SPRAWOZDANIA
Z POSIEDZEŃ WYDZIAŁU III T.N.W.

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SOCIÉTÉ DES SCIENCES ET DES LETTRES DE VARSOVIE

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XLIV[e] ANNÉE 1951

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1952

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Z ZASIŁKU MINISTERSTWA SZKOLNICTWA WYŻSZEGO
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Michał Kamieński

Researches on the origin of the Comet P/Wolf I

Part IX

Influence of Jupiter and Saturn on the motion of the Comet during the period

1784 Oct. 24.5 — 1776 June 18.5

Mémoire présenté à la séance du 7 mai 1951.

1. The system of elements P_{-18} deduced in Part VIII of the author's Researches on the Origin of the Comet Wolf I was taken as a basis for the investigations given below:

1784 Oct. 24.5 Greenwich Mean Time

$$P_{-18} \ldots \begin{array}{ll} M = 358^0\ 6'11''.6 & \Omega = 212^0 57'39''.5 \\ \varphi = 23^0 45'22''.4 & \pi = 12^0 34'55''.5 \\ \mu = 422''.2868 & i = 26^0 46'56''.1 \end{array} \Bigg\} \ 1950.0$$

With the help of it the backward perturbations in the comet's motion, due to Jupiter and Saturn during the whole period of its revolution 1784—1776, were carried on. The actual computations were performed by applying the method of variation of arbitrary constants on changing permanently the systems of elements every 50 days, in order to avoid of complementary calculations of the second order perturbations. The 50-day interval appeared as quite adequate for this period because the Comet did not approach close to Jupiter. Namely, its distance from this planet varied from 6.4 to 3.7, as it can be seen from the enclosed Tables. The minimum distance $\triangle = 2.08$ of these bodies took place at the end of 1779. Nevertheless, even so faint an approach of the Comet to Jupiter provoked considerable variations of its motion, surpassing 4^0 in the mean anomaly and attaining nearly $8''$ in its mean daily motion.

After the integration of the differentials of perturbations given in the Tables below, the author obtained the following results:

1784 Oct. 24.5 — 1776 June 18.5			
	Jupiter	Saturn	Total
δM	$-14556''.5$	$-194''.4$	$-245'50''.9$
$\delta\varphi$	$+2340''.3$	$+45''.4$	$+39'45''.7$
$\delta\Omega$	$+2261''.2$	$+17''.4$	$+37'59''.0$
$\delta\pi$	$+1258''.5$	$+89''.2$	$+22'27''.7$
δi	$-161''.2$	$+15''.7$	$-2'25''.5$
$\delta\mu$	$+7''.6892$	$+0''.0314$	$+7''.7206$

2. Adding the above totals to the system P_{-18} of elements, the author got the following perturbed system P_{-19}:

1776 June 18.5 Greenwich Mean Time

$$P_{-19} \dots \left.\begin{array}{ll} M = 356^0 14' 6''.0 & \Omega = 213^0 35' 38''.5 \\ \varphi = 24^0 25' 8''.1 & \pi = 12^0 57' 23''.2 \\ \mu = 430''.0074 & i = 26^0 44' 30''.6 \end{array}\right\} 1950.0$$

This last system will serve as a basis for the backwards computation of the perturbations in the Comet's motion up to the beginning of 1750. Thus, the path of the comet will be investigated during the period of 200 years, 1750—1950, because it was rediscovered in June 1950 when returning the tenth time to its perihelion since its first discovery in 1884. Let us note en passant that in the author's researches on the motion of this comet for the period 1884—1950, the influence of all the planets was taken into consideration. As to the backward period 1884—1750, the influences of Jupiter and Saturn only were taken into account, what is quite sufficiet for our purpose.

Cracow, August 28, 1950.

JUPITER

1784 Oct. 24.5 — 1776 June 18.5

	$d\delta\Omega$	$'f$	$d\delta i$	$'f$	$d\delta\varphi$	$'f$	Δ
	″	″	″	″	″	″	
1776 March 10.5	— 3·1	+2263·7	+ 0·8	—162·5	— 8·7	+2350·5	6·442
Apr. 29.5	— 2·0	+2261·7	+ 0·9	—161·6	— 7·4	+2343·1	6·069
June 18.5	— 0·9	+2260·8	+ 0·7	—160·9	— 5·1	+2338·0	5·685
Aug. 7.5	— 0·1	+2260·7	+ 0·2	—160·7	— 2·1	+2335·9	5·296
Sept. 26.5	0·0	+2260·7	— 0·6	—161·3	+ 1·4	+2337·3	4·916
Nov. 15.5	— 1·1	+2259·6	— 1·8	—163·1	+ 4·6	+2341·9	4·558
1777 Jan. 4.5	— 3·8	+2255·8	— 3·1	—166·2	+ 6·9	+2348·8	4·234
Feb. 23.5	— 8·4	+2247·4	— 4·4	—170·6	+ 8·2	+2357·0	3·945
Apr. 14.5	— 15·0	+2232·4	— 5·4	—176·0	+ 8·1	+2365·1	3·695
June 3.5	— 23·4	+2209·0	— 5·8	—181·8	+ 6·9	+2372·0	3·481
July 23.5	— 33·2	+2175·8	— 5·6	—187·4	+ 4·5	+2376·5	3·301
Sept. 11.5	— 44·0	+2131·8	— 4·6	—192·0	+ 1·4	+2377·9	3·147
Oct. 31.5	— 55·1	+2076·7	— 2·8	—194·8	— 2·4	+2375·5	3·016
Dec. 20.5	— 66·1	+2010·6	— 0·1	—194·9	— 6·8	+2368·7	2·901
1778 Feb. 8.5	— 76·7	+1933·9	+ 3·2	—191·7	— 11·7	+2357·0	2·799
March 30.5	— 86·6	+1847·3	+ 7·2	—184·5	— 17·1	+2339·9	2·709
May 19.5	— 95·3	+1752·0	+ 11·6	—172·9	— 23·1	+2316·8	2·625
July 8.5	—102·9	+1649·1	+ 16·5	—156·4	— 29·7	+2287·1	2·549
Aug. 27.5	—108·8	+1540·3	+ 21·6	—134·8	— 37·2	+2249·9	2·477
Oct. 16.5	—113·2	+1427·1	+ 26·8	—108·0	— 45·6	+2204·3	2·410
Dec. 5.5	—115·6	+1311·5	+ 31·8	— 76·2	— 54·7	+2149·6	2·347
1779 Jan. 24.5	—115·8	+1195·7	+ 36·5	— 39·7	— 64·9	+2084·7	2·288
March 15.5	—113·4	+1082·3	+ 40·5	+ 0·8	— 75·7	+2009·0	2·236
May 4.5	—108·7	+ 973·6	+ 43·6	+ 44·4	— 86·8	+1922·2	2·188
June 23.5	—101·3	+ 872·3	+ 45·4	+ 89·8	— 98·9	+1823·3	2·147
Aug. 12.5	— 91·1	+ 781·2	+ 45·3	+ 135·1	—110·0	+1713·3	2·116
Oct. 1.5	— 78·7	+ 702·5	+ 43·7	+ 178·8	—119·9	+1593·4	2·096
Nov. 20.5	— 65·1	+ 637·4	+ 40·2	+ 219·0	—128·1	+1465·3	2·083
1780 Jan. 9.5	— 50·6	+ 586·8	+ 34·8	+ 253·8	—133·7	+1331·6	2·084
Feb. 28.5	— 36·5	+ 550·3	+ 28·1	+ 281·9	—136·0	+1195·6	2·097
Apr. 18.5	— 23·7	+ 526·6	+ 20·4	+ 302·3	—135·2	+1060·4	2·123
June 7.5	— 12·9	+ 513·7	+ 12·5	+ 314·8	—131·2	+ 929·2	2·161
July 27.5	— 4·4	+ 509·3	+ 4·9	+ 319·7	—124·6	+ 804·6	2·209
Sept. 15.5	+ 1·6	+ 510·9	— 1·2	+ 318·5	—115·9	+ 688·7	2·268

JUPITER

1784 Oct. 24.5 — 1776 June 18.5

	dδπ	'f	λdδμ	'f	''f	P	'f
	″	″	″	″	″	″	″
1776 March 10.5	+ 15·6	+1239·6	+ 2·463	+381·723	—12168·130	— 13·1	—3142·4
Apr. 29.5	+ 13·1	+1252·7	+ 2·006	+383·729	—11786·407	— 8·6	—3151·0
June 18.5	+ 11·6	+1264·3	+ 1·331	+385·060	—11402·678	— 5·4	—3156·4
Aug. 7.5	+ 12·2	+1276·5	+ 0·445	+385·505	—11017·618	— 4·4	—3160·8
Sept. 26.5	+ 15·5	+1292·0	— 0·600	+384·905	—10632·113	— 6·6	—3167·4
Nov. 15.5	+ 21·6	+1313·6	— 1·660	+383·245	—10247·208	— 12·1	—3179·5
1777 Jan. 4.5	+ 29·6	+1343·2	— 2·650	+380·595	— 9863·963	— 20·2	—3199·7
Feb. 23.5	+ 38·7	+1381·9	— 3·499	+377·096	— 9483·368	— 30·0	—3229·7
Apr. 14.5	+ 47·7	+1429·6	— 4·147	+372·949	— 9106·272	— 40·0	—3269·7
June 3.5	+ 55·7	+1485·3	— 4·618	+368·331	— 8733·323	— 49·3	—3319·0
July 23.5	+ 62·2	+1547·5	— 4·897	+363·434	— 8364·922	— 57·1	—3376·1
Sept. 11.5	+ 67·3	+1614·8	— 5·045	+358·389	— 8001·558	— 63·2	—3439·3
Oct. 31.5	+ 71·1	+1685·9	— 5·114	+353·275	— 7643·169	— 67·3	—3506·6
Dec. 20.5	+ 73·6	+1759·5	— 5·143	+348·132	— 7289·894	— 69·7	—3576·3
1778 Feb. 8.5	+ 75·9	+1835·4	— 5·181	+342·951	— 6941·762	— 70·7	—3647·0
March 30.5	+ 77·4	+1912·8	— 5·253	+337·698	— 6598·811	— 70·4	—3717·4
May 19.5	+ 78·9	+1991·7	— 5·384	+332·314	— 6261·113	— 68·9	—3786·3
July 8.5	+ 80·4	+2072·1	— 5·614	+326·700	— 5928·799	— 66·6	—3852·9
Aug. 27.5	+ 81·4	+2153·5	— 5·927	+320·773	— 5602·099	— 63·2	—3916·1
Oct. 16.5	+ 82·0	+2235·5	— 6·336	+314·437	— 5281·326	— 58·6	—3974·7
Dec. 5.5	+ 82·2	+2317·7	— 6·850	+307·587	— 4966·889	— 52·7	—4027·4
1779 Jan. 24.5	+ 81·2	+2398·9	— 7·461	+300·126	— 4659·302	— 45·0	—4072·4
March 15.5	+ 79·3	+2478·2	— 8·171	+291·955	— 4359·176	— 35·3	—4107·7
May 4.5	+ 75·3	+2553·5	— 8·952	+283·003	— 4067·221	— 23·3	—4131·0
June 23.5	+ 68·9	+2622·4	— 9·779	+273·224	— 3784·218	— 8·7	—4139·7
Aug. 12.5	+ 59·4	+2681·8	—10·580	+262·644	— 3510·994	+ 9·3	—4130·4
Oct. 1.5	+ 46·9	+2728·7	—11·311	+251·333	— 3248·350	+ 29·6	—4100·8
Nov. 20.5	+ 31·6	+2760·3	—11·968	+239·365	— 2997·017	+ 51·5	—4049·3
1780 Jan. 9.5	+ 14·0	+2774·3	—12·447	+226·918	— 2757·652	+ 74·3	—3975·0
Feb. 28.5	— 5·6	+2768·7	—12·720	+214·198	— 2530·734	+ 97·6	—3877·4
Apr. 18.5	— 25·6	+2743·1	—12·741	+201·457	— 2316·536	+119·4	—3758·0
June 7.5	— 44·8	+2698·3	—12·563	+188·894	— 2115·079	+138·3	—3619·7
July 27.5	— 62·4	+2635·9	—12·210	+176·684	— 1926·185	+153·9	—3465·8
Sept. 15.5	— 77·9	+2558·0	—11·708	+164·976	— 1749·501	+166·3	—3299·5

	$d\delta\Omega$	'f	$d\delta i$	'f	$d\delta\varphi$	'f	Δ
	″	″	″	″	″	″	
1780 Sept. 15.5	+ 1·6	+ 510·9	— 1·2	+ 318·5	— 115·9	+ 688·7	2·268
Nov. 4.5	+ 5·2	+ 516·1	— 7·9	+ 310·6	— 106·1	+ 582·6	2·336
Dec. 24.5	+ 7·0	+ 523·1	— 12·7	+ 297·9	— 95·4	+ 487·2	2·412
1781 Feb. 12.5	+ 7·1	+ 530·2	— 16·2	+ 281·7	— 84·9	+ 402·3	2·494
Apr. 3.5	+ 5·9	+ 536·1	— 18·7	+ 263·0	— 74·3	+ 328·0	2·580
May 23.5	+ 4·4	+ 540·5	— 20·4	+ 242·6	— 64·3	+ 263·7	2·669
July 12.5	+ 2·2	+ 542·7	— 21·3	+ 221·3	— 55·2	+ 208·5	2·760
Aug. 31.5	— 0·2	+ 542·5	— 21·5	+ 199·8	— 46·9	+ 161·6	2·851
Oct. 20.5	— 2·7	+ 539·8	— 21·3	+ 178·5	— 39·4	+ 122·2	2·939
Dec. 9.5	— 5·1	+ 534·7	— 20·8	+ 157·7	— 32·6	+ 89·6	3·023
1782 Jan. 28.5	— 7·4	+ 527·3	— 20·0	+ 137·7	— 26·6	+ 63·0	3·104
March 19.5	— 9·6	+ 517·7	— 19·2	+ 118·5	— 21·4	+ 41·6	3·176
May 8.5	— 11·7	+ 506·0	— 18·2	+ 100·3	— 16·7	+ 24·9	3·240
June 27.5	— 13·7	+ 492·3	— 17·2	+ 83·1	— 12·6	+ 12·3	3·294
Aug. 16.5	— 15·7	+ 476·6	— 16·3	+ 66·8	— 8·9	+ 3·4	3·335
Oct. 5.5	— 17·7	+ 458·9	— 15·3	+ 51·5	— 5·7	— 2·3	3·365
Nov. 24.5	— 19·7	+ 439·2	— 14·3	+ 37·2	— 2·8	— 5·1	3·384
1783 Jan. 13.5	— 21·8	+ 417·4	— 13·4	+ 23·8	— 0·1	— 5·2	3·390
March 4.5	— 24·0	+ 393·4	— 12·4	+ 11·4	+ 2·3	— 2·9	3·380
Apr. 23.5	— 26·7	+ 366·7	— 11·5	— 0·1	+ 4·5	+ 1·6	3·352
June 12.5	— 29·6	+ 337·1	— 10·5	— 10·6	+ 6·6	+ 8·2	3·306
Aug. 1.5	— 32·8	+ 304·3	— 9·2	— 19·8	+ 8·4	+ 16·6	3·249
Sept. 20.5	— 36·4	+ 267·9	— 7·7	— 27·5	+ 10·0	+ 26·6	3·174
Nov. 9.5	— 40·2	+ 227·7	— 5·7	— 33·2	+ 11·0	+ 37·6	3·086
Dec. 29.5	— 43·5	+ 184·2	— 3·1	— 36·3	+ 11·0	+ 48·6	2·995
1784 Feb. 17.5	— 45·3	+ 138·9	+ 0·1	— 36·2	+ 8·9	+ 57·5	2·912
Apr. 7.5	— 44·3	+ 94·6	+ 3·8	— 32·4	+ 3·9	+ 61·4	2·850
May 27.5	— 39·5	+ 55·1	+ 7·5	— 24·9	— 4·9	+ 56·5	2·823
July 16.5	— 30·5	+ 24·6	+ 9·9	— 15·0	— 15·9	+ 40·6	2·860
Sept. 4.5	— 19·3	+ 5·3	+ 10·4	— 4·6	— 25·6	+ 15·0	2·974
Oct. 24.5	— 9·3	— 4·0	+ 8·9	+ 4·3	— 30·3	— 15·3	3·167
Dec. 13.5	— 2·6	— 6·6	+ 6·5	+ 10·8	— 29·4	— 44·7	3·429
1785 Feb. 1.5	+ 0·7		+ 4·1		— 24·6		3·742

