A UNIFIED APPROACH TO ADAPTVE MODELLING AND SIMULATION IN COUPLED AND SOLID MECHANICS PROBLEMS

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1. Introduction

The main goal of this paper is to present some compuational technology for the adaptive analysis of the problems of solid mechanics and the coupled problems as well. The adaptation concerns both the modelling and solution of the problems. The main feature of the presented methodology is its unifying character, which means that the same methods are applied to hierarchical modelling and hierarchical approximations of the problems under consideration. Furthermore, the analogous a priori error estimation methods, serving the same a posteriori error estimation algorithms and analogous adaptivity control schemes, are applied to the mentioned problems.

The presented methodology is assigned for complex problems in which three types of complexity are allowed. The physical complexity consists in the possibility to consider the coupled problems where the fields of various physical character are analysed together. The geometrical complexity lies in the possibility to apply structures or domains composed of different geometrical parts, i.e. solid parts, thin- or thick-walled symmetric-thickness members, and transition parts as well. Finally, the model complexity is related to the possible application of more than one mathematical model for the decription of the uncoupled physical phenomenon or coupled physical phenomena under consideration regardless of the division of the structure or domain into geometrical parts. In this paper the expemplification of the approach includes the uncoupled problems of elasticity and dielectricity and the coupled problem of piezoelectricity.

2. State-of-the-art issues

The 3D-based hierarchical models considered here conform to the primal displacement variational formulation. Such models were proposed first in the initiating work [3] concerning shell structures. The models were generalized in [11] for a wider class of 3D-based elastic theories. The a priori error estimation method for the modelling error is based on the general dimensional reduction method of [1,2], exemplified by the conventional shell models [6]. The a priori error estimation for the approximation error is based on the general considerations for the hp-approximations of the elliptic problems. The a posteriori error estimation takes advantage of the equilibrated residual method, elaborated for the approximation error in the uncoupled elliptic problems [8]. The method was applied to 3D-elasticity in [5] and the conventional hierarchical shell models in [7]. The method was also adopted for the 3D-based elastic models in [12]. The adaptivity control method uses the three-step strategy [4]. Such an adaptive strategy was used for the conventional plate- and shell-like structures in [7]. The method was extended to the 3D-based complex elastic structeres in [12].

Application of the above mentioned methods to dielectricity and piezoelectricty is less common. Hierarchical models of dielectrics and piezoelectrics were elaborated in [9] up to the fourth order for the case of the mixed variational formulation. The 3D-based models within the primal variational formulations were presented in [13, 14]. Therein, also the hierarchical approximations, analogous to the elasticity case, were proposed. The a priori modelling error estimation for the case of dielectricity were performed in the unpublished work of this author. The a priori approximation error estimation can be based on the general considerations concerning elliptic problems. The a posteriori error estimation and adaptivity control algorithms based on the equilibrated residual methods for the cases of dielectricity and piezoelectricity were introduced in [13, 14]. The same works include also the three-step adaptivity control algorithms for both the cases. The alternative way of adaptivity control for the case of piezoelectricity can be found in [10].

3. Findings and attainments of the research

The following novely elements are presented in the paper. The idea of the 3D-based hierarchy of models of linear elasticity, including 3D-elasticity, first- and higher-order hierarchical shell models, and solid-to-shell transition models, has been adopted for dielectricity. The hierarchy of 3D-based dielelectric models consists of the 3D-dielectricity model and the hierarchical symmetric-thickness models of dielectricty. In the case of the piezoelectricity, both hierachies are applied in such a way that any combination of the elastic and dielectric models is possible. As far as the hierachical hp-approximations are concerned, the same hierarchical shape functions, constrained approximations, and transition approximations as well, have been applied to the above elastic, dielectric and piezoelectric hierarchies of models. Furthermore, for any of three media, the hierachies of the corresponding models have been combined with the related hierarchical approximations. This way, three hierarchies of numerical models of elasticity, dielectricity, and piezoelectricity have been generated.

In the case of the a priori error estimation, the existing results for elasticity have been extended onto dielelectricty. The a posteriori error estimation methods, based on the equilibrated residual method applied earlier to both conventional and 3D-based models, have been applied to the 3D-based models of the uncoupled dielectricity and 3D-based models of the coupled piezoelectricity. The related algorithms for the splitting functions determination and for solution of the local problems have been elaborated and theoretically substantiated.

The methods of model- and hp-adaptivity control based on the three-step strategy have been extended to the complex problems where the physical, geometrical and model complexity may appear. The methods cover elastic, dielectric and piezoelectric problems, and their electro-mechanical combinations as well.

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