FRACTION HYPERELASTIC DAMAGE MODEL FOR ROOF MEMBRANES MATERIALS

Bartosz Łuczak[1], Wojciech Sumelka[1], and George Z. Voyiadjis[2]

[1]*Institute of Structural Engineering, Poznan University of Technology, Piotrowo 5 street, 60-965, Poznan, Poland*
[2]*Boyd Professor, Department of Civil and Environmental Engineering, Louisiana State University, Baton Rouge, LA 70803, USA*

e-mail: {bartosz.luczak, wojciech.sumelka}@put.poznan.pl, voyiadjis@eng.lsu.edu

Roofing are an important constituent of each building structure. Due to their meaningful function the materials which are used to construct this element should meet very rigorous requirements, especially to manage extreme weather conditions like moisture penetration, high temperatures gradients, rainfalls, snowfalls or UV radiation. Therefore, the time in which this materials meet their requirements are relatively short in comparison to the life cycle of the whole building. This results in the need to make repairs and renovations which usually are expensive, hence even small elongation of the life time of these materials will cause significant savings.

To make the life time predictions of roofing materials possible the computer-aided engineering tools are needed. Hence, in this article the modelling of roof membranes in the framework of the hyperelastic fractional damage material model with memory presented by W. Sumelka and G.Z. Voyiadjis in [5] is applied. Furthermore, the extension of this model to include healing effects will be discussed [6]. In the model, the original idea of extension of the classical scalar hyperelastic damage model (energy equivalence formulation) utilizing fractional calculus [1, 3, 4] was postulated. The evolution of damage variable ϕ has obtained the fractional form

$$ {}^{C}{}_{t-l_t}D^{\alpha}{}_{t}\phi = \frac{1}{T^{\alpha}}\Phi\left\langle \frac{I_\phi}{\tau_\phi} - 1 \right\rangle, \tag{1}$$

where the bracket $\langle \, . \, \rangle$ defines the ramp function and the ${}^{C}D$ is the left-sided Caputo derivative

$$ {}^{C}{}_{a}D^{\alpha}{}_{t}f(t) = \frac{1}{\Gamma(n-\alpha)} \int_a^t \frac{f^{(n)}(\tau)}{(t-\tau)^{\alpha-n+1}} d\tau \qquad \text{for} \quad t > a, \tag{2}$$

where t denotes time variable, α is an order of derivative (order of fractional velocity), T stand for the characteristic time, Φ is the overstress function, I_ϕ is the stress intensity invariant, τ_ϕ is the threshold stress for damage evolution, Γ is the Euler gamma and $n = [\alpha] + 1$ function.

The investigations are divided into three parts. First one aimed to reconstruct the results presented in [5] utilizing software for symbolic mathematical calculations *Wolfram Mathematica*. Next, the implementation of the concept of hyperelastic fractional damage material model with memory in *AceFem* software is considered. The choice of *AceFem* software was dictated by its structure (the system combines symbolic and numeric approaches) and because its environment is designed to solve multi-physics and multi-field problems. Finally, the last step covers the preparation of a simple 3D model with "memory". Simulations, include a cubic geometry induced for basic deformation modes [2].

References

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