	$d\delta\pi$	$'f$	$\lambda d\delta\mu$	$'f$	$''f$	P	$'f$
	″	″	″	″	″	″	″
1780 Sept. 15.5	— 77·9	+2558·0	—11·708	+164·976	— 1749·501	+ 166·3	—3299·5
Nov. 4.5.	— 90·3	+2467·7	—11·123	+153·853	— 1584·525	+ 174·7	—3124·8
Dec. 24.5	—100·0	+2367·7	—10·483	+143·370	— 1430·672	+ 179·6	—2945·2
1781 Feb. 12.5	—106·9	+2260·8	— 9·833	+133·537	— 1287·302	+ 181·4	—2763·8
Apr. 3.5	—111·5	+2149·3	— 9·205	+124·332	— 1153·765	+ 181·0	—2582·8
May 23.5	—113·8	+2035·5	— 8·608	+115·724	— 1029·433	+ 178·2	—2404·6
July 12.5	—114·6	+1920·9	— 8·052	+107·672	— 913·709	+ 174·3	—2230·3
Aug. 31.5	—113·9	+1807·0	— 7·560	+100·112	— 806·037	+ 169·1	—2061·2
Oct. 20.5	—112·4	+1694·6	— 7·128	+ 92·984	— 705·925	+ 163·4	—1897·8
Dec. 9.5	—110·2	+1584·4	— 6·766	+ 86·218	— 612·941	+ 157·3	—1740·5
1782 Jan. 28.5	—107·4	+1477·0	— 6·452	+ 79·766	— 526·723	+ 150·8	—1589·7
March 19.5	—104·6	+1372·4	— 6·207	+ 73·559	— 446·957	+ 144·4	—1445·3
May 8.5	—101·6	+1270·8	— 6·013	+ 67·546	— 373·398	+ 138·2	—1307·1
June 27.5	— 98·7	+1172·1	— 5·875	+ 61·671	— 305·852	+ 132·1	—1175·0
Aug. 16.5	— 96·1	+1076·0	— 5·786	+ 55·885	— 244·181	+ 126·3	—1048·7
Oct. 5.5	— 93·6	+ 982·4	— 5·747	+ 50·138	— 188·296	+ 120·8	— 927·9
Nov. 24.5	— 91·4	+ 891·0	— 5·747	+ 44·391	— 138·158	+ 115·4	— 812·5
1783 Jan. 13.5	— 89·4	+ 801·6	— 5·784	+ 38·607	— 93·767	+ 110·1	— 702·4
March 4.5	— 87·9	+ 713·7	— 5·870	+ 32·737	— 55·160	+ 105·2	— 597·2
Apr. 23.5	— 86·8	+ 626·9	— 5·991	+ 26·746	— 22·423	+ 100·3	— 496·9
June 12.5	— 86·1	+ 540·8	— 6·125	+ 20·621	+ 4·323	+ 95·4	— 401·5
Aug. 1.5	— 85·0	+ 455·8	— 6·253	+ 14·368	+ 24·944	+ 89·6	— 311·9
Sept. 20.5	— 83·8	+ 372·0	— 6·330	+ 8·038	+ 39·312	+ 83·2	— 228·7
Nov. 9.5	— 81·7	+ 290·3	— 6·261	+ 1·777	+ 47·350	+ 75·0	— 153·7
Dec. 29.5	— 77·2	+ 213·1	— 5·844	— 4·067	+ 49·127	+ 63·9	— 89·8
1784 Feb. 17.5	— 68·7	+ 144·4	— 4·832	— 8·899	+ 45·060	+ 49·0	— 40·8
Apr. 7.5	— 56·0	+ 88·4	— 3·003	— 11·902	+ 36·161	+ 31·0	— 9·8
May 27.5	— 40·5	+ 47·9	— 0·260	— 12·162	+ 24·259	+ 12·9	+ 3·1
July 16.5	— 25·5	+ 22·4	+ 2·963	— 9·199	+ 12·097	— 0·1	+ 3·0
Sept. 4.5	— 15·9	+ 6·5	+ 5·704	— 3·495	+ 2·898	— 3·4	— 0·4
Oct. 24.5	— 12·8	— 6·3	+ 7·096	+ 3·601	— 0·597	+ 2·0	+ 1·6
Dec. 13.5	— 13·0	— 19·3	+ 6·981	+ 10·582	+ 3·004	+ 10·3	+ 11·9
1785 Feb. 1.5	— 12·1		+ 5·852		+ 13·586	+ 16·0	

SATURN

1784 Oct. 24.5 — 1776 June 18.5

	$d\delta\Omega$	*'f*	$d\delta i$	*'f*	$d\delta\varphi$	*'f*	Δ
	″	″	″	″	″	″	
1776 March 10.5	— 0·1	+ 17·9	0·0	+ 15·7	+ 0·4	+ 44·8	11·50
Apr. 29.5	— 0·1	+ 17·8	0·0	+ 15·7	+ 0·4	+ 45·2	11·75
June 18.5	0·0	+ 17·8	0·0	+ 15·7	+ 0·3	+ 45·5	11·95
Aug. 7.5	0·0	+ 17·8	0·0	+ 15·7	+ 0·2	+ 45·7	12·06
Sept. 26.5	0·0	+ 17·8	0·0	+ 15·7	0·0	+ 45·7	12·13
Nov. 15.5	0·0	+ 17·8	0·0	+ 15·7	— 0·1	+ 45·6	12·13
1777 Jan. 4.5	0·0	+ 17·8	0·0	+ 15·7	— 0·2	+ 45·4	12·11
Feb. 23.5	0·0	+ 17·8	0·0	+ 15·7	— 0·2	+ 45·2	12·00
Apr. 14.5	0·0	+ 17·8	0·0	+ 15·7	— 0·2	+ 45·0	11·88
June 3.5	0·0	+ 17·8	0·0	+ 15·7	— 0·2	+ 44·8	11·74
July 23.5	0·0	+ 17·8	0·0	+ 15·7	— 0·2	+ 44·6	11·58
Sept. 11.5	0·0	+ 17·8	0·0	+ 15·7	— 0·2	+ 44·4	11·40
Oct. 31.5	0·0	+ 17·8	0·0	+ 15·7	— 0·2	+ 44·2	11·20
Dec. 20.5	0·0	+ 17·8	0·0	+ 15·7	— 0·2	— 44·0	11·02
1778 Feb. 8.5	0·0	+ 17·8	0·0	+ 15·7	— 0·2	+ 43·8	10·86
March 30.5	0·0	+ 17·8	0·0	+ 15·7	— 0·2	+ 43·6	10·73
May 19.5	0·0	+ 17·8	0·0	+ 15·7	— 0·2	+ 43·4	10·58
July 8.5	0·0	+ 17·8	0·0	+ 15·7	— 0·2	+ 43·2	10·41
Aug. 27.5	0·0	+ 17·8	0·0	+ 15·7	— 0·2	+ 43·0	10·24
Oct. 16.5	0·0	+ 17·8	0·0	+ 15·7	— 0·3	+ 42·7	10·09
Dec. 5.5	0·0	+ 17·8	0·0	+ 15·7	— 0·3	+ 42·4	9·95
1779 Jan. 24.5	0·0	+ 17·8	0·0	+ 15·7	— 0·4	+ 42·0	9·81
March 15.5	0·0	+ 17·8	0·0	+ 15·7	— 0·4	+ 41·6	9·68
May 4.5	0·0	+ 17·8	0·0	+ 15·7	— 0·5	+ 41·1	9·55
June 23.5	+ 0·1	+ 17·9	0·0	+ 15·7	— 0·6	+ 40·5	9·40
Aug. 12.5	+ 0·1	+ 18·0	0·0	+ 15·7	— 0·6	+ 39·9	9·25
Oct. 1.5	+ 0·1	+ 18·1	— 0·1	+ 15·6	— 0·7	+ 39·2	9·12
Nov. 20.5	+ 0·1	+ 18·2	— 0·1	+ 15·5	— 0·8	+ 38·4	9·01
1780 Jan. 9.5	+ 0·2	+ 18·4	— 0·1	+ 15·4	— 0·9	+ 37·5	8·89
Feb. 28.5	+ 0·2	+ 18·6	— 0·1	+ 15·3	— 0·9	+ 36·6	8·78
Apr. 18.5	+ 0·2	+ 18·8	— 0·2	+ 15·1	— 1·0	+ 35·6	8·66
June 7.5	+ 0·2	+ 19·0	— 0·2	+ 14·9	— 1·1	+ 34·5	8·54
July 27.5	+ 0·2	+ 19·2	— 0·2	+ 14·7	— 1·1	+ 33·4	8·43
Sept. 15.5	+ 0·2	+ 19·4	— 0·3	+ 14·4	— 1·2	+ 32·2	8·31

SATURN

1784 Oct. 24.5 — 1776 June 18.5

	$d\delta\pi$	'f	$\lambda d\delta\mu$	'f	"f	P	'f
	"	"	"	"	"	"	"
1776 March 10.5	— 1·0	+ 90·5	— 0·139	+ 1·732	—90·290	+ 0·8	— 108·2
Apr. 29.5	— 0·9	+ 89·6	— 0·117	+ 1·615	—88·558	+ 0·5	— 107 7
June 18.5	— 0·8	+ 88·8	— 0·083	+ 1·532	—86·943	+ 0·4	— 107·3
Aug. 7.5	— 0·8	+ 88·0	— 0·039	+ 1·493	—85·411	+ 0·3	— 107·0
Sept. 26.5	— 0·9	+ 87·1	+ 0·008	+ 1·501	—83·918	+ 0·3	— 106·7
Nov. 15.5	— 1·0	+ 86·1	+ 0·051	+ 1·552	—82·417	+ 0·5	— 106·2
1777 Jan. 4.5	— 1·2	+ 84·9	+ 0·086	+ 1·638	—80·865	+ 0·7	— 105·5
Feb. 23.5	— 1·4	+ 83·5	+ 0·111	+ 1.749	—79·227	+ 0·9	— 104·6
Apr. 14.5	— 1·5	+ 82·0	+ 0·127	+ 1·876	—77·478	+ 1·2	— 103·4
June 3.5	— 1·6	+ 80·4	+ 0·136	+ 2·012	—75·602	+ 1·4	— 102·0
July 23.5	— 1·7	+ 78·7	+ 0·139	+ 2·151	—73·590	+ 1·6	— 100·4
Sept. 11.5	— 1·7	+ 77·0	+ 0·137	+ 2·288	—71·439	+ 1·8	— 98·6
Oct. 31.5	— 1.6	+ 75·4	+ 0·132	+ 2·420	—69·151	+ 1·9	— 96·7
Dec. 20.5	— 1·6	+ 73·8	+ 0·124	+ 2·544	—66·731	+ 1·9	— 94·8
1778 Feb. 8.5	— 1·5	+ 72·3	+ 0·115	+ 2·659	—64·187	+ 2·0	— 92·8
March 30.5	— 1·4	+ 70·9	+ 0·106	+ 2·765	—61·528	+ 2·0	— 90·8
May 19.5	— 1·3	+ 69·6	+ 0·095	+ 2·860	—58·763	+ 2·0	— 88·8
July 8.5	— 1·2	+ 68·4	+ 0·084	+ 2·944	—55·903	+ 2·1	— 86·7
Aug. 27.5	— 1·1	+ 67·3	+ 0·072	+ 3·016	—52·959	+ 2·1	— 84·6
Oct. 16.5	— 1·0	+ 66·3	+ 0·059	+ 3·075	—49·943	+ 2·1	— 82·5
Dec. 5.5	— 0·9	+ 65·4	+ 0·047	+ 3·122	—46·868	+ 2·1	— 80·4
1779 Jan. 24.5	— 0·9	+ 64·5	+ 0·035	+ 3·157	—43·746	+ 2·2	— 78·2
March 15.5	— 0·8	+ 63·7	+ 0·022	+ 3·179	—40·589	+ 2·2	— 76·0
May 4.5	— 0·8	+ 62·9	+ 0·010	+ 3·189	—37·410	+ 2·2	— 73·8
June 23.5	— 0·7	+ 62·2	— 0·004	+ 3·185	—34·221	+ 2·2	— 71·6
Aug. 12.5	— 0·7	+ 61·5	— 0·017	+ 3·168	—31·036	+ 2·3	— 69·3
Oct. 1.5	— 0·7	+ 60·8	— 0·031	+ 3·137	—27·868	+ 2·4	— 66·9
Nov. 20.5	— 0·8	+ 60·0	— 0·044	+ 3·093	—24·731	+ 2·4	— 64·5
1780 Jan. 9.5	— 0·8	+ 59·2	— 0·056	+ 3·037	—21·638	+ 2·5	— 62·0
Feb. 28.5	— 0·9	+ 58·3	— 0·069	+ 2·968	—18·601	+ 2·6	— 59·4
Apr. 18.5	— 1·0	+ 57·3	— 0·083	+ 2·885	—15·633	+ 2·7	— 56·7
June 7.5	— 1·1	+ 56·2	— 0·096	+ 2·789	—12·748	+ 2·8	— 53·9
July 27.5	— 1·2	+ 55·0	— 0·109	+ 2·680	— 9·959	+ 2·8	— 51·1
Sept. 15.5	— 1·3	+ 53·7	— 0·123	+ 2·557	— 7·279	+ 2·9	— 48·2

	$d\delta\Omega$	$'f$	$d\delta i$	$'f$	$d\delta\varphi$	$'f$	Δ
	″	″	″	″	″	″	
1780 Sept. 15.5	+ 0·2	+ 19·4	— 0·3	+ 14·4	— 1·2	+ 32·2	8·31
Nov. 4.5	+ 0·2	+ 19·6	— 0·3	+ 14·1	— 1·3	+ 30·9	8·20
Dec. 24.5	+ 0·2	+ 19·8	— 0·4	+ 13.7	— 1·3	+ 29·6	8·10
1781 Feb. 12.5	+ 0·2	+ 20·0	— 0·4	+ 13·3	— 1·4	+ 28·2	8·00
Apr. 3.5	+ 0·2	+ 20·2	— 0·5	+ 12·8	— 1·4	+ 26·8	7·89
May 23.5	+ 0·1	+ 20·3	— 0·5	+ 12·3	— 1·5	+ 25·3	7·79
July 12.5	+ 0·1	+ 20·4	— 0·6	+ 11·7	— 1·5	+ 23·8	7·68
Aug. 31.5	0·0	+ 20·4	— 0·6	+ 11·1	— 1·5	+ 22·3	7·59
Oct. 20.5	— 0·1	+ 20·3	— 0·7	+ 10·4	— 1·5	+ 20·8	7·50
Dec. 9.5	— 0·2	+ 20·1	— 0·7	+ 9·7	— 1·5	+ 19·3	7·40
1782 Jan. 28.5	— 0·3	+ 19·8	— 0·8	+ 8·9	— 1·5	+ 17·8	7·29
March 19.5	— 0·4	+ 19·4	— 0·8	+ 8·1	— 1·5	+ 16·3	7·20
May 8.5	— 0·6	+ 18·8	— 0·9	+ 7·2	— 1·4	+ 14·9	7·12
June 27.5	— 0·7	+ 18·1	— 0·9	+ 6·3	— 1·3	+ 13·6	7·06
Aug. 16.5	— 0·9	+ 17·2	— 0·9	+ 5·4	— 1·3	+ 12·3	6·99
Oct. 5.5	— 1·0	+ 16·2	— 0·9	+ 4·5	— 1·2	+ 11·1	6·92
Nov. 24.5	— 1·2	+ 15·0	— 0·9	+ 3·6	— 1·1	+ 10·0	6·86
1783 Jan. 13.5	— 1·4	+ 13·6	— 0·8	+ 2·8	— 1·0	+ 9·0	6·83
March 4.5	— 1·5	+ 12·1	— 0·8	+ 2·0	— 0·9	+ 8·1	6·81
Apr. 23.5	— 1·6	+ 10·5	— 0·7	+ 1·3	— 0·8	+ 7·3	6·81
June 12.5	— 1·7	+ 8·8	— 0·6	+ 0·7	— 0·8	+ 6·5	6·82
Aug. 1.5	— 1·7	+ 7·1	— 0·5	+ 0·2	— 0·7	+ 5·8	6·88
Sept. 20.5	— 1·6	+ 5·5	— 0·3	— 0·1	— 0·7	+ 5·1	6·99
Nov. 9.5	— 1·5	+ 4·0	— 0·2	— 0·3	— 0·9	+ 4·2	7·16
Dec. 29.5	— 1·3	+ 2·7	— 0·1	— 0·4	— 0·8	+ 3·4	7·36
1784 Feb. 17.5	— 1·0	+ 1·7	0·0	— 0·4	— 0·8	+ 2·6	7·60
Apr. 7.5	— 0·8	+ 0·9	+ 0·1	— 0·3	— 0·8	+ 1·8	7·89
May 27.5	— 0·5	+ 0·4	+ 0·1	— 0·2	— 0·7	+ 1·1	8·26
July 16.5	— 0·3	+ 0·1	+ 0·1	— 0·1	— 0·6	+ 0·5	8·69
Sept. 4.5	— 0·1	0·0	+ 0·1	0·0	— 0·4	+ 0·1	9·16
Oct. 24.5	0·0	0·0	0·0	0·0	— 0·2	— 0·1	9·66
Dec. 13.5	0·0	0·0	0·0	0·0	0·0	— 0·1	10·15
1785 Feb. 1.5	0·0		— 0·1		+ 0·2		10·63

	$d\delta\pi$	$'f$	$\lambda d\delta\mu$	$'f$	$''f$	P	$'f$
	″	″	″	″	″	″	″
1780 Sept. 15.5	— 1·3	+ 53·7	—0·123	+ 2·557	— 7·279	+ 2·9	—48·2
Nov. 4.5	— 1·5	+ 52·2	—0·137	+ 2·420	— 4·722	+ 3·0	—45·2
Dec. 24.5	— 1·7	+ 50.5	—0·150	+ 2·270	— 2·302	+ 3·1	—42·1
1781 Feb. 12.5	— 1·9	+ 48 6	—0·162	+ 2·108	— 0·032	+ 3·3	—38·8
Apr. 3.5	— 2·1	+ 46·5	—0·175	+ 1·933	+ 2·076	+ 3·3	—35·5
May 23.5	— 2·3	+ 44·2	—0·188	+ 1·745	+ 4·009	+ 3·4	—32·1
July 12.5	— 2·5	+ 41·7	—0·200	+ 1·545	+ 5·754	+ 3·8	—28·3
Aug. 31.5	— 2·7	+ 39·0	—0·211	+ 1·334	+ 7·299	+ 3·6	—24·7
Oct. 20.5	— 2·9	+ 36·1	—0·221	+ 1·113	+ 8·633	+ 3·6	—21·1
Dec. 9.5	— 3·1	+ 33·0	—0·229	+ 0·884	+ 9·746	+ 3·6	—17·5
1782 Jan. 28.5	— 3·4	+ 29·6	—0·239	+ 0·645	+10·630	+ 3·7	—13·8
March 19.5	— 3·6	+ 26·0	—0·242	+ 0·403	+11·275	+ 3·6	—10·2
May 8.5	— 3·7	+ 22·3	—0·242	+ 0·161	+11·678	+ 3·4	— 6·8
June 27.5	— 3·8	+ 18·5	—0·241	—0·080	+11·839	+ 3·4	— 3·4
Aug. 16.5	— 3·8	+ 14·7	—0·233	—0·313	+11·759	+ 3·1	— 0·3
Oct. 5.5	— 3·7	+ 11·0	—0·219	—0·532	+11·446	+ 2·8	+ 2·5
Nov. 24.5	— 3·6	+ 7·4	—0·201	—0·733	+10·914	+ 2.4	+ 4·9
1783 Jan. 13.5	— 3·3	+ 4·1	—0·175	—0·908	+10·181	+ 2·0	+ 6·9
March 4.5	— 3·0	+ 1·1	—0·141	—1·049	+ 9·273	+ 1·4	+ 8·3
Apr. 23.5	— 2·5	— 1·4	—0·100	—1·149	+ 8·224	+ 0·8	+ 9·1
June 12.5	— 2·0	— 3·4	—0·052	—1·201	+ 7·075	+ 0·2	+ 9·3
Aug. 1.5	— 1·3	— 4·7	+ 0·002	—1·199	+ 5·874	— 0·3	+ 9·0
Sept. 20.5	— 0·7	— 5·4	+ 0·057	—1·142	+ 4·675	— 0·9	+ 8·1
Nov. 9.5	0·0	— 5·4	+ 0·109	—1·033	+ 3·533	— 1·3	+ 6·8
Dec. 29.5	+ 0·5	— 4·9	+ 0·156	—0·877	+ 2·500	— 1·5	+ 5·3
1784 Feb. 17.5	+ 0·9	— 4·0	+ 0·189	—0·688	+ 1·623	— 1·6	+ 3·7
Apr. 7.5	+ 1·1	— 2·9	+ 0·204	—0·484	+ 0·935	— 1·3	+ 2·4
May 27.5	+ 1·0	— 1·9	+ 0·192	—0·292	+ 0·451	— 1·1	+ 1·3
July 16.5	+ 0·9	— 1·0	+ 0·157	—0·135	+ 0·159	— 0·8	+ 0·5
Sept. 4.5	+ 0·7	— 0·3	+ 0·107	—0·028	+ 0·024	— 0·4	+ 0·1
Oct. 24.5	+ 0·6	+ 0·3	+ 0·047	+ 0·019	— 0·004	— 0·2	— 0·1
Dec. 13.5	+ 0·6	+ 0·9	—0·010	+ 0·009	+ 0·015	— 0·2	— 0·3
1785 Feb. 1.5	+ 0·6		—0·058		+ 0·024	— 0·4	

Michał Kamieński

Researches on the origin of the Comet P/Wolf I
Part X

Heliocentric perturbations in the motion of the Comet due to Jupiter and Saturn during the period

1776 June 18.5 — 1768 Apr. 1.5

Mémoire présenté à la séance du 7 mai 1951.

