.

The implementation and accuracy of the procedure, which may be particularly useful in providing a unique means for active reduction of vibrations in the systems for which internal/external passive damping is almost absent or not available, will be discussed in details.

2. References

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[2] W. Szyszkowski and E. Sharbati, On the FEM modeling of mechanical system controlled by relative motion of a member: A pendulum-mass interaction test case, *Finite Elements in Analysis and Design*, 45:730, 2009