THREE DIMENSIONAL REISSNER BEAM WITH NON-LINEAR CONTACT BETWEEN LAYERS

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1. Introduction

In this contribution, an initially curved composite Reissner beam is considered, [2]. The proposed numerical model enables simulations of arbitrary contact between laminae, modelled as a distributed load as a function of displacements. Delaminations of any size and direction can be simulated. We use quaternions for parametrization of rotations [4]. Rotation of an arbitrary vector using quaternion can be expressed as $\overline{a} = \hat{q} \circ \hat{a} \circ \hat{q}^* = \Phi_L(\hat{q})\, \Phi_R(\hat{q}^*)\hat{a} = Q(\hat{q})\hat{a}$. We use Q to denote the 4×4 rotational matrix.

2. Formulation

A three-dimensional beam element is uniquely described by the position vector $r_g(x)$ of the beam centroid axis and the orthonormal base vectors $G_1(x)$, $G_2(x)$, $G_3(x)$, where G_2 and G_3 span the plane of its cross-section and G_1 is its normal vector. Parameter $x \in [0, L]$ is the arc-length of elements centroid axis. We assume that the cross-sections are rigid and conserve their shape during deformation. Another set of fixed orthonormal vectors g_1, g_2, g_3 define physical space from reference point $\mathcal{O}$ of global coordinate system X, Y, Z, Figure 1.

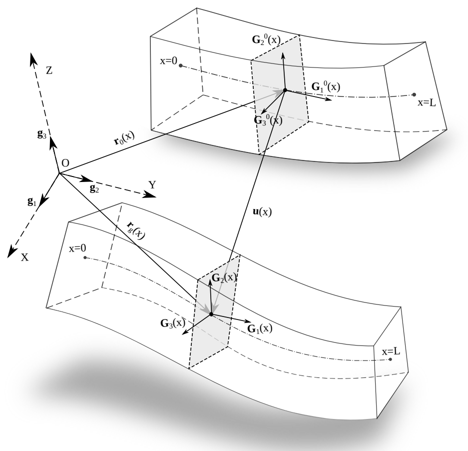

Figure 1: Undeformed and deformed arbitrary beam element in space.

The equilibrium equations of an element of a beam are given by the following set of differential equations:

$$N_g'(x) + n_g(x) - k\, r_g(x) = 0, \tag{1}$$

$$M_g'(x) + r_g'(x) \times N_g(x) + m_g(x) = 0. \tag{2}$$

The force $N_g(x)$ and the moment $M_g(x)$ stress resultants depend on the external distributed force and moment vectors $n_g(x)$ and $m_g(x)$, respectively, both measured per unit of the undeformed length of the axis. We add to this formulation a distributed contact force $k = \text{diag}\,[k_x, k_y, k_z]$ to model general nonlinear behavior between the beam and the foundation or bonds between adjacent layers in composite beams.

Constitutive equations describe a connection of stress resultants with strain vectors γ_G and κ_G. The translational strain vector γ_g is expressed with respect to global basis and rotational strain vector κ_G with respect to material basis. Both are assumed constant throughout the length of the finite element of the beam, as in Refs. [1], [3],

$$\gamma_g = [\hat{q} \circ \hat{\gamma}_G \circ \hat{q}^*] = r_g' + c_g, \tag{3}$$

$$\hat{\kappa}_G = 2\hat{q}^* \circ \hat{q}' + \hat{d}_G, \tag{4}$$

where c_G and d_G are constants, given in the initial configuration. System of equations is constructed with constitutive, equilibrium, kinematic equations and boundary conditions.

Solution of a linearized system $K^{[n]}\delta y = -f^{[n]}$ in each step results in a vector of corrections $\delta y = \left[\delta r_g^0, \delta\hat{q}^0, \delta r_g^L, \delta\hat{q}^L, \delta N_g^0, \delta M_g^0, \delta\gamma_g, \delta\kappa_g\right]^T$. Update procedure for rotational quaternions requires special attention. Calculated correction rotational quaternion $\delta\hat{q}$ will be used to obtain multiplicative incremental rotational quaternion $\Delta\hat{q}^p = \cos(\delta\hat{q}^p \circ \hat{q}^{*p}) + \frac{[\delta\hat{q}^p \circ \hat{q}^{*p}]_{\mathbb{R}3}}{|\delta\hat{q}^p \circ \hat{q}^{*p}|}\sin(\delta\hat{q}^p \circ \hat{q}^{*p})$, that is combined with the current rotational quaternion as $\hat{q}^{p[n+1]} = \Delta\hat{q}^p \circ \hat{q}^{p[n]}$.

3. Numerical example

We solved several examples and compared results with other authors to confirm the accuracy of our procedure. Here, we present an example from an adhesive shear stiffness test.

Adhesive stiffness is described with hyperbolic tangent function of spatial distance between two FE. To simulate shear test, two beams are held together by an adhesive and then pulled apart while measuring the force. Sketch of the test and the stiffness function is shown in Figure 2. Simulation results are shown as a relation between force and displacement of both beams. It is evident how beams return to initial configuration after the bond breaks, since only elastic deformation is considered.

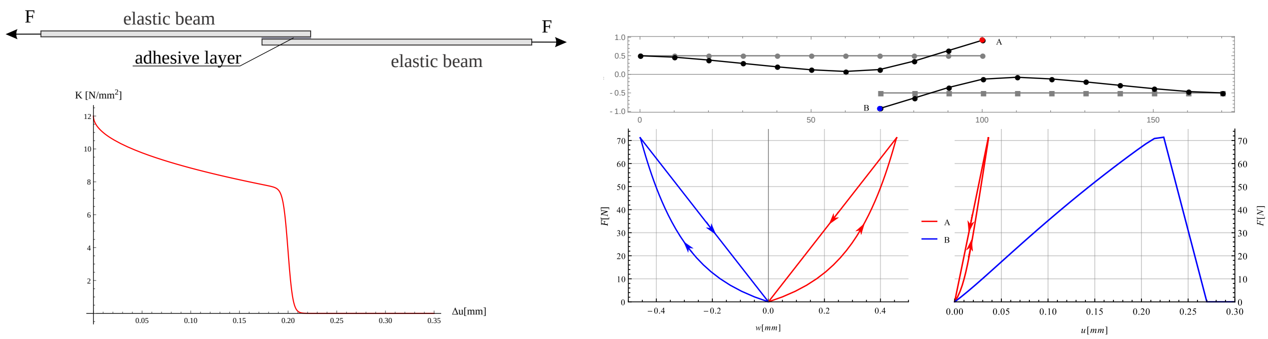

Figure 2: Shear test of two beams connected by adhesive with defined properties.

References

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