1. The system P_{-19} of elements of the Comet P/Wolf I for 1776 June 18.5 G. M. T., published in Part IX of the author's „Researches", was taken as a basis for the computation of the heliocentric perturbations in the motion of the comet for its previous revolution round the Sun in the period 1776—1768:

1776 June 18.5 Greenwich Mean Time

$$P_{-19} \dots \begin{matrix} M = 356^{0}14'6''.0 \\ \varphi = 24^{0}25'8''.1 \\ n = 430''.0074 \end{matrix} \qquad \left.\begin{matrix} \Omega = 213^{0}35'38''.5 \\ \pi = 12^{0}57'23''.2 \\ i = 26^{0}44'30''.6 \end{matrix}\right\} 1950.0$$

The system P_{-19} differs from the main fundamental system P_1 for 1884 Sept. 24.0 only by taking into consideration the perturbations due to Jupiter and Saturn for the period 1884—1776. The numerical values of these perturbations as well as the corresponding systems of elements are to be found in the author's articles concerning his researches on the origin of the Comet P/Wolf I. Of all of them only Parts I—II—III—IV relating to the period 1884 Sept. 24.0 — 1839 Apr. 20.5 were published in the „Bulletin de l'Académie Polonaise des Sciences et des Lettres. Série A. Sciences Mathématiques. Cracovie, 1939—1949". The manuscripts of the analogical investigations for the period 1839—1776 containing Parts V — VI — VII — VIII — IX of the author's „Researches", are in print in the „Reports of the Warsaw Scientific Society".

The actual computations were performed by applying the Method of Variation of Arbitrary Constants on changing permanently the systems of elements every 50 days, in order to take into consideration the higher order perturbations. The distances $\triangle$ of the comet from Jupiter were in the given period 1776—1768 comparatively large, varying within the limits:

$$5.68 < \triangle < 9.95.$$

Consequently, the interval $\lambda = 50$ days appeared to be quite adequate for our purposes.

The formulae used by the author for the computation of perturbations are widely known. They were given in Part VIII of his „Researches on the origin of the Comet P/Wolf I", now under publication (l. c.).

After the integration of the differentials of perturbations given in the adjacent Tables, the author got the following results:

1776 June 18.5 — 1768 Apr. 1.5			
	Jupiter	Saturn	Total
δM	$+4527''.7$	$+87''.8$	$+4615''.5$
$\delta \varphi$	$-271''.8$	$-86''.7$	$-368''.5$
$\delta \Omega$	$+225''.7$	$+80''.4$	$+306''.1$
$\delta \pi$	$-175''.0$	$+46''.5$	$-128''.5$
δi	$-144''.0$	$-33''.3$	$-177''.7$
δn	$-2''.2521$	$-0''.0940$	$-2''.3461$

2. Adding these totals to the system of elements P_{-19} given above, the author deduced the following perturbed system P_{-20}:

1768 Apr. 1.5 Gr. M. T.

$$P_{-20} \ldots \begin{array}{ll} M = 359^{0}10'39''.3 & \Omega = 213^{0}40'44''.6 \\ \varphi = 24^{0}19'\ 9''.6 & \pi = 12^{0}55'14''.7 \\ n = 427''.6613 & i = 26^{0}41'32''.9 \end{array} \Bigg\} \ 1950.0$$

The system P_{-20} will be used as a basis for further backward computations of the perturbations in the motion of the Comet up to the beginning of 1750. The author considers it to be exact enough for the calculations of the perturbations in question, the more so as they are not too large, with the exception of those for the period 1759—1755. During this period, and more precisely — up to the beginning of 1757, the distance $\triangle$ of the comet from Jupiter was rapidly diminishing, attaining its minimum value $\triangle \cong 0.1$. It produced comparatively large variations in the elements of the Comet P/Wolf I.

JUPITER

1776 June 18.5 — 1768 Apr. 1.5

	$d\delta\Omega$	'f	$d\delta i$	'f	$d\delta\varphi$	'f	Δ
	"	"	"	"	"	"	
1767 Dec. 23.5	— 1·6	+ 227·0	+ 0·5	— 145·3	+ 5·1	— 277·5	7·642
1768 Feb. 11.5	— 1·0	+ 226·0	+ 0·6	— 144·7	+ 4·2	— 273·3	7·741
Apr. 1.5	— 0·5	+ 225·5	+ 0·6	— 144·1	+ 2·7	— 270·6	7·822
May 21.5	— 0·1	+ 225·4	+ 0·5	— 143·6	+ 1·2	— 269·4	7·878
July 10.5	+ 0·1	+ 225·5	+ 0·3	— 143·3	— 0·3	— 269·7	7·903
Aug. 29.5	+ 0·1	+ 225·6	+ 0·2	— 143·1	— 1·4	— 271·1	7·907
Oct. 18.5	0·0	+ 225·6	0·0	— 143·1	— 1·9	— 273·0	7·898
Dec. 7.5	— 0·2	+ 225·4	— 0·1	— 143·2	— 2·0	— 275·0	7·883
1769 Jan. 26.5	— 0·6	+ 224·8	— 0·2	— 143·4	— 1·6	— 276·6	7·869
March 17.5	— 1·2	+ 223·6	— 0·3	— 143·7	— 1·0	— 277·6	7·858
May 6.5	— 1·8	+ 221·8	— 0·3	— 144·0	— 0·2	— 277·8	7·852
June 25.5	— 2·5	+ 219·3	— 0·2	— 144·2	+ 0·8	— 277·0	7·858
Aug. 14.5	— 3·2	+ 216·1	— 0·1	— 144·3	+ 1·9	— 275·1	7·878
Oct. 3.5	— 4·0	+ 212·1	+ 0·1	— 144·2	+ 3·1	— 272·0	7·910
Nov. 22.5	— 4·8	+ 207·3	+ 0·3	— 143·9	+ 4·3	— 267·7	7·956
1770 Jan. 11.5	— 5·6	+ 201·7	+ 0·5	— 143·4	+ 5·5	— 262·2	8·013
March 2.5	— 6·4	+ 195·3	+ 0·9	— 142·5	+ 6·8	— 255·4	8·077
Apr. 21.5	— 7·1	+ 188·2	+ 1·2	— 141·3	+ 8·0	— 247·4	8·153
June 10.5	— 7·8	+ 180·4	+ 1·6	— 139·7	+ 9·2	— 238·2	8·239
July 30.5	— 8·4	+ 172·0	+ 2·1	— 137·6	+ 10·4	— 227·8	8·333
Sept. 18.5	— 9·0	+ 163·0	+ 2·6	— 135·0	+ 11·5	— 216·3	8·433
Nov. 7.5	— 9·4	+ 153·6	+ 3·1	— 131·9	+ 12.5	— 203·8	8·537
Dec. 27.5	— 9·8	+ 143·8	+ 3·6	— 128·3	+ 13·4	— 190·4	8·644
1771 Feb. 15.5	— 10·0	+ 133·8	+ 4·1	— 124·2	+ 14·2	— 176·2	8·752
Apr. 6.5	— 10·1	+ 123·7	+ 4·5	— 119·7	+ 14·9	— 161·3	8·863
May 26.5	— 10·1	+ 113·6	+ 5·1	— 114·6	+ 15·5	— 145·8	8·977
July 15.5	— 10·0	+ 103·6	+ 5·6	— 109·0	+ 15·9	— 129·9	9·088
Sept. 3.5	— 9·7	+ 93·9	+ 6·1	— 102·9	+ 16·2	— 113·7	9·198
Oct. 23.5	— 9·3	+ 84·6	+ 6·5	— 96·4	+ 16·4	— 97·3	9·305
Dec. 12.5	— 8·8	+ 75·8	+ 6·8	— 89·6	+ 16·6	— 80·7	9·405
1772 Jan. 31.5	— 8·1	+ 67·7	+ 7·1	— 82·5	+ 16·3	— 64·4	9·500
March 21.5	— 7·4	+ 60·3	+ 7·3	— 75·2	+ 16·0	— 48·4	9·588
May 10.5	— 6·6	+ 53·7	+ 7·4	— 67·8	+ 15·6	— 32·8	9·668
June 29.5	— 5·7	+ 48·0	+ 7·4	— 60·4	+ 15·1	— 17·7	9·740

JUPITER

1776 June 18.5 — 1768 Apr. 1.5

	$d\delta\pi$	$'f$	$\lambda d\delta n$	$'f$	$''f$	P	$'f$
	"	"	"	"	"	"	"
1767 Dec. 23.5	— 16·1	— 154·6	— 1·586	—111·040	+ 4013·045	+ 9·6	+ 728·7
1768 Feb. 11.5	— 14·0	— 168·6	— 1·182	—112·222	+ 3902·005	+ 6·7	+ 735·4
Apr. 1.5	— 12·8	— 181·4	— 0·682	—112·904	+ 3789·783	+ 5·0	+ 740·4
May 21.5	— 12·9	— 194·3	— 0·139	—113·043	+ 3676·879	+ 4·7	+ 745·1
July 10.5	— 14·4	— 208·7	+ 0·391	—112·652	+ 3563·836	+ 6·0	+ 751·1
Aug. 29.5	— 16·8	— 225·5	+ 0·855	—111·797	+ 3451·184	+ 8·5	+ 759·6
Oct. 18.5	— 19·7	— 245·2	+ 1·229	—110·568	+ 3339·387	+ 11·7	+ 771·3
Dec. 7.5	— 22·6	— 267·8	+ 1·513	—109·055	+ 3228·819	+ 15·0	+ 786·3
1769 Jan. 26.5	— 25·4	— 293·2	+ 1·718	—107·337	+ 3119·764	+ 18·2	+ 804·5
March 17.5	— 27·7	— 320·9	+ 1·859	—105·478	+ 3012·427	+ 21·1	+ 825·6
May 6.5	— 29·5	— 350·4	+ 1·946	—103·532	+ 2906·949	+ 23·6	+ 849·2
June 25.5	— 31·0	— 381·4	+ 2·004	—101·528	+ 2803·417	+ 25·7	+ 874·9
Aug. 14.5	— 32·1	— 413·5	+ 2·035	— 99·493	+ 2701·889	+ 27·3	+ 902·2
Oct. 3.5	— 32·8	— 446·3	+ 2·045	— 97·448	+ 2602·396	+ 28·4	+ 930·6
Nov, 22.5	— 33·1	— 479·4	+ 2·037	— 95·411	+ 2504·948	+ 29·1	+ 959·7
1770 Jan. 11.5	— 33·0	— 512·4	+ 2·018	— 93·393	+ 2409.537	+ 29·3	+ 989·0
March 2.5	— 32·6	— 545·0	+ 1·991	— 91·402	+ 2316·144	+ 29·0	+1018·0
Apr. 21.5	— 31·8	— 576·8	+ 1·961	— 89 441	+ 2224·742	+ 28·3	+1046·3
June 10.5	— 30·7	— 607·5	+ 1·928	— 87·513	+ 2135·301	+ 27·2	+1073·5
July 30.5	— 29·2	— 636·7	+ 1·892	— 85·621	+ 2047·788	+ 25·6	+1099·1
Sept. 18.5	— 27·4	— 664·1	+ 1·853	— 83·768	+ 1962·167	+ 23·4	+1122·5
Nov. 7.5	— 25·5	— 689·6	+ 1·813	— 81·955	+ 1878·399	+ 21·1	+1143·6
Dec. 27.5	— 23·0	— 712·6	+ 1·775	— 80·180	+ 1796·444	+ 18·2	+1161·8
1771 Feb. 15.5	— 20·6	— 733·2	+ 1·737	— 78·443	+ 1716·264	+ 15·1	+1176·9
Apr. 6.5	— 17.7	— 750·9	+ 1·701	— 76·742	+ 1637·821	+ 11·6	+1188·5
May 26.5	— 14·7	— 765·6	+ 1·668	— 75·074	+ 1561·079	+ 7·9	+1196·4
July 15.5	— 11.5	— 771·1	+ 1·637	— 73·437	+ 1486·005	+ 3·9	+1200·3
Sept. 3.5	— 8·2	— 785·3	+ 1·611	— 71·826	+ 1412·568	— 0·2	+1200·1
Oct. 23.5	— 4·9	— 790·2	+ 1·589	— 70·237	+ 1340·742	— 4·6	+1195·5
Dec. 12.5	— 1·4	— 791·6	+ 1·571	— 68·666	+ 1270·505	— 8·9	+1186·6
1772 Jan. 31.5	+ 2·0	— 789·6	+ 1·558	— 67·108	+ 1201·839	— 13·3	+1173·3
March 21.5	+ 5·4	— 784·2	+ 1·552	— 65·556	+ 1134·731	— 17·7	+1155·6
May 10.5	+ 8·9	— 775·3	+ 1·551	— 64·005	+ 1069·175	— 22·0	+1133·6
June 29.5	+ 12·1	— 763·2	+ 1·556	— 62·449	+ 1005·170	— 26·1	+1107·5

	$d\delta\Omega$	'f	$d\delta i$	'f	$d\delta\varphi$	'f	Δ
	″	″	″	″	″	″	
1772 June 29.5	— 5·7	+ 48·0	+ 7·4	— 60·4	+ 15·1	— 17·7	9·740
Aug. 18.5	— 4·8	+ 43·2	+ 7·4	— 53·0	+ 14·5	— 3·2	9·803
Oct. 7.5	— 3·9	+ 39·3	+ 7·2	— 45·8	+ 13·7	+ 10·5	9·856
Nov. 26.5	— 3·0	+ 36·3	+ 7·0	— 38·8	+ 12·8	+ 23·3	9·897
1773 Jan. 15.5	— 2·1	+ 34·2	+ 6·6	— 32·2	+ 11·9	+ 35·2	9·928
March 6.5	— 1·3	+ 32·9	+ 6·2	— 26·0	+ 10·8	+ 46·0	9·945
Apr. 25.5	— 0·6	+ 32·3	+ 5·7	— 20·3	+ 9·7	+ 55·7	9·950
June 14.5	+ 0·1	+ 32·4	+ 5·2	— 15·1	+ 8·5	+ 64·2	9·940
Aug. 3.5	+ 0·6	+ 33·0	+ 4·6	— 10·5	+ 7·3	+ 71·5	9·917
Sept. 22.5	+ 1·0	+ 34·0	+ 3·9	— 6·6	+ 6·1	+ 77·6	9·879
Nov. 11.5	+ 1·3	+ 35·3	+ 3·3	— 3·3	+ 4·9	+ 82·5	9·827
Dec. 31.5	+ 1·4	+ 36·7	+ 2·6	— 0·7	+ 3·7	+ 86·2	9·761
1774 Feb. 19.5	+ 1·3	+ 38·0	+ 2·0	+ 1·3	+ 2·4	+ 88·6	9·681
Apr, 10.5	+ 1·1	+ 39·1	+ 1·3	+ 2·6	+ 1·3	+ 89·9	9·585
May 30.5	+ 0·7	+ 39·8	+ 0·7	+ 3·3	+ 0·1	+ 90·0	9·471
July 19.5	+ 0·3	+ 40·1	+ 0·2	+ 3·5	— 1·0	+ 89·0	9·341
Sept. 7.5	— 0·3	+ 39·8	— 0·2	+ 3·3	— 2·1	+ 86·9	9·196
Oct. 27.5	— 1·0	+ 38·8	— 0·6	+ 2·7	— 3·1	+ 83·8	9·034
Dec. 16.5	— 1·7	+ 37·1	— 0·9	+ 1·8	— 4·1	+ 79·7	8·855
1775 Feb. 4.5	— 2·5	+ 34·6	— 1·0	+ 0·8	— 5·0	+ 74·7	8·664
March 26.5	— 3·2	+ 31·4	— 1·1	— 0·3	— 5·9	+ 68·8	8·459
May 15.5	— 3·8	+ 27·6	— 1·0	— 1·3	— 6·8	+ 62·0	8·235
July 4.5	— 4·3	+ 23·3	— 0·8	— 2·1	— 7·5	+ 54·5	7·987
Aug. 23.5	— 4·6	+ 18·7	— 0·5	— 2·6	— 8·2	+ 46·3	7·720
Oct. 12.5	— 4·7	+ 14·0	— 0·2	— 2·8	— 8·9	+ 37·4	7·432
Dec. 1.5	— 4·5	+ 9.5	+ 0·2	— 2·6	— 9·2	+ 28·2	7·124
1776 Jan. 20.5	— 3·9	+ 5·6	+ 0·5	— 2·1	— 9·3	+ 18·9	6·795
March 10.5	— 3·1	+ 2·5	+ 0·8	— 1·3	— 8·7	+ 10·2	6·444
Apr. 29.5	— 2·0	+ 0·5	+ 0·9	— 0·4	— 7·4	+ 2·8	6·072
June 18.5	— 0·9	— 0·4	+ 0·7	+ 0·3	— 5·1	— 2·3	5·685
Aug. 7.5	— 0·1	— 0·5	+ 0·2	+ 0·5	— 2·1	— 4·4	5·296
Sept. 26.5	0·0		— 0·6		+ 1·4		4·916

	dδπ	'f	λdδn	'f	''f	P	'f
	"	"	"	"	"	"	"
1772 June 29.5	+12·1	—763·2	+ 1·556	—62·449	+ 1005·170	—26·1	+1107·5
Aug. 18.5	+15·2	—748·0	+ 1·567	—60·882	+ 942·721	—30·1	+1077·4
Oct. 7.5	+18·2	—729·8	+ 1·582	—59·300	+ 881·839	—34·0	+1043·4
Nov. 26.5	+20·9	—708·9	+ 1·605	—57·695	+ 822·539	—37·4	+1006·0
1773 Jan. 15.5	+23·4	—685·5	+ 1·636	—56·059	+ 764·844	—40·6	+ 965·4
March 6.5	+25·8	—659·7	+ 1·669	—54·390	+ 708·785	—43·6	+ 921·8
Apr. 25.5	+27·9	—631·8	+ 1·711	—52·679	+ 654·395	—46·2	+ 875·6
June 14.5	+29·6	—602·2	+ 1·757	—50·922	+ 601·716	—48·2	+ 827·4
Aug. 3.5	+31·1	—571·1	+ 1·808	—49·114	+ 550·794	—49·9	+ 777·5
Sept, 22.5	+32·3	—538·8	+ 1·862	—47·252	+ 501·680	—51 2	+ 726·3
Nov. 11.5	+33·2	—505·6	+ 1·921	—45·331	+ 454·428	—52·0	+ 674·3
Dec. 31.5	+33·8	—471·8	+ 1·986	—43·345	+ 409·097	—52·5	+ 621·8
1774 Feb. 19.5	+34·2	—437·6	+ 2·053	—41·292	+ 365·752	—52·4	+ 569·4
Apr. 10.5	+34·3	—403·3	+ 2·123	—39·169	+ 324·460	—52·0	+ 517·4
May 30.5	+34·2	—369·1	+ 2·197	—36·972	+ 285·291	—51·3	+ 466·1
July 19.5	+33·8	—335·3	+ 2·271	—34·701	+ 248·319	—50·1	+ 416·0
Sept. 7.5	+33·3	—302·0	+ 2·349	—32·352	+ 213·618	—48·6	+ 367·4
Oct. 27.5	+32·5	—269·5	+ 2·429	—29·923	+ 181·266	—46·9	+ 320·5
Dec. 16.5	+31·6	—237·9	+ 2·508	—27·415	+ 151·343	—44·8	+ 275·7
1775 Feb. 4.5	+30·6	—207·3	+ 2·591	—24·824	+ 123·928	—42·3	+ 233·4
March 26.5	+29·5	—177·8	+ 2·675	—22·149	+ 99·104	—39·9	+ 193·5
May 15.5	+28·3	—149·5	+ 2·756	—19·393	+ 76·955	—37·2	+ 156·3
July 4.5	+26·9	—122·6	+ 2·826	—16·567	+ 57·562	—34·0	+ 122·3
Aug. 23.5	+25·2	— 97 4	+ 2·877	—13·690	+ 40·995	—30·6	+ 91·7
Oct. 12.5	+23·3	— 74·1	+ 2·895	—10·795	+ 27·305	—26·7	+ 65·0
Dec. 1.5	+21·1	— 53·0	+ 2·857	— 7·938	+ 16·510	—22·5	+ 42·5
1776 Jan. 20.5	+18·5	— 34·5	+ 2·730	— 5·208	+ 8·572	—17·9	+ 24·6
March 10.5	+15·6	— 18·9	+ 2·464	— 2·744	+ 3·364	—13·1	+ 11·5
Apr. 29.5	+13·1	— 5·8	+ 2·012	— 0·732	+ 0·620	— 8·6	+ 2·9
June 18.5	+11·6	+ 5·8	+ 1·334	+ 0·602	— 0·112	— 5·4	— 2·5
Aug. 7.5	+12·2	+ 18·0	+ 0·445	+ 1·047	+ 0·490	— 4·4	— 6·9
Sept. 26.5	+15·5		—0·600		+ 1·537	— 6·6	

SATURN

1776 June 18.5 — 1768 Apr. 1.5

	$d\delta\Omega$	'f	$d\delta i$	'f	$d\delta\varphi$	'f	Δ
	"	"	"	"	"	"	
1767 Dec. 23.5	— 0·2	+ 80·5	+ 0·1	— 33·3	— 0·5	— 86·4	10·15
1758 Feb. 11.5	— 0·1	+ 80.4	0·0	— 33·3	— 0·3	— 86·7	9·62
Apr. 1.5	0·0	+ 80.4	0·0	— 33·3	0·0	— 86·7	9·08
May 21.5	0·0	+ 80.4	0·0	— 33·3	+ 0·3	— 86·4	8·55
July 10.5	0·0	+ 80.4	— 0·1	— 33·4	+ 0·5	— 85·9	8·05
Aug. 29.5	— 0·2	+ 80·2	— 0·2	— 33·6	+ 0·8	— 85·1	7·59
Oct. 18.5	— 0·5	+ 79·7	— 0·3	— 33·9	+ 1·0	— 84·1	7·19
Dec. 7.5	— 0·9	+ 78·8	— 0·4	— 34·3	+ 1·0	— 83·1	6·84
1769 Jan. 26.5	— 1·3	+ 77·5	— 0·4	— 34·7	+ 1·1	— 82·0	6·55
March 17.5	— 1·9	+ 75·6	— 0·4	— 35·1	+ 1·0	— 81·0	6·31
May 6.5	— 2·4	+ 73·2	— 0·3	— 35·4	+ 1·0	— 80·0	6·11
June 25.5	— 3·0	+ 70·2	— 0·2	— 35·6	+ 1·0	— 79·0	5·96
Aug. 14.5	— 3·5	+ 66·7	— 0·1	— 35·7	+ 1·0	— 78·0	5·85
Oct. 3.5	— 3·8	+ 62·9	+ 0·1	— 35·6	+ 1·1	— 76·9	5·78
Nov. 22.5	— 4·1	+ 58·8	+ 0·2	— 35·4	+ 1·2	— 75·7	5·74
1770 Jan. 11.5	— 4·4	+ 54·4	+ 0·4	— 35·0	+ 1·3	— 74·4	5·73
March 2.5	— 4·4	+ 50·0	+ 0·6	— 34·4	+ 1·4	— 73·0	5·73
Apr. 21.5	— 4·4	+ 45·6	+ 0·7	— 33·7	+ 1·5	— 71·5	5·74
June 10.5	— 4·3	+ 41·3	+ 0·9	— 32·8	+ 1·6	— 69·9	5·77
July 30.5	— 4.2	+ 37·1	+ 1·0	— 31·8	+ 1·8	— 68·1	5·81
Sept. 18.5	— 4·0	+ 33·1	+ 1·1	— 30·7	+ 1·9	— 66·2	5·85
Nov. 7.5	— 3·9	+ 29·2	+ 1·2	— 29·5	+ 2·0	— 64·2	5·88
Dec. 27.5	— 3·7	+ 25·5	+ 1·3	— 28·2	+ 2·1	— 62·1	5·92
1771 Feb. 15.5	— 3·4	+ 22·1	+ 1·4	— 26·8	+ 2·2	— 59·9	5·96
Apr. 6.5	— 3·2	+ 18·9	+ 1·4	— 25·4	+ 2·3	— 57·6	6·00
May 26.5	— 3·0	+ 15·9	+ 1·5	— 23·9	+ 2·4	— 55·2	6·04
July 15.5	— 2·7	+ 13·2	+ 1·5	— 22·4	+ 2·4	— 52·8	6·09
Sept. 3.5	— 2·5	+ 10·7	+ 1·5	— 20·9	+ 2·5	— 50·3	6·15
Oct. 23.5	— 2·2	+ 8·5	+ 1·5	— 19·4	+ 2·5	— 47·8	6·20
Dec. 12.5	— 2·0	+ 6·5	+ 1·5	— 17·9	+ 2·5	— 45·3	6·26
1772 Jan. 31.5	— 1·7	+ 4·8	+ 1·5	— 16·4	+ 2·6	— 42·7	6·31
March 21.5	— 1·5	+ 3·3	+ 1·5	— 14·9	+ 2·6	— 40·1	6·36
May 10.5	— 1·3	+ 2·0	+ 1·5	— 13·4	+ 2·6	— 37·5	6·41
June 29.5	— 1·1	+ 0·9	+ 1·4	— 12·0	+ 2·6	— 34·9	6·46

SATURN

1776 June 18.5 — 1768 Apr. 1.5

	$d\delta\pi$	'f	$\lambda d\delta n$	'f	"f	P	'f
	"	"	"	"	"	"	"
1767 Dec. 23.5	+ 0·9	+ 45·4	+ 0·134	— 4·799	+154·116	— 0·7	— 56·2
1768 Feb. 11.5	+ 0·8	+ 46·2	+ 0·086	— 4·713	+149·317	— 0·4	— 56·6
Apr. 1.5	+ 0·7	+ 46·9	+ 0·010	— 4·703	+144·604	— 0·3	— 56·9
May 21.5	+ 0·9	+ 47·8	— 0·078	— 4·781	+139·901	— 0·4	— 57·3
July 10.5	+ 1·2	+ 49·0	— 0·164	— 4·945	+135·120	— 0·8	— 58·1
Aug. 29.5	+ 1·6	+ 50·6	— 0·238	— 5·183	+130·175	— 1·3	— 59·4
Oct. 18.5	+ 1·8	+ 52·4	— 0·283	— 5·466	+124·992	— 1·9	— 61·3
Dec. 7.5	+ 1·9	+ 54·3	— 0·301	— 5·767	+119·526	— 2·4	— 63·7
1769 Jan. 26.5	+ 1·7	+ 56·0	— 0·288	— 6·055	+113·759	— 2·7	— 66·4
March 17.5	+ 1·3	+ 57·3	— 0·254	— 6·309	+107·704	— 2·7	— 69·1
May 6.5	+ 0·7	+ 58·0	— 0·198	— 6·507	+101·395	— 2·5	— 71·6
June 25.5	+ 0·1	+ 58·1	— 0·133	— 6·640	+ 94·888	— 2·1	— 73·7
Aug. 14.5	— 0·9	+ 57·2	— 0·061	— 6·701	+ 88·248	— 1·5	— 75·2
Oct. 3.5	— 1·7	+ 55·5	+ 0·009	— 6·692	+ 81·547	— 0·8	— 76·0
Nov. 22.5	— 2·5	+ 53·0	+ 0·074	— 6·618	+ 74·855	— 0·1	— 76·1
1770 Jan. 11.5	— 3·1	+ 49·9	+ 0·132	— 6·486	+ 68·237	+ 0·7	— 75·4
March 2.5	— 3·6	+ 46·3	+ 0·178	— 6·308	+ 11·751	+ 1·4	— 74·0
Apr. 21.5	— 4·0	+ 42·3	+ 0·216	— 6·092	+ 55·443	+ 2·0	— 72·0
June 10.5	— 4·2	+ 38·1	+ 0·245	— 5·847	+ 49·351	+ 2·5	— 69·5
July 30.5	— 4·4	+ 33·7	+ 0·265	— 5·582	+ 43·504	+ 2·9	— 66·6
Sept. 18.5	— 4·4	+ 29·3	+ 0·279	— 5·303	+ 37·922	+ 3·1	— 63·5
Nov. 7.5	— 4·3	+ 25·0	+ 0·292	— 5·011	+ 32·619	+ 3·4	— 60·1
Dec. 27.5	— 4·2	+ 20·8	+ 0·299	— 4·712	+ 27·608	+ 3·6	— 56·5
1771 Feb. 15.5	— 4·1	+ 16·7	+ 0·302	— 4·410	+ 22·896	+ 3·7	— 52·8
Apr. 6.5	— 3·7	+ 13·0	+ 0·303	— 4·107	+ 18·486	+ 3·7	— 49·1
May 26.5	— 3·7	+ 9·3	+ 0·302	— 3·805	+ 14·379	+ 3·7	— 45·4
July 15.5	— 3·4	+ 5·9	+ 0·298	— 3·507	+ 10·574	+ 3·6	— 41·8
Sept. 3.5	— 3·1	+ 2·8	+ 0·292	— 3·215	+ 7·067	+ 3·5	— 38·3
Oct. 23.5	— 2·7	+ 0·1	+ 0·285	— 2·930	+ 3·852	+ 3·4	— 34·9
Dec. 12.5	— 2·4	— 2·3	+ 0·279	— 2·651	+ 0·922	+ 3·2	— 31·7
1772 Jan. 31.5	— 2·1	— 4·4	+ 0·274	— 2·377	— 1·729	+ 3·0	— 28·7
March 21.5	— 1·7	— 6·1	+ 0·269	— 2·108	— 4·106	+ 2·8	— 25·9
May 10.5	— 1·4	— 7·5	+ 0·263	— 1·845	— 6·214	+ 2·5	— 23·4
June 29.5	— 1·0	— 8·5	+ 0·256	— 1·589	— 8·059	+ 2·3	— 21·1

	$d\delta\Omega$	'f	$d\delta i$	'f	$d\delta\varphi$	'f	Δ
	″	″	″	″	″	″	
1772 June 29.5	— 1·1	+ 0·9	+ 1·4	— 12·0	+ 2·6	— 34·9	6·46
Aug. 18.5	— 0·9	0·0	+ 1·4	— 10·6	+ 2·6	— 32·3	6·52
Oct. 7.5	— 0·7	— 0·7	+ 1·3	— 9·3	+ 2·5	— 29·8	6·59
Nov. 26.5	— 0·5	— 1·2	+ 1·2	— 8·1	+ 2·4	— 27·4	6·66
1773 Jan. 15.5	— 0·4	— 1·6	+ 1·1	— 7·0	+ 2·4	— 25·0	6·73
March 6.5	— 0·2	— 1·8	+ 1·0	— 6·0	+ 2·3	— 22·7	6·81
Apr. 25.5	— 0·1	— 1·9	+ 1·0	— 5·0	+ 2·2	— 20·5	6·90
June 14.5	0·0	— 1·9	+ 0·9	— 4·1	+ 2·1	— 18·4	7·00
Aug. 3.5	+ 0·1	— 1·8	+ 0·8	— 3·3	+ 2·0	— 16·4	7·11
Sept. 22.5	+ 0·2	— 1·6	+ 0·7	— 2·6	+ 1·8	— 14·6	7·23
Nov. 11.5	+ 0·2	— 1·4	+ 0·6	— 2·0	+ 1·7	— 12·9	7·36
Dec. 31.5	+ 0·3	— 1·1	+ 0·5	— 1·5	+ 1·5	— 11·4	7·51
1774 Feb. 19.5	+ 0·3	— 0·8	+ 0·4	— 1·1	+ 1·4	— 10·0	7·65
Apr. 10.5	+ 0·3	— 0·5	+ 0·3	— 0·8	+ 1·2	— 8·8	7·82
May 30.5	+ 0·3	— 0·2	+ 0·3	— 0·5	+ 1·1	— 7·7	7·99
July 19.5	+ 0·2	0·0	+ 0·2	— 0·3	+ 0·9	— 6·8	8·18
Sept. 7.5	+ 0·2	+ 0·2	+ 0·1	— 0·2	+ 0·8	— 6·0	8·39
Oct. 27.5	+ 0·2	+ 0·4	+ 0·1	— 0·1	+ 0·7	— 5·3	8·62
Dec. 16.5	+ 0·1	+ 0·5	+ 0·1	0·0	+ 0·6	— 4·7	8·87
1775 Feb. 4.5	+ 0·1	+ 0·6	0·0	0·0	+ 0·5	— 4·2	9·13
March 26.5	0·0	+ 0·6	0·0	0·0	+ 0·4	— 3·8	9·40
May 15.5	0·0	+ 0·6	0·0	0·0	+ 0·5	— 3·3	9·70
July 4.5	0·0	+ 0·6	0·0	0·0	+ 0·4	— 2·9	10·00
Aug. 23.5	— 0·1	+ 0·5	0·0	0·0	+ 0·4	— 2·5	10·30
Oct. 12.5	— 0·1	+ 0·4	0·0	0·0	+ 0·4	— 2·1	10·59
Dec. 1.5	— 0·1	+ 0·3	0·0	0·0	+ 0·5	— 1·6	10·89
1776 Jan. 20.5	— 0·1	+ 0·2	0·0	0·0	+ 0·5	— 1·1	11·21
March 10.5	— 0·1	+ 0·1	0·0	0·0	+ 0·5	— 0·6	11·50
Apr. 29.5	— 0·1	0·0	0·0	0·0	+ 0·4	— 0·2	11·75
June 18.5	0·0	0·0	0·0	0·0	+ 0·3	+ 0·1	11·95
Aug. 7.5	0·0	0·0	0·0	0·0	+ 0·2	+ 0·3	12·06
Sept. 26.5	0·0		0·0		0·0		12·13

	$d\delta\pi$	'f	$\lambda d\delta n$	'f	''f	P	'f
	″	″	″	″	″	″	″
1772 June 29.5	— 1·0	— 8·5	+ 0·256	— 1·589	— 8·059	+ 2·3	— 21·1
Aug. 18.5	— 0·7	— 9·2	+ 0·249	— 1·340	— 9·648	+ 2·0	— 19·1
Oct. 7.5	— 0·4	— 9·6	+ 0·240	— 1·100	—10·988	+ 1·8	— 17·3
Nov. 26.5	0·0	— 9·6	+ 0·230	— 0·870	—12·088	+ 1·5	— 15·8
1773 Jan. 15.5	+ 0·3	— 9·3	+ 0·222	— 0·648	—12·958	+ 1·3	— 14·5
March 6.5	+ 0·6	— 8·7	+ 0·214	— 0·434	—13·606	+ 1·0	— 13·5
Apr. 25.5	+ 0·9	— 7·8	+ 0·206	— 0·228	—14·040	+ 0·7	— 12·8
June 14.5	+ 1·1	— 6·7	+ 0·195	— 0·033	—14·268	+ 0·6	— 12·2
Aug. 3.5	+ 1·3	— 5·4	+ 0·193	+ 0·150	—14·301	+ 0·4	— 11·8
Sept. 22.5	+ 1·4	— 4·0	+ 0·169	+ 0·319	—14·151	+ 0·2	— 11·6
Nov. 11.5	+ 1·5	— 2·5	+ 0·154	+ 0·473	—13·832	+ 0·1	— 11·5
Dec. 31.5	+ 1·5	— 1·0	+ 0·139	+ 0·612	—13·359	+ 0·1	— 11·4
1774 Feb. 19.5	+ 1·6	+ 0·6	+ 0·125	+ 0·737	—12·747	0·0	— 11·4
Apr. 10.5	+ 1·5	+ 2·1	+ 0·107	+ 0·844	—12·010	0·0	— 11·4
May 30.5	+ 1·4	+ 3·5	+ 0·088	+ 0·932	—11·166	+ 0·1	— 11·3
July 19.5	+ 1·2	+ 4·7	+ 0·067	+ 0·999	—10·234	+ 0·2	— 11·1
Sept. 7.5	+ 1·0	+ 5·7	+ 0·045	+ 1·044	— 9·235	+ 0·3	— 10·8
Oct. 27.5	+ 0·8	+ 6·5	+ 0·023	+ 1·067	— 8·191	+ 0·4	— 10·4
Dec. 16.5	+ 0·5	+ 7·0	0·000	+ 1·067	— 7·124	+ 0·6	— 9·8
1775 Feb. 4.5	+ 0·2	+ 7·2	— 0·024	+ 1·043	— 6·057	+ 0·7	— 9·1
March 26.5	— 0·1	+ 7·1	— 0·048	+ 0·995	— 5·014	+ 0·9	— 8·2
May 15.5	— 0·3	+ 6·8	— 0·071	+ 0·924	— 4·019	+ 1·0	— 7·2
July 3.5	— 0·6	+ 6·2	— 0·092	+ 0·832	— 3·095	+ 1·1	— 6·1
Aug. 23.5	— 0·8	+ 5·4	— 0·112	+ 0·720	— 2·263	+ 1·2	— 4·9
Oct. 12.5	— 0·9	+ 4·5	— 0·129	+ 0·591	— 1·543	+ 1·2	— 3·7
Dec. 1.5	— 1·1	+ 3·4	— 0·142	+ 0·449	— 0·952	+ 1·1	— 2·6
1776 Jan. 20.5	— 1·1	+ 2·3	— 0·147	+ 0·302	— 0·503	+ 1·0	— 1·6
March 10.5	— 1·0	+ 1·3	— 0·139	+ 0·163	— 0·201	+ 0·8	— 0·8
Apr. 29.5	— 0·9	+ 0·4	— 0·118	+ 0·045	— 0·038	+ 0·6	— 0·2
June 18.5	— 0·8	— 0·4	— 0·083	— 0·038	+ 0·007	+ 0·4	+ 0·2
Aug. 7.5	— 0·8	— 1·2	— 0·039	— 0·077	— 0·031	+ 0·3	+ 0·5
Sept. 26.5	— 0·9		+ 0·008		— 0·108	+ 0·3	

Michał Kamieński

Researches on the Motion of the Comet P/Wolf I
Part XVI

Perturbations due to Venus, the Earth, Mars and Uranus during the period

1942 June 10.0 — 1950 Oct. 6.0

Mémoire présenté à la séance du 7 mai 1951.

§ 1. The present paper is a direct continuation of the author's investigations published in Part XV ot his „Researches". Jointly with these, it gives a complete image of the comet's motion under the influence of the Sun and of planets during the period 1942—1950. The perturbations caused by Mercury and Neptune are small and consequently can be neglected.

The Tables of differentials of perturbations given below were computed under permanent change of elements every $\lambda = 20$ days for Venus and the Earth, and every 40 days — for Mars and Uranus. The Method of Variation of Arbitrary Constants was used throughout. It is to be noted that the application of this method to the computation of the Earth and especially of Venus perturbations presents some difficulties because of the rapid motion of these planets on their orbits. The diminution of the interval to 10 days, however, would necessitate a considerable increase of labour, with only a small increasing of the accuracy of computations.

With this in view, in course of the work several checks were applied. For the most part, they consisted in the examination of the course of differences of several functions computed. This course was checked both numerically and graphically. It is to be noted that the differentials of perturbations vary almost cyclically, especially for Venus. Exempli gratia, those of $\lambda d \delta n$ for the period 1947 Feb. 24 — 1947 Oct. 22, consequently for appr. 240 days, are nearly repeated for the next period 1947 Oct. 22 — 1948 June 18, as can be seen from the Tables below. This period is close to that of sidereal revolutions — 225 days — of Venus round the Sun. Many analogical checks performed during the actual computations gave evidence that the enclosed Tables of differentials do not contain such errors as could in any way negatively influence the exactness of the results.

§ 2. Yet, the best check of the accuracy of computations of the perturbations due to the Earth and especially to Venus is the application of a quite different independent method to these computations. The author applied the Barocentric Method, on referring the motion of the comet to the common gravity centre of the Sun and of the perturbing planet. In spite of the considerable distance of the Earth from the Sun, the results obtained with this other method are very close to those found by the Method of Variation of Arbitrary Constants. Much better results were obtained on application of the Barocentric Method to the motion of Venus given in Table I below.

However, since the computations by the Barocentric Method cannot give such accuracy as that obtained by the Method of Variation of Arbitrary Constants, they were not taken into consideration while deducing the systems of perturbed elements.

TABLE I.

1942 June 10 — 1950 Oct. 6

Venus

	Var. of Arb. Const.	Baroc. Meth.
δM	+ 44″.35	+ 44″.68
$\delta\varphi$	— 4″.47	— 4″.15
$\delta\Omega$	— 0″.34	— 0″.26
$\delta\pi$	+ 2″.37	+ 2″.37
δi	+ 0″.59	+ 0″.53
δn	+ 0″.0223	+ 0″.0225

The Earth

	Var. of Arb. Const.	Baroc. Meth.
δM	+ 8″.96	+ 6″.88
$\delta\varphi$	+ 2″.85	+ 3″.18
$\delta\Omega$	+ 0″.41	+ 0″.37
$\delta\pi$	+ 2″.26	+ 1″.97
δi	— 0″.52	— 0″.21
δn	— 0″.0149	— 0″.0156

§ 3. The system Q_4 of elements for 1942 June 10.0 U. T. given below was taken as a basis of all the computations. This system was published in the author's paper „The New Path of the Comet Wolf I", printed in the „Bulletin de l'Académie

Polonaise des Sciences et des Lettres, Cracovie, 1948". Il represents perfectly all the observations of the comet in the period 1925—1933/1934—1942.

Epoch and Oscul.

1942 June 10.0 U. T.

$$Q_4 \ldots \begin{array}{ll} M & 358^\circ 23' 12''.60 \\ \varphi & 23^\circ 52' 37''.48 \\ \Omega & 204^\circ 20' 31''.30 \\ \pi & 5^\circ 18' 37''.97 \\ i & 27^\circ 18' 11''.29 \\ n & 428''.1947 \end{array}$$

Aequin 1950.0

With this system of elements, the differentials of perturbations due to the influence of the four planets in question were computed. The higher order perturbations were taken into account as well. After their integration, the author got Table II given below.

TABLE II.

	1942 June 10.0 — 1950 Oct. 6.0			
	Venus	The Earth	Mars	Uranus
δM	+ 44″.35	+ 8″.96	— 0″.56	+ 0″.75
$\delta \varphi$	— 4″.47	+ 2″.85	+ 0″.25	— 0″.61
$\delta \Omega$	— 0″.34	+ 0″.41	— 0″.12	— 0″.25
$\delta \pi$	+ 2″.37	+ 2″.26	— 0″.34	— 1″.06
δi	+ 0″.59	— 0″.52	— 0″.02	— 0″.25
δn	+ 0″.0223	— 0″.0149	— 0″.0014	— 0″.0001

On joining these values of perturbations with the analogical ones due to Jupiter and Saturn for the same period (v. Researches on the Motion of the Comet P/Wolf I, Part XV), and adding the totals to the system Q_4, the author got the definitive system Q_5 of elements for 1950 Oct. 6.0 U. T.

Epoch and Oscul.

1950 Oct. 6.0

	M	357°56′ 4″.81
	φ	23°21′ 7″.68
Q_5 ...	Ω	203°52′46″.10
	π	5° 1′30″.08
	i	27°18′58″.79
	n	421″.5795

Aequin 1950.0

In such a way, system Q_5 is the direct continuation of the equalized systems Q_1, Q_2, Q_3, Q_4 (l. c.). Il represents the observations of the comet in 1950 well enough, as it was evidenced by the author in his paper concerning the apparition of the comet in 1950.

VENUS
1942 June 10.0 — 1950 Oct. 6.0

		dδΩ	'f	dδi	'f	dδφ	'f	Δ
		"	"	"	"	"	"	
1942 May	21	+ 0·22	— 0·08	— 0·18	+ 0·08	+ 0·90	— 0·27	2·111
June	10	+ 0·14	+ 0·06	— 0·15	— 0·07	+ 0·45	+ 0·18	1·844
June	30	+ 0·04	+ 0·10	— 0·07	— 0·14	— 0·12	+ 0·06	1·731
July	20	— 0·02	+ 0·08	+ 0·04	— 0·10	— 0·66	— 0·60	1·830
Aug.	9	— 0·03	+ 0·05	+ 0·15	+ 0·05	— 1·04	— 1·64	2·105
Aug.	29	+ 0·01	+ 0·06	+ 0·21	+ 0·26	— 1·15	— 2·79	2·457
Sept.	18	+ 0·06	+ 0·12	+ 0·21	+ 0·47	— 0·97	— 3·76	2·806
Oct.	8	+ 0·07	+ 0·19	+ 0·14	+ 0·61	— 0·54	— 4·30	3·094
Oct.	28	+ 0·02	+ 0·21	+ 0·03	+ 0·64	0·00	— 4·30	3·281
Nov.	17	— 0·09	+ 0·12	— 0·09	+ 0·55	+ 0·50	— 3·80	3·348
Dec.	7	— 0·20	— 0·08	— 0·17	+ 0·38	+ 0·82	— 2·98	3·294
Dec.	27	— 0·29	— 0·37	— 0·20	+ 0·18	+ 0·90	— 2·08	3·132
1943 Jan.	16	— 0·28	— 0·65	— 0·16	+ 0·02	+ 0·73	— 1·35	2·892
Feb.	5	— 0·17	— 0·82	— 0·08	— 0·06	+ 0·37	— 0·98	2·627
Feb.	25	+ 0·03	— 0·79	+ 0·01	— 0·05	— 0·09	— 1·07	2·417
Mar.	17	+ 0·26	— 0·53	+ 0·10	+ 0·05	— 0·51	— 1·58	2·357
Apr.	6	+ 0·45	— 0·08	+ 0·14	+ 0·19	— 0·76	— 2·34	2·497
Apr.	26	+ 0·51	+ 0·43	+ 0·14	+ 0·33	— 0·77	— 3·11	2·807
May	16	+ 0·40	+ 0·83	+ 0·10	+ 0·43	— 0·56	— 3·67	3·197
June	5	+ 0·14	+ 0·97	+ 0·03	+ 0·46	— 0·20	— 3·87	3·583
June	25	— 0·18	+ 0·79	— 0·03	+ 0·43	+ 0·19	— 3·68	3·898
July	15	— 0·48	+ 0·31	— 0·07	+ 0·36	+ 0·47	— 3·21	4·100
Aug.	4	— 0·64	— 0·33	— 0·08	+ 0·28	+ 0·58	— 2·63	4·174
Aug.	24	— 0·62	— 0·95	— 0·06	+ 0·22	+ 0·48	— 2·15	4·116
Sept.	13	— 0·40	— 1·35	— 0·03	+ 0·19	+ 0·21	— 1·94	3·950
Oct.	3	— 0·04	— 1·39	0·00	+ 0·19	— 0·13	— 2·07	3·715
Oct.	23	+ 0·35	— 1·04	+ 0·01	+ 0·20	— 0·44	— 2·51	3·482
Nov,	12	+ 0·66	— 0·38	+ 0·01	+ 0·21	— 0·60	— 3·11	3·343
Dec.	2	+ 0·77	+ 0·39	— 0·01	+ 0·20	— 0·56	— 3·67	3·373
Dec.	22	+ 0·63	+ 1·02	— 0·02	+ 0·18	— 0·32	— 3·99	3·589
1944 Jan.	11	+ 0·29	+ 1·31	— 0·01	+ 0·17	+ 0·04	— 3·95	3·937
Jan.	31	— 0·14	+ 1·17	+ 0·01	+ 0·18	+ 0·38	— 3·57	4·325
Feb.	20	— 0·54	+ 0·63	+ 0·04	+ 0·22	+ 0·59	— 2·98	4·671
Mar.	11	— 0·77	— 0·14	+ 0·07	+ 0·29	+ 0·60	— 2·38	4·914

VENUS

1942 June 10.0 — 1950 Oct. 6.0

	$d\delta\pi$	$'f$	$\lambda d\delta n$	$'f$	$''f$	P	$'f$
	″	″	″	″	″	″	″
1942 May 21	+ 0·76	— 0·70	— 0·084	+ 0·024	— 0·021	— 0·12	+ 0·25
June 10	+ 1·49	+ 0·79	— 0·040	— 0·016	+ 0·003	— 0·54	— 0·29
June 30	+ 1·74	+ 2·53	+ 0·011	— 0·005	— 0·013	— 0·67	— 0·96
July 20	+ 1·42	+ 3·95	+ 0·059	+ 0·054	— 0·018	— 0·46	— 1·42
Aug. 9	+ 0·61	+ 4·56	+ 0·096	+ 0·150	+ 0·036	+ 0·02	— 1·40
Aug. 29	— 0·39	+ 4·17	+ 0·111	+ 0·261	+ 0·186	+ 0·57	— 0·83
Sept. 18	— 1·27	+ 2·90	+ 0·100	+ 0·361	+ 0·447	+ 0·98	+ 0·15
Oct. 8	— 1·73	+ 1·17	+ 0·066	+ 0·427	+ 0·808	+ 1·04	+ 1·19
Oct. 28	— 1·60	— 0·43	+ 0·018	+ 0·445	+ 1·235	+ 0·70	+ 1·89
Nov. 17	— 0·92	— 1·35	— 0·032	+ 0·413	+ 1·680	+ 0·03	+ 1·92
Dec. 7	+ 0·13	— 1·22	— 0·072	+ 0·341	+ 2·093	— 0·78	+ 1·14
Dec. 27	+ 1·23	+ 0·01	— 0·095	+ 0·246	+ 2·434	— 1·49	— 0·35
1943 Jan. 16	+ 2·08	+ 2·09	— 0·096	+ 0·150	+ 2·680	— 1·86	— 2·21
Feb. 5	+ 2·42	+ 4·51	— 0·074	+ 0·076	+ 2·830	— 1·76	— 3·97
Feb. 25	+ 2·09	+ 6·60	— 0·035	+ 0·041	+ 2·906	— 1·07	— 5·04
Mar. 17	+ 1·14	+ 7·74	+ 0·013	+ 0·054	+ 2·947	0·00	— 5·04
Apr. 6	— 0·22	+ 7·52	+ 0·058	+ 0·112	+ 3·001	+ 1·21	— 3·83
Apr. 26	— 1·61	+ 5·91	+ 0·089	+ 0·201	+ 3.113	+ 2·20	— 1·63
May 16	— 2·59	+ 3·32	+ 0·097	+ 0·298	+ 3·314	+ 2·64	+ 1·01
June 5	— 2·84	+ 0·48	+ 0·081	+ 0·379	+ 3·612	+ 2·34	+ 3·35
June 25	— 2·26	— 1·78	+ 0·045	+ 0·424	+ 3·991	+ 1·35	+ 4·70
July 15	— 1·04	— 2·82	0·000	+ 0·424	+ 4·415	— 0·04	+ 4·66
Aug. 4	+ 0·49	— 2·33	— 0·044	+ 0·380	+ 4·839	— 1·48	+ 3·18
Aug. 24	+ 1·90	— 0·43	— 0·075	+ 0·305	+ 5·219	— 2·52	+ 0·66
Sept. 13	+ 2·82	+ 2·39	— 0·086	+ 0·219	+ 5·524	— 2·92	— 2·26
Oct. 3	+ 2·97	+ 5·36	— 0·075	+ 0·144	+ 5·743	— 2·49	— 4·75
Oct. 23	+ 2·28	+ 7·64	— 0·044	+ 0·100	+ 5·887	— 1·30	— 6·05
Nov. 12	+ 0·90	+ 8·54	— 0·001	+ 0·099	+ 5·987	+ 0·36	— 5·69
Dec. 2	— 0·77	+ 7·77	+ 0·042	+ 0·141	+ 6·086	+ 1·99	— 3·70
Dec. 22	— 2·23	+ 5·54	+ 0·072	+ 0·213	+ 6·227	+ 3·08	— 0·62
1944 Jan. 11	— 3·03	+ 2·51	+ 0·083	+ 0·296	+ 6·440	+ 3·28	+ 2·66
Jan. 31	— 2·93	— 0·42	+ 0·071	+ 0·367	+ 6·736	+ 2·54	+ 5·20
Feb. 20	— 2·01	— 2·43	+ 0·041	+ 0·408	+ 7·103	+ 1·08	+ 6·28
Mar. 11	— 0·56	— 2·99	+ 0·002	+ 0·410	+ 7·511	— 0·66	+ 5·62

		$d\delta\Omega$	'f	$d\delta i$	'f	$d\delta\varphi$	'f	Δ
		″	″	″	″	″	″	
1944 Mar.	11	— 0·77	— 0·14	+ 0·07	+ 0·29	+ 0·60	— 2·38	4·914
Mar.	31	— 0·78	— 0·92	+ 0·09	+ 0·38	+ 0·41	— 1·97	5·020
Apr.	20	— 0·55	— 1·47	+ 0·07	+ 0·45	+ 0·06	— 1·91	4·980
May	10	— 0·14	— 1·61	+ 0·02	+ 0·47	— 0·32	— 2·23	4·813
May	30	+ 0·30	— 1·31	— 0·05	+ 0·42	— 0·64	— 2·87	4·556
June	19	+ 0·67	— 0·64	— 0·12	+ 0·30	— 0·77	— 3·64	4·293
July	9	+ 0·84	+ 0·20	— 0·16	+ 0·14	— 0·65	— 4·29	4·118
July	29	+ 0·74	+ 0·94	— 0·15	— 0·01	— 0·31	— 4·60	4·114
Aug.	18	+ 0·41	+ 1·35	— 0·09	— 0·10	+ 0·15	— 4·45	4·310
Sept.	7	— 0·04	+ 1·31	+ 0·01	— 0·09	+ 0·57	— 3·88	4·647
Sept.	27	— 0·47	+ 0·84	+ 0·12	+ 0·03	+ 0·82	— 3·06	5·035
Oct.	17	— 0·74	+ 0·10	+ 0·20	+ 0·23	+ 0·82	— 2·24	5·380
Nov.	6	— 0·79	— 0·69	+ 0·23	+ 0·46	+ 0·57	— 1·67	5·610
Nov.	26	— 0·61	— 1·30	+ 0·18	+ 0·64	+ 0·13	— 1·54	5·692
Dec.	16	— 0·25	— 1·55	+ 0·08	+ 0·72	— 0·37	— 1·91	5·615
1945 Jan.	5	+ 0·19	— 1·36	— 0·06	+ 0·66	— 0·76	— 2·67	5·401
Jan.	25	+ 0·57	— 0·79	— 0·20	+ 0·46	— 0·98	— 3·65	5·104
Feb.	14	+ 0·77	— 0·02	— 0·29	+ 0·17	— 0·88	— 4·53	4·814
Mar.	6	+ 0·73	+ 0·71	— 0·28	— 0·11	— 0·50	— 5·03	4·637
Mar.	26	+ 0·46	+ 1·17	— 0·19	— 0·30	+ 0·05	— 4·98	4·654
Apr.	15	+ 0·06	+ 1·23	— 0·03	— 0·33	+ 0·59	— 4·39	4·875
May	5	— 0·34	+ 0·89	+ 0·15	— 0·18	+ 0·96	— 3·43	5·230
May	25	— 0·62	+ 0·27	+ 0·29	+ 0·11	+ 1·04	— 2·39	5·612
June	14	— 0·70	— 0·43	+ 0·34	+ 0·45	+ 0·82	— 1·57	5·933
July	4	— 0·58	— 1·01	+ 0·30	+ 0·75	+ 0·35	— 1·22	6·121
July	24	— 0·30	— 1·31	+ 0·16	+ 0·91	— 0·23	— 1·45	6·142
Aug.	13	+ 0·07	— 1·24	— 0·04	+ 0·87	— 0·77	— 2·22	5·998
Sept.	2	+ 0·41	— 0·83	— 0·24	+ 0·63	— 1·10	— 3·32	5·723
Sept.	22	+ 0·61	— 0·22	— 0·37	+ 0·26	— 1·10	— 4·42	5·382
Oct.	12	+ 0·62	+ 0·40	— 0·39	— 0·13	— 0·77	— 5·19	5·084
Nov.	1	+ 0·43	+ 0·83	— 0·29	— 0·42	— 0·19	— 5·38	4·938
Nov.	21	+ 0·13	+ 0·96	— 0·09	— 0·51	+ 0·44	— 4·94	5·009
Dec.	11	— 0·20	+ 0·76	+ 0·14	— 0·37	+ 0·94	— 4·00	5·275
Dec.	31	— 0·44	+ 0·32	+ 0·33	— 0·04	+ 1·16	— 2·84	5·648
1946 Jan.	20	— 0·54	— 0·22	+ 0·42	+ 0·38	+ 1·04	— 1·80	6·015
Feb.	9	— 0·47	— 0·69	+ 0·39	+ 0·77	+ 0·62	— 1·18	6·289

	$d\delta\pi$	'f	$\lambda d\delta n$	'f	''f	P	'f
	"	"	"	"	"	"	"
1944 Mar. 11	— 0·56	— 2·99	+ 0·002	+ 0·410	+ 7·511	— 0·66	+ 5·62
Mar. 31	+ 1·02	— 1·97	— 0·037	+ 0·373	+ 7·921	— 2·21	+ 3·41
Apr. 20	+ 2·29	+ 0·32	— 0·064	+ 0·309	+ 8·294	— 3·14	+ 0·27
May 10	+ 2·90	+ 3·22	— 0·074	+ 0·235	+ 8·603	— 3·18	— 2·91
May 30	+ 2·67	+ 5·89	— 0·064	+ 0·171	+ 8·838	— 2·25	— 5·16
June 19	+ 1·65	+ 7·54	— 0·036	+ 0·135	+ 9·009	— 0·61	— 5·77
July 9	+ 0·13	+ 7·67	+ 0·002	+ 0·137	+ 9·144	+ 1·27	— 4·50
July 29	— 1·41	+ 6·26	+ 0·038	+ 0·175	+ 9·281	+ 2·79	— 1·71
Aug. 18	— 2·49	+ 3·77	+ 0·063	+ 0·238	+ 9·456	+ 3·48	+ 1·77
Sept. 7	— 2·84	+ 0·93	+ 0·070	+ 0·308	+ 9·694	+ 3·10	+ 4·87
Sept. 27	— 2·29	— 1·36	+ 0·058	+ 0·366	+ 10·002	+ 1·87	+ 6·74
Oct. 17	— 1·15	— 2·51	+ 0·031	+ 0·397	+ 10·368	+ 0·12	+ 6·86
Nov. 6	+ 0·26	— 2·25	— 0·003	+ 0·394	+ 10·765	— 1·64	+ 5·22
Nov. 26	+ 1·55	— 0·70	— 0·035	+ 0·359	+ 11·159	— 2·91	+ 2·31
Dec. 16	+ 2·36	+ 1·66	— 0·058	+ 0·301	+ 11·518	— 3·33	— 1·02
1945 Jan. 5	+ 2·46	+ 4·12	— 0·064	+ 0·237	+ 11·819	— 2·74	— 3·76
Jan. 25	+ 1·82	+ 5·94	— 0·053	+ 0·184	+ 12·056	— 1·30	— 5·06
Feb. 14	+ 0·59	+ 6·53	— 0·027	+ 0·157	+ 12·240	+ 0·58	— 4·48
Mar. 6	— 0·75	+ 5·78	+ 0·006	+ 0·163	+ 12·397	+ 2·32	— 2·16
Mar. 26	— 1·83	+ 3·95	+ 0·037	+ 0·200	+ 12·560	+ 3·33	+ 1·17
Apr. 15	— 2·31	+ 1·64	+ 0·056	+ 0·256	+ 12·760	+ 3·10	+ 4·27
May 5	— 2·08	— 0·44	+ 0·059	+ 0·315	+ 13·016	+ 2·33	+ 6·60
May 25	— 1·25	— 1·69	+ 0·047	+ 0·362	+ 13·331	+ 0·69	+ 7·29
June 14	— 0·11	— 1·80	+ 0·022	+ 0·384	+ 13·693	— 1·11	+ 6·18
July 4	+ 1·01	— 0·79	— 0·007	+ 0·377	+ 14·077	— 2·56	+ 3·62
July 24	+ 1·79	+ 1·00	— 0·034	+ 0·343	+ 14·454	— 3·24	+ 0·38
Aug. 13	+ 2·00	+ 3·00	— 0·052	+ 0·291	+ 14·797	— 2·95	— 2·57
Sept. 2	+ 1·59	+ 4·59	— 0·055	+ 0·236	+ 15·088	— 1·74	— 4·31
Sept. 22	+ 0·67	+ 5·26	— 0·043	+ 0·193	+ 15·324	+ 0·06	— 4·25
Oct. 12	— 0·47	+ 4·79	— 0·019	+ 0·174	+ 15·517	+ 1·86	— 2·39
Nov. 1	— 1·38	+ 3·41	+ 0·010	+ 0·184	+ 15·691	+ 3·10	+ 0·71
Nov. 21	— 1·82	+ 1·59	+ 0·035	+ 0·219	+ 15·875	+ 3·33	+ 4·04
Dec. 11	— 1·69	— 0·10	+ 0·050	+ 0·269	+ 16·094	+ 2·59	+ 6·63
Dec. 31	— 1·04	— 1·14	+ 0·049	+ 0·313	+ 16·363	+ 1·09	+ 7·72
1946 Jan. 20	— 0·10	— 1·24	+ 0·039	+ 0·357	+ 16·681	— 0·69	+ 7·03
Feb. 9	+ 0·81	— 0·43	+ 0·017	+ 0·374	+ 17·038	— 2·23	+ 4·80

	$d\delta\Omega$	$'f$	$d\delta i$	$'f$	$d\delta\varphi$	$'f$	Δ
	"	"	"	"	"	"	
1946 Feb. 9	— 0·47	— 0·69	+ 0·39	+ 0·77	+ 0·62	— 1·18	6·289
March 1	— 0·28	— 0·97	+ 0·24	+ 1·01	+ 0·02	— 1·16	6·408
March 21	— 0·02	— 0·99	+ 0·02	+ 1·03	— 0·60	— 1·76	6·350
Apr. 10	+ 0·24	— 0·75	— 0·23	+ 0·80	— 1·06	— 2·82	6·126
Apr. 30	+ 0·41	— 0·34	— 0·41	+ 0·39	— 1·21	— 4·03	5·790
May. 20	+ 0·44	+ 0·10	— 0·47	— 0·08	— 1·00	— 5·03	5·419
June 9	+ 0·33	+ 0·43	— 0·32	— 0·45	— 0·48	— 5·51	5·144
June 29	+ 0·14	+ 0·57	— 0·16	— 0·61	+ 0·18	— 5·33	5·071
July 19	— 0·08	+ 0·49	+ 0·10	— 0·51	+ 0·78	— 4·55	5·198
Aug. 8	— 0·25	+ 0·24	+ 0·33	— 0·18	+ 1·13	— 3·42	5·511
Aug. 28	— 0·32	— 0·08	+ 0·46	+ 0·28	+ 1·16	— 2·26	5·888
Sept. 17	— 0·30	— 0·38	+ 0·46	+ 0·74	+ 0·86	— 1·40	6·220
Oct. 7	— 0·19	— 0·57	+ 0·31	+ 1·05	+ 0·31	— 1·09	6.428
Oct. 27	— 0·04	— 0·61	+ 0·07	+ 1·12	— 0·33	— 1·42	6·460
Nov. 16	+ 0·10	— 0·51	— 0·19	+ 0·93	— 0·88	— 2·30	6·312
Dec. 6	+ 0·19	— 0·32	— 0·41	+ 0·52	— 1·17	— 3·47	6·010
Dec. 26	+ 0·22	— 0·10	— 0·50	+ 0·02	— 1·11	— 4·58	5.624
1947 Jan. 15	+ 0·17	+ 0·07	— 0·43	— 0·41	— 0·72	— 5·30	5·253
Feb. 4	+ 0·08	+ 0·15	— 0·23	— 0·64	— 0·12	— 5·42	5·038
Feb. 24	— 0·01	+ 0·14	+ 0·04	— 0·60	+ 0·52	— 4·90	5·014
March 16	— 0·08	+ 0·06	+ 0·30	— 0·30	+ 0·97	— 3·93	5·224
Apr. 5	— 0·10	— 0·04	+ 0·46	+ 0·16	+ 1·14	— 2·79	5·567
Apr. 25	— 0·08	— 0·12	+ 0·48	+ 0·64	+ 1·00	— 1·79	5·927
May 15	— 0·05	— 0·17	+ 0·36	+ 1·00	+ 0·55	— 1·24	6·202
June 4	— 0·01	— 0·18	+ 0·14	+ 1·14	— 0·03	— 1·27	6·327
June 24	0·00	— 0·18	— 0·13	+ 1·01	— 0·61	— 1·88	6·270
July 14	0·00	— 0·18	— 0·36	+ 0·65	— 1·00	— 2·88	6·035
Aug. 3	— 0·03	— 0·21	— 0·48	+ 0·17	— 1·09	— 3·97	5·668
Aug. 23	— 0·04	— 0·25	— 0·45	— 0·28	— 0·84	— 4·81	5·253
Sept. 12	— 0·04	— 0·29	— 0·28	— 0·56	— 0·35	— 5·16	4·907
Oct. 2	0·00	— 0·29	— 0·02	— 0·58	+ 0·24	— 4·92	4·741
Oct. 22	+ 0·06	— 0·23	+ 0·23	— 0·35	+ 0·73	— 4·19	4·809
Nov. 11	+ 0·12	— 0·11	+ 0·41	+ 0·06	+ 0·99	— 3·20	5·074
Dec. 1	+ 0·16	+ 0·05	+ 0·46	+ 0·52	+ 0·96	— 2·24	5·417
Dec. 21	+ 0·15	+ 0·20	+ 0·38	+ 0·90	+ 0·68	— 1·56	5·738
1948 Jan. 10	+ 0·08	+ 0·28	+ 0·18	+ 1·08	+ 0·18	— 1·38	5·943

	$d\delta\pi$	$'f$	$\lambda d\delta n$	$'f$	$''f$	P	$'f$
	″	″	″	″	″	″	″
1946 Feb. 9	+ 0·81	— 0·43	+ 0·017	+ 0·374	+ 17·038	— 2·23	+ 4·80
March 1	+ 1·44	+ 1·01	— 0·009	+ 0·365	+ 17·412	— 3·10	+ 1·70
March 21	+ 1·60	+ 2·61	— 0·033	+ 0·332	+ 17·777	— 3·04	— 1·34
Apr. 10	+ 1·22	+ 3·83	— 0·048	+ 0·284	+ 18·109	— 2·02	— 3·36
Apr. 30	+ 0·44	+ 4·27	— 0·050	+ 0·234	+ 18·393	— 0·34	— 3·70
May. 20	— 0·50	+ 3·77	— 0·038	+ 0·196	+ 18·627	+ 1·49	— 2·21
June 9	— 1·25	+ 2·52	— 0·015	+ 0·181	+ 18·823	+ 2·87	+ 0·66
June 29	— 1·57	+ 0·95	+ 0·011	+ 0·192	+ 19·004	+ 3·34	+ 4·00
July 19	— 1·36	— 0·41	+ 0·033	+ 0·225	+ 19·196	+ 2·78	+ 6·78
Aug. 8	— 0·71	— 1·12	+ 0·046	+ 0·271	+ 19·421	+ 1·41	+ 8·19
Aug. 28	+ 0·14	— 0·98	+ 0·047	+ 0·318	+ 19·692	— 0·35	+ 7·84
Sept. 17	+ 0·93	— 0·05	+ 0·036	+ 0·354	+ 20·010	— 1·96	+ 5·88
Oct. 7	+ 1·41	+ 1·36	+ 0·015	+ 0·369	+ 20·364	— 2·99	+ 2·89
Oct. 27	+ 1·42	+ 2·78	— 0·009	+ 0·360	+ 20·733	— 3·11	— 0·22
Nov. 16	+ 0·95	+ 3·73	— 0·032	+ 0·328	+ 21·093	— 2·27	— 2·49
Dec. 6	+ 0·13	+ 3·86	— 0·046	+ 0·282	+ 21·421	— 0·68	— 3·17
Dec. 26	— 0·77	+ 3·09	— 0·048	+ 0·234	+ 21·703	+ 1·17	— 2·00
1947 Jan 15	— 1·44	+ 1·65	— 0·036	+ 0·198	+ 21·937	+ 2·66	+ 0·66
Feb. 4	— 1·63	+ 0·02	— 0·015	+ 0·183	+ 22·135	+ 3·27	+ 3·93
Feb. 24	— 1·27	— 1.25	+ 0·010	+ 0·193	+ 22·318	+ 2·98	+ 6·91
March 16	— 0·50	— 1·75	+ 0·032	+ 0·225	+ 22·511	+ 1·73	+ 8·64
Apr. 5	+ 0·44	— 1·31	+ 0·046	+ 0·271	+ 22·736	0·00	+ 8·64
Apr. 25	+ 1·24	— 0·07	+ 0·047	+ 0·318	+ 23·007	— 1·71	+ 6·93
May 15	+ 1·67	+ 1·60	+ 0·037	+ 0·355	+ 23·325	— 2·90	+ 4·03
June 4	+ 1·57	+ 3·17	+ 0·016	+ 0·371	+ 23·680	— 3·22	+ 0·81
June 24	+ 0·95	+ 4·12	— 0·009	+ 0·362	+ 24·051	— 2·59	— 1·78
July 14	— 0·03	+ 4·09	— 0·033	+ 0·329	+ 24·413	— 1.06	— 2·84
Aug. 3	— 1·07	+ 3·02	— 0·049	+ 0·280	+ 24·742	+ 0·81	— 2·03
Aug. 23	— 1·81	+ 1·21	— 0.051	+ 0·229	+ 25·022	+ 2·47	+ 0·44
Sept. 12	— 1·99	— 0·78	— 0·039	+ 0·190	+ 25·251	+ 3·37	+ 3·81
Oct. 2	— 1·52	— 2·30	— 0·017	+ 0·173	+ 25·441	+ 3·29	+ 7·10
Oct. 22	— 0·56	— 2·86	+ 0·010	+ 0·183	+ 25.614	+ 2·13	+ 9·23
Nov. 11	+ 0·56	— 2·30	+ 0·034	+ 0·217	+ 25·797	+ 0·45	+ 9·68
Dec. 1	+ 1·56	— 0·74	+ 0·050	+ 0·267	+ 26·014	— 1·38	+ 8·30
Dec. 21	+ 2·10	+ 1·36	+ 0·053	+ 0·320	+ 26·281	— 2·76	+ 5·54
1948 Jan. 10	+ 2·03	+ 3·39	+ 0·042	+ 0·362	+ 26·601	— 3·33	+ 2·21

	$d\delta\Omega$	'f	$d\delta i$	'f	$d\delta\varphi$	'f	Δ
	"	"	"	"	"	"	
1948 Jan. 10	+ 0·08	+ 0·28	+ 0·18	+ 1·08	+ 0·18	− 1·38	5·943
Jan. 30	− 0·03	+ 0·25	− 0·06	+ 1·02	− 0·34	− 1·72	5·983
Feb. 19	− 0·16	+ 0.09	− 0·29	+ 0·73	− 0·76	− 2·48	5·841
March 10	− 0·25	− 0·16	− 0·42	+ 0·31	− 0·93	− 3·41	5·535
March 30	− 0·28	− 0·44	− 0·42	− 0·11	− 0·81	− 4·22	5·127
Apr. 19	− 0·21	− 0·65	− 0·29	− 0·40	− 0·45	− 4·67	4·710
May 9	− 0·06	− 0·71	− 0·07	− 0·47	+ 0·04	− 4·63	4·402
May 29	+ 0·13	− 0·58	+ 0·15	− 0·32	+ 0·49	− 4·14	4·300
June 18	+ 0·29	− 0·29	+ 0·32	0·00	+ 0·77	− 3·37	4·424
July 8	+ 0·38	+ 0·09	+ 0·39	+ 0·39	+ 0·81	− 2·56	4·704
July 28	+ 0·35	+ 0·44	+ 0·34	+ 0·73	+ 0·62	− 1·94	5·030
Aug. 17	+ 0·22	+ 0·66	+ 0·19	+ 0.92	+ 0·26	− 1·68	5·297
Sept. 6	0·00	+ 0·66	0·00	+ 0·92	− 0·17	− 1·85	5·431
Sept. 26	− 0·25	+ 0·41	− 0·19	+ 0·73	− 0·53	− 2·38	5·399
Oct. 16	− 0·43	− 0·02	− 0·32	+ 0·41	− 0·71	− 3·09	5·196
Nov. 5	− 0·49	− 0·51	− 0·34	+ 0·07	− 0·65	− 3·74	4·851
Nov. 25	− 0·39	− 0·90	− 0·25	− 0·18	− 0·37	− 4·11	4·428
Dec. 15	− 0·16	− 1·06	− 0·10	− 0·28	+ 0·02	− 4·09	4·025
1949 Jan. 4	+ 0·13	− 0·93	+ 0·08	− 0·20	+ 0·38	− 3·71	3·750
Jan. 24	+ 0·38	− 0·55	+ 0·20	0·00	+ 0·61	− 3·10	3·682
Feb. 13	+ 0·54	− 0·01	+ 0·26	+ 0·26	+ 0·63	− 2·47	3·819
March 5	+ 0·53	+ 0·52	+ 0·24	+ 0·50	+ 0·45	− 2·02	4·084
March 25	+ 0·36	+ 0·88	+ 0·15	+ 0·65	+ 0·14	− 1·88	4·366
Apr. 14	+ 0·07	+ 0·95	+ 0·03	+ 0·68	− 0·21	− 2·09	4·581
May 4	− 0·26	+ 0·69	− 0·09	+ 0·59	− 0·48	− 2·57	4·669
May 24	− 0·52	+ 0·17	− 0·17	+ 0·42	− 0·59	− 3·16	4·609
June 13	− 0·62	− 0·45	− 0·18	+ 0·24	− 0·47	− 3·63	4·396
July 3	− 0·53	− 0·98	− 0·15	+ 0·09	− 0·16	− 3·79	4·063
July 23	− 0·27	− 1·25	− 0·07	+ 0·02	+ 0·21	− 3·58	3·662
Aug. 12	+ 0·07	− 1·18	+ 0·02	+ 0·04	+ 0·53	− 3·05	3·277
Sept. 1	+ 0·39	− 0·79	+ 0·08	+ 0·12	+ 0·68	− 2·37	2·996
Sept. 21	+ 0·58	− 0·21	+ 0·10	+ 0·22	+ 0·62	− 1·75	2·893
Oct. 11	+ 0·60	+ 0·39	+ 0·08	+ 0·30	+ 0·35	− 1·40	2·974
Oct. 31	+ 0·44	+ 0·83	+ 0·05	+ 0·35	− 0·04	− 1·44	3·178
Nov. 20	+ 0·15	+ 0·98	+ 0·01	+ 0·36	− 0·44	− 1·88	3·417
Dec. 10	− 0·18	+ 0·80	− 0·01	+ 0·35	− 0·73	− 2·61	3·616

	dδπ	'f	λdδn	'f	''f	P	'f
	″	″	″	″	″	″	″
1948 Jan. 10	+2·03	+3·39	+0·042	+0·362	+26·601	−3·33	+2·21
Jan. 30	+1·30	+4·69	+0·018	+0·380	+26·963	−2·89	−0·68
Feb. 19	+0·13	+4·82	−0·011	+0·369	+27·343	−1·56	−2·24
March 10	−1·16	+3·66	−0·038	+0·331	+27·712	+0·32	−1·92
March 30	−2·16	+1·50	−0·057	+0·274	+28·043	+2·14	+0·22
Apr. 19	−2·51	−1·01	−0·059	+0·215	+28·317	+3·32	+3·54
May 9	−2·10	−3·11	−0·045	+0·170	+28·532	+3·49	+7·03
May 29	−1·05	−4·16	−0·019	+0·151	+28·702	+2·62	+9·65
June 18	+0·32	−3·84	+0·012	+0·163	+28·853	+1.02	+10·67
July 8	+1·62	−2·22	+0·041	+0·204	+29·016	−0·85	+9·82
July 28	+2·47	+0·25	+0·060	+0·264	+29·220	−2·45	+7·37
Aug. 17	+2·63	+2·88	+0·063	+0·327	+29·484	−3·34	+4·03
Sept. 6	+2·01	+4·89	+0·049	+0·376	+29·811	−3·24	+0·79
Sept. 26	+0·75	+5·64	+0·021	+0·397	+30·187	−2·17	−1·38
Oct. 16	−0·80	+4·84	−0·015	+0·382	+30·584	−0·40	−1·78
Nov. 5	−2·18	+2·66	−0·049	+0·333	+30·966	+1·53	−0·25
Nov. 25	−2·92	−0·26	−0·070	+0·263	+31·299	+3·01	+2·76
Dec. 15	−2·80	−3·06	−0·072	+0·191	+31·562	+3·58	+6·34
1949 Jan. 4	−1·85	−4·91	−0·055	+0·136	+31·753	+3·10	+9·44
Jan. 24	−0·39	−5·30	−0·022	+0·114	+31·889	+1·76	+11·20
Feb. 13	+1·19	−4·11	+0·017	+0·131	+32·003	−0·04	+11·16
March 5	+2·44	−1·67	+0·052	+0·183	+32·134	−1·78	+9·38
March 25	+3·02	+1·35	+0·075	+0·258	+32·317	−2·99	+6·39
Apr. 14	+2·76	+4·11	+0·082	+0·340	+32·575	−3·35	+3·04
May 4	+1·69	+5·80	+0·061	+0·401	+32·915	−2·73	+0·31
May 24	+0·10	+5·90	+0·024	+0·425	+33·316	−1·29	−0·98
June 13	−1·56	+4·34	−0·022	+0·403	+33·741	+0·53	−0·45
July 3	−2·77	+1·57	−0·064	+0·339	+34·144	+2·18	+1·73
July 23	−3·10	−1·53	−0·089	+0·250	+34·483	+3·16	+4·89
Aug. 12	−2·62	−4·15	−0·091	+0·159	+34·733	+3·20	+8·09
Sept. 1	−1·38	−5·53	−0·069	+0·090	+34·892	+2·37	+10·46
Sept. 21	+0·20	−5·33	−0·029	+0·061	+34·982	+0·96	+11·42
Oct. 11	+1·67	−3·66	+0·019	+0·080	+35·043	−0·61	+10·81
Oct. 31	+2·69	−0·97	+0·064	+0·144	+35·123	−1·97	+8·84
Nov. 20	+2·95	+1·98	+0·093	+0·237	+35·267	−2·71	+6·13
Dec. 10	+2·40	+4·38	+0·100	+0·337	+35·504	−2·67	+3·46

3

	$d\delta\Omega$	'f	$d\delta i$	'f	$d\delta\varphi$	'f	Δ
	″	″	″	″	″	″	
1949 Dec. 10	− 0·18	+ 0·80	− 0·01	+ 0·35	− 0·73	− 2·61	3·616
Dec. 30	− 0·46	+ 0·34	− 0·01	+ 0·34	− 0·79	− 3·40	3·728
1950 Jan. 19	− 0·59	− 0·25	0·00	+ 0·34	− 0·58	− 3·98	3·729
Feb. 8	− 0·52	− 0·77	+ 0·02	+ 0·36	− 0·15	− 4·13	3·607
Feb. 28	− 0·30	− 1·07	+ 0·02	+ 0·38	+ 0·36	− 3·77	3·379
March 20	− 0·01	− 1.08	+ 0·01	+ 0·39	+ 0·80	− 2·97	3·071
Apr. 9	+ 0·26	− 0·82	− 0·04	+ 0·35	+ 1·04	− 1·93	2·724
Apr. 29	+ 0·42	− 0·40	− 0·08	+ 0·27	+ 0·99	− 0·94	2·391
May 19	+ 0·44	+ 0·04	− 0·10	+ 0·17	+ 0·68	− 0·26	2·143
June 8	+ 0·33	+ 0·37	− 0·09	+ 0·08	+ 0·19	− 0·07	2·044
June 28	+ 0·14	+ 0·51	− 0·05	+ 0·03	− 0·37	− 0·44	2·121
July 18	− 0·06	+ 0·45	+ 0·03	+ 0·06	− 0·87	− 1·31	2·331
Aug. 7	− 0·22	+ 0·23	+ 0·11	+ 0·17	− 1·14	− 2·45	2·603
Aug. 27	− 0·27	− 0·04	+ 0·17	+ 0·34	− 1·11	− 3·56	2·866
Sept. 16	− 0·23	− 0·27	+ 0·18	+ 0·52	− 0·76	− 4·32	3·072
Oct. 6	− 0·13	− 0·40	+ 0·13	+ 0·65	− 0·21	− 4·53	3·188
Oct. 26	− 0·02	− 0·42	+ 0·03	+ 0·68	+ 0·39	− 4·14	3·195
Nov. 15	+ 0·04		− 0·09		+ 0·87		3·090

THE EARTH

1942 June 10.0 — 1950 Oct. 6.0

	$d\delta\Omega$	'f	$d\delta i$	'f	$d\delta\varphi$	'f	Δ
	″	″	″	″	″	″	
1942 May 21	+ 0·08	− 0·04	− 0·06	+ 0·04	+ 0·66	− 0·36	3·053
June 10	+ 0·08	+ 0·04	− 0·09	− 0·05	+ 0·72	+ 0·36	2·869
June 30	+ 0·07	+ 0·11	− 0·11	− 0·16	+ 0·71	+ 1·07	2·664
July 20	+ 0·05	+ 0·16	− 0·12	− 0·28	+ 0·65	+ 1·72	2·442
Aug. 9	+ 0·02	+ 0·18	− 0·11	− 0·39	+ 0·53	+ 2·25	2·214
Aug. 29	0·00	+ 0·18	− 0·09	− 0·48	+ 0·36	+ 2·61	1·992
Sept. 18	− 0·01	+ 0·17	− 0·05	− 0·53	+ 0·15	+ 2·76	1·799
Oct. 8	− 0·01	+ 0·16	− 0·02	− 0·55	− 0·07	+ 2·69	1·665
Oct. 28	+ 0·01	+ 0·17	+ 0·02	− 0·53	− 0·27	+ 2·42	1·623
Nov. 17	+ 0·05	+ 0·22	+ 0·05	− 0·48	− 0·43	+ 1·99	1·693
Dec. 7	+ 0·10	+ 0·32	+ 0·08	− 0·40	− 0·55	+ 1·44	1·868

		$d\delta\pi$	$'f$	$\lambda d\delta n$	$'f$	$''f$	P	$'f$
		″	″	″	″	″	″	″
1949 Dec.	10	+2·40	+4·38	+0·100	+0·337	+35·504	−2·67	+3·46
Dec.	30	+1·19	+5·57	+0·078	+0·415	+35·841	−1·89	+1·57
1950 Jan.	19	−0·33	+5·24	+0·035	+0·450	+36·256	−0·61	+0·96
Feb.	8	−1·69	+3·55	−0·021	+0·429	+36·706	+0·75	+1·71
Feb.	28	−2·47	+1·08	−0·072	+0·357	+37·135	+1·77	+3·48
March	20	−2·45	−1·37	−0·104	+0·253	+37·492	+2·18	+5·66
Apr.	9	−1·74	−3·11	−0·111	+0·142	+37·745	+1·93	+7·59
Apr.	29	−0·57	−3·68	−0·091	+0·051	+37·887	+1·20	+8·79
May	19	+0·66	−3·02	−0·050	+0·001	+37·938	+0·27	+9·06
June	8	+1·61	−1·41	+0·002	+0·003	+37·939	−0·57	+8·49
June	28	+2·05	+0·64	+0·053	+0·056	+37·942	−1·12	+7·37
July	18	+1·87	+2·51	+0·094	+0·150	+37·998	−1·25	+6·12
Aug.	7	+1·17	+3·68	+0·112	+0·262	+38·148	−0·97	+5·15
Aug.	27	+0·16	+3·84	+0·103	+0·365	+38·410	−0·42	+4·73
Sept.	16	−0·80	+3·04	+0·068	+0·433	+38·775	+0·16	+4·89
Oct.	6	−1·39	+1·65	+0·017	+0·450	+39·208	+0·53	+5·42
Oct.	26	−1·42	+0·23	−0·036	+0·414	+39·658	+0·56	+5·98
Nov.	15	−0·91		−0·079		+40·072	+0·26	

THE EARTH

1942 June 10.0 — 1950 Oct. 6.0

		$d\delta\pi$	$'f$	$\lambda d\delta n$	$'f$	$''f$	P	$'f$
		″	″	″	″	″	″	″
1942 May	21	−0·57	+0·14	−0·065	+0·034	−0·028	+0·35	−0·09
June	10	−0·22	−0·08	−0·069	−0·035	+0·006	+0·14	+0·05
June	30	+0·16	+0·08	−0·068	−0·103	−0·029	−0·08	−0·03
July	20	+0·54	+0·62	−0·063	−0·166	−0·132	−0·30	−0·33
Aug.	9	+0·87	+1·49	−0·054	−0·220	−0·298	−0·48	−0·81
Aug.	29	+1·14	+2·63	−0·041	−0·261	−0·518	−0·59	−1·40
Sept.	18	+1·30	+3·93	−0·024	−0·285	−0·779	−0·60	−2·00
Oct.	8	+1·33	+5·26	−0·007	−0·292	−1·064	−0·53	−2·53
Oct.	28	+1·21	+6·47	+0·010	−0·282	−1·356	−0·35	−2·88
Nov.	17	+0·92	+7·39	+0·026	−0·256	−1·638	−0·08	−2·96
Dec.	7	+0·47	+7·86	+0·040	−0·216	−1·894	+0·26	−2·70

	$d\delta\Omega$	$'f$	$d\delta i$	$'f$	$d\delta\varphi$	$'f$	Δ
	″	″	″	″	″	″	
1942 Dec. 7	+0·10	+0·32	+0·08	−0·40	−0·55	+1·44	1·868
Dec. 27	+0·16	+0·48	+0·11	−0·29	−0·63	+0·81	2·123
1943 Jan. 16	+0·20	+0·68	+0·11	−0·18	−0·65	+0·16	2·424
Feb. 5	+0·21	+0·89	+0·10	−0·08	−0·61	−0·45	2·745
Feb. 25	+0·19	+1·08	+0·08	0·00	−0·50	−0·95	3·066
March 17	+0·13	+1·21	+0·05	+0·05	−0·35	−1·30	3·370
Apr. 6	+0·04	+1·25	+0·01	+0·06	−0·18	−1·48	3·646
Apr. 26	−0·06	+1·19	−0·02	+0·04	0·00	−1·48	3·885
May 16	−0·16	+1·03	−0·04	0·00	+0·14	−1·34	4·080
June 5	−0·26	+0·77	−0·05	−0·05	+0·27	−1·07	4·228
June 25	−0·33	+0·44	−0·06	−0·11	+0·35	−0·72	4·322
July 15	−0·37	+0·07	−0·05	−0·16	+0·37	−0·35	4·362
Aug. 4	−0·37	−0·30	−0·04	−0·20	+0·35	0·00	4·347
Aug. 24	−0·32	−0.62	−0·03	−0·23	+0·28	+0·28	4·279
Sept. 13	−0·24	−0·86	−0·02	−0·25	+0·16	+0·44	4·162
Oct. 3	−0·11	−0·97	0·00	−0·25	+0·02	+0·46	4·005
Oct. 23	+0·03	−0·94	0·00	−0·25	−0·13	+0·33	3·822
Nov. 12	+0·19	−0·75	0·00	−0·25	−0·27	+0·06	3·630
Dec. 2	+0·33	−0·42	0·00	−0·25	−0·38	−0·32	3·456
Dec. 22	+0·43	+0·01	−0·01	−0·26	−0·43	−0·75	3·331
1944 Jan. 11	+0·48	+0·49	−0·02	−0·28	−0·43	−1·18	3·286
Jan. 31	+0·48	+0·97	−0·03	−0·31	−0·36	−1·54	3·341
Feb. 20	+0·39	+1·36	−0·03	−0·34	−0·23	−1·77	3·497
March 11	+0·27	+1·63	−0·03	−0·37	−0·08	−1·85	3·740
March 31	+0·12	+1·75	−0·01	−0·38	+0·09	−1·76	4·040
Apr. 20	−0·05	+1·70	+0·01	−0·37	+0·24	−1·52	4·369
May 10	−0·21	+1·49	+0·03	−0·34	+0·36	−1·16	4·699
May 30	−0·34	+1·15	+0·05	−0·29	+0·43	−0·73	5·011
June 19	−0·43	+0·72	+0·08	−0·21	+0·46	−0·27	5·277
July 9	−0·48	+0·24	+0·09	−0·12	+0·43	+0·16	5·492
July 29	−0·47	−0·23	+0·10	−0·02	+0·34	+0·50	5·642
Aug. 18	−0·42	−0·65	+0·09	+0·07	+0·21	+0·71	5·720
Sept. 7	−0·31	−0·96	+0·07	+0·14	+0·04	+0·75	5·727
Sept. 27	−0·17	−1·13	+0·04	+0·18	−0·15	+0·60	5·658
Oct. 17	0·00	−1·13	0·00	+0·18	−0·33	+0·27	5·520
Nov. 6	+0·17	−0·96	−0·05	+0·13	−0·49	−0·22	5·322

	$d\delta\pi$	'f	$\lambda d\delta n$	'f	''f	P	'f
	″	″	″	″	″	″	″
1942 Dec. 7	+0·47	+7·86	+0·040	—0·216	—1·894	+0·26	—2·70
Dec. 27	—0·07	+7·79	+0·054	—0·162	—2·110	+0·69	—2·05
1943 Jan. 16	—0·61	+7·18	+0·064	—0·098	—2·272	+1·01	—1·04
Feb. 5	—1·11	+6·07	+0·068	—0·030	—2·370	+1·30	+0·26
Feb. 25	—1·47	+4·60	+0·067	+0·037	—2·400	+1·46	+1·72
March 17	—1·66	+2·94	+0·060	+0·097	—2·363	+1·45	+3·17
Apr. 6	—1·66	+1·28	+0·049	+0·146	—2·266	+1·28	+4·45
Apr. 26	—1·49	—0·21	+0·034	+0·180	—2·120	+0·97	+5·42
May 16	—1·15	—1·36	+0·017	+0·197	—1·940	+0·54	+5·96
June 5	—0·69	—2·05	0·000	+0·197	—1·743	+0·04	+6·00
June 25	—0·16	—2·21	—0·017	+0·180	—1·546	—0·48	+5·52
July 15	+0·39	—1·82	—0·031	+0·149	—1·366	—0·97	+4·55
Aug. 4	+0·95	—0·87	—0·043	+0·106	—1·217	—1·43	+3·12
Aug. 24	+1·42	+0·55	—0·052	+0·054	—1·111	—1·73	+1·39
Sept. 13	+1·77	+2·32	—0·055	—0·001	—1·057	—1·89	—0·50
Oct. 3	+1·95	+4·27	—0·054	—0·055	—1·058	—1·85	—2·35
Oct. 23	+1·93	+6·20	—0·050	—0·105	—1·113	—1·60	—3·95
Nov. 12	+1·68	+7·88	—0·036	—0·141	—1·218	—1·13	—5·08
Dec. 2	+1·25	+9·13	—0·021	—0·162	—1·359	—0·51	—5·59
Dec. 22	+0·65	+9·78	—0·004	—0·166	—1·521	+0·22	—5·37
1944 Jan. 11	—0·07	+9·71	+0·012	—0·154	—1·687	+0·94	—4·43
Jan. 31	—0·69	+9·02	+0·029	—0·125	—1·841	+1·57	—2·86
Feb. 20	—1·27	+7·75	+0·041	—0·084	—1·966	+2·02	—0·84
March 11	—1·69	+6·06	+0·049	—0·035	—2·050	+2·25	+1·41
March 31	—1·92	+4·14	+0·051	+0·016	—2·085	+2·23	+3·64
Apr. 20	—1·93	+2·21	+0·049	+0·065	—2·069	+1·98	+5·62
May 10	—1·75	+0·46	+0·042	+0·107	—2·004	+1·53	+7·15
May 30	—1·39	—0·93	+0·032	+0·139	—1·897	+0·94	+8·09
June 19	—0·92	—1·85	+0·020	+0·159	—1·758	+0·28	+8·37
July 9	—0·37	—2·22	+0·006	+0·165	—1·599	—0·42	+7·95
July 29	+0·21	—2·01	—0·007	+0·158	—1·434	—1·06	+6·89
Aug. 18	+0·76	—1·25	—0·020	+0·138	—1·276	—1·61	+5·28
Sept. 7	+1·22	—0·03	—0·031	+0·107	—1·138	—1·99	+3·29
Sept. 27	+1·56	+1·53	—0·039	+0·068	—1·031	—2·16	+1·13
Oct. 17	+1·72	+3·25	—0·043	+0·025	—0·963	—2·08	—0·95
Nov. 6	+1·68	+4·93	—0·043	—0·018	—0·938	—1·75	—2·70

	$d\delta\Omega$	'f	$d\delta i$	'f	$d\delta\varphi$	'f	Δ
	"	"	"	"	"	"	
1944 Nov. 6	+ 0·17	— 0·96	— 0·05	+ 0·13	— 0·49	— 0·22	5·322
Nov. 26	+ 0·32	— 0·64	— 0·10	+ 0·03	— 0·60	— 0·82	5·084
Dec. 16	+ 0·43	— 0·21	— 0·14	— 0·11	— 0·63	— 1·45	4·832
1945 Jan. 5	+ 0·49	+ 0·28	— 0·16	— 0·27	— 0·59	— 2·04	4·596
Jan. 25	+ 0·47	+ 0·75	— 0·17	— 0·44	— 0·47	— 2·51	4·415
Feb. 14	+ 0·41	+ 1·16	— 0·15	— 0·59	— 0·29	— 2·80	4·322
March 6	+ 0·29	+ 1·45	— 0·12	— 0·71	— 0·07	— 2·87	4·342
March 26	+ 0·15	+ 1·60	— 0·06	— 0·77	+ 0·16	— 2·71	4·476
Apr. 15	— 0·01	+ 1·59	0·00	— 0·77	+ 0·36	— 2·35	4·706
May 5	— 0·15	+ 1·44	+ 0·07	— 0·70	+ 0·53	— 1·82	5·000
May 25	— 0·27	+ 1·17	+ 0·13	— 0·57	+ 0·63	— 1·19	5·327
June 14	— 0·35	+ 0·82	+ 0·17	— 0·40	+ 0·67	— 0·52	5·655
July 4	— 0·40	+ 0·42	+ 0·20	— 0·20	+ 0·64	+ 0·12	5·954
July 24	— 0·40	+ 0·02	+ 0.21	+ 0·01	+ 0·54	+ 0·66	6·204
Aug. 13	— 0·36	— 0·34	+ 0·20	+ 0·21	+ 0·38	+ 1·04	6·390
Sept. 2	— 0·28	— 0·62	+ 0·16	+ 0·37	+ 0·18	+ 1·22	6·499
Sept. 22	— 0·16	— 0·78	+ 0·10	+ 0·47	— 0·06	+ 1·16	6·522
Oct. 12	— 0·04	— 0·82	+ 0·02	+ 0·49	— 0·30	+ 0·86	6·460
Nov. 1	+ 0·09	— 0·73	— 0·06	+ 0·43	— 0·52	+ 0.34	6·316
Nov. 21	+ 0·21	— 0·52	— 0·14	+ 0·29	— 0·69	— 0·35	6·100
Dec. 11	+ 0·29	— 0·23	— 0·21	+ 0·08	— 0·78	— 1·13	5·826
Dec. 31	+ 0·34	+ 0·11	— 0·26	— 0·18	— 0·77	— 1·90	5·526
1946 Jan. 20	+ 0·34	+ 0·45	— 0·27	— 0·45	— 0·67	— 2·57	5·230
Feb. 9	+ 0·29	+ 0·74	— 0·24	— 0·69	— 0·49	— 3·06	4·980
March 1	+ 0·22	+ 0·96	— 0·19	— 0·88	— 0·26	— 3·32	4·807
March 21	+ 0·12	+ 1·08	— 0·11	— 0·99	+ 0·01	— 3·31	4·748
Apr. 10	+ 0·02	+ 1·10	— 0·02	— 1·01	+ 0·26	— 3·05	4·814
Apr. 30	— 0·07	+ 1·03	+ 0·07	— 0·94	+ 0·48	— 2·57	4·990
May 20	— 0·14	+ 0·89	+ 0·15	— 0·79	+ 0·64	— 1·93	5·249
June 9	— 0·20	+ 0·69	+ 0·22	— 0·57	+ 0·75	— 1·18	5·556
June 29	— 0·22	+ 0·47	+ 0·26	— 0·31	+ 0·75	— 0·43	5·875
July 19	— 0·22	+ 0·25	+ 0·28	— 0·03	+ 0·70	+ 0·27	6·170
Aug. 8	— 0·20	+ 0·05	+ 0·26	+ 0·23	+ 0·57	+ 0·84	6·422
Aug. 28	— 0·15	— 0·10	+ 0·22	+ 0·45	+ 0·38	+ 1·22	6·616
Sept. 17	— 0·10	— 0·20	+ 0·15	+ 0·60	+ 0·15	+ 1·37	6·727
Oct. 7	— 0·03	— 0·23	+ 0·06	+ 0·66	— 0·11	+ 1·26	6·748

	$d\delta\pi$	$'f$	$\lambda d\delta n$	$'f$	$''f$	P	$'f$
	"	"	"	"	"	"	"
1944 Nov. 6	+ 1·68	+ 4·93	— 0·043	— 0·018	— 0·938	— 1·75	— 2·70
Nov. 26	+ 1·43	+ 6·36	— 0·038	— 0·056	— 0·956	— 1·18	— 3·88
Dec. 16	+ 1·01	+ 7·37	— 0·029	— 0·085	— 1·012	— 0·45	— 4·33
1945 Jan. 5	+ 0·46	+ 7·83	— 0·017	— 0·102	— 1·097	+ 0·36	— 3·97
Jan. 25	— 0·12	+ 7·71	— 0·004	— 0·106	— 1·199	+ 1·13	— 2·84
Feb. 14	— 0·67	+ 7·04	+ 0·010	— 0·096	— 1·305	+ 1·76	— 1·08
March 6	— 1·11	+ 5·93	+ 0·021	— 0·075	— 1·401	+ 2·16	+ 1·08
March 26	— 1·39	+ 4·54	+ 0·030	— 0·045	— 1·476	+ 2·30	+ 3·38
Apr. 15	— 1·50	+ 3·04	+ 0·036	— 0·009	— 1·521	+ 2·18	+ 5·56
May 5	— 1·44	+ 1·60	+ 0·038	+ 0·029	— 1·530	+ 1·83	+ 7·39
May 25	— 1·22	+ 0·38	+ 0·036	+ 0·065	— 1·501	+ 1·30	+ 8·69
June 14	— 0·89	— 0·51	+ 0·032	+ 0·097	— 1·436	+ 0·66	+ 9·35
July 4	— 0·48	— 0·99	+ 0·025	+ 0·122	— 1·339	— 0·04	+ 9·31
July 24	— 0·04	— 1·03	+ 0·016	+ 0·138	— 1·217	— 0·72	+ 8·59
Aug. 13	+ 0·39	— 0·64	+ 0·005	+ 0·143	— 1·079	— 1·33	+ 7·26
Sept. 2	+ 0·77	+ 0·13	— 0·006	+ 0·137	— 0·936	— 1·80	+ 5·46
Sept. 22	+ 1·05	+ 1·18	— 0·016	+ 0·121	— 0·799	— 2·07	+ 3·39
Oct. 12	+ 1·20	+ 2·38	— 0·025	+ 0·096	— 0·678	— 2·09	+ 1·30
Nov. 1	+ 1·19	+ 3·57	— 0·031	+ 0·065	— 0·582	— 1·86	— 0·56
Nov. 21	+ 1·03	+ 4·60	— 0·035	+ 0·030	— 0·517	— 1·39	— 1·95
Dec. 11	+ 0·71	+ 5·31	— 0·035	— 0·005	— 0·487	— 0·71	— 2·66
Dec. 31	+ 0·31	+ 5·62	— 0·031	— 0·036	— 0·492	+ 0·07	— 2·59
1946 Jan. 20	— 0·13	+ 5·49	— 0·024	— 0·060	— 0·528	+ 0·85	— 1·74
Feb. 9	— 0·53	+ 4·96	— 0·014	— 0·074	— 0·588	+ 1·51	— 0·23
March 1	— 0·85	+ 4·11	— 0·004	— 0·078	— 0·662	+ 2·00	+ 1·77
March 21	— 1·04	+ 3·07	+ 0·007	— 0·071	— 0·740	+ 2·22	+ 3·99
Apr. 10	— 1·08	+ 1·99	+ 0·016	— 0·055	— 0·811	+ 2·18	+ 6·17
Apr. 30	— 0·99	+ 1·00	+ 0·023	— 0·032	— 0·866	+ 1·89	+ 8·06
May 20	— 0·78	+ 0·22	+ 0·028	— 0·004	— 0·898	+ 1·41	+ 9·47
June 9	— 0·48	— 0·26	+ 0·031	+ 0·027	— 0·902	+ 0·79	+ 10·26
June 29	— 0·14	— 0·40	+ 0·031	+ 0·058	— 0·875	+ 0·11	+ 10·37
July 19	+ 0·21	— 0·19	+ 0·028	+ 0·086	— 0·817	— 0·58	+ 9·79
Aug. 8	+ 0·53	+ 0·34	+ 0·022	+ 0·108	— 0·731	— 1·20	+ 8·59
Aug. 28	+ 0·78	+ 1·12	+ 0·016	+ 0·124	— 0·623	— 1·69	+ 6·90
Sept. 17	+ 0·93	+ 2·05	+ 0·007	+ 0·131	— 0·499	— 2·00	+ 4·90
Oct. 7	+ 0·96	+ 3·01	— 0·003	+ 0·128	— 0·368	— 2·07	+ 2·83

	$d\delta\Omega$	'f	$d\delta i$	'f	$d\delta\varphi$	'f	Δ
	"	"	"	"	"	"	
1946 Oct. 7	— 0·03	— 0·23	+ 0·06	+ 0·66	— 0·11	+ 1·26	6·748
Oct. 27	+ 0·03	— 0·20	— 0·05	+ 0·61	— 0·36	+ 0·90	6·677
Nov. 16	+ 0·08	— 0·12	— 0·15	+ 0·46	— 0·58	+ 0·32	6·521
Dec. 6	+ 0·11	— 0·01	— 0·23	+ 0·23	— 0·73	— 0·41	6·285
Dec. 26	+ 0·12	+ 0·11	— 0·29	— 0·06	— 0·79	— 1·20	5·986
1947 Jan. 15	+ 0·12	+ 0·23	— 0·31	— 0·37	— 0·76	— 1·96	5·652
Feb. 4	+ 0·10	+ 0·33	— 0·29	— 0·66	— 0·63	— 2·59	5·315
Feb. 24	+ 0·07	+ 0·40	— 0·23	— 0·89	— 0·44	— 3·03	5·012
March 16	+ 0·04	+ 0·44	— 0·15	— 1·04	— 0·19	— 3·22	4·789
Apr. 5	+ 0·01	+ 0·45	— 0·05	— 1·09	+ 0·06	— 3·16	4·672
Apr. 25	— 0·01	+ 0·44	+ 0·05	— 1·04	+ 0·29	— 2·87	4·682
May 15	— 0·02	+ 0·42	+ 0·14	— 0·90	+ 0·49	— 2·38	4·809
June 4	— 0·02	+ 0·40	+ 0·22	— 0·68	+ 0·62	— 1·76	5·028
June 24	— 0·01	+ 0·39	+ 0·26	— 0·42	+ 0·69	— 1·07	5·304
July 14	0·00	+ 0·39	+ 0·28	— 0·14	+ 0·70	— 0·37	5·600
Aug. 3	+ 0·01	+ 0·40	+ 0·27	+ 0·13	+ 0·64	+ 0·27	5·882
Aug. 23	+ 0·02	+ 0·42	+ 0·23	+ 0·36	+ 0·52	+ 0·79	6·124
Sept. 12	+ 0·02	+ 0·44	+ 0·17	+ 0·53	+ 0·31	+ 1·10	6·305
Oct. 2	+ 0·02	+ 0·46	+ 0·08	+ 0·61	+ 0·09	+ 1·19	6·408
Oct. 22	0·00	+ 0·46	— 0·02	+ 0·59	— 0·14	+ 1·05	6·425
Nov. 11	— 0·04	+ 0·42	— 0·12	+ 0·47	— 0·36	+ 0·69	6·353
Dec. 1	— 0·07	+ 0·35	— 0·20	+ 0·27	— 0·53	+ 0·16	6·191
Dec. 21	— 0·10	+ 0·25	— 0·26	+ 0·01	— 0·64	— 0·48	5·948
1948 Jan. 10	— 0·12	+ 0·13	— 0·28	— 0·27	— 0·66	— 1·14	5·639
Jan. 30	— 0·13	0·00	— 0·27	— 0·54	— 0·60	— 1·74	5·291
Feb. 19	— 0·12	— 0·12	— 0·22	— 0·76	— 0·46	— 2·20	4·928
March 10	— 0·09	— 0·21	— 0·15	— 0·91	— 0·28	— 2·48	4·589
March 30	— 0·04	— 0·25	— 0·07	— 0·98	— 0·08	— 2·56	4·312
Apr. 19	+ 0·02	— 0·23	+ 0·02	— 0·96	+ 0·12	— 2·44	4·131
May 9	+ 0·08	— 0·15	+ 0·10	— 0·86	+ 0·30	— 2·14	4·067
May 29	+ 0·14	— 0·01	+ 0·16	— 0·70	+ 0·43	— 1·71	4·123
June 18	+ 0·19	+ 0·18	+ 0·21	— 0·49	+ 0·51	— 1·20	4·280
July 8	+ 0·22	+ 0·40	+ 0·22	— 0·27	+ 0·53	— 0·67	4·502
July 28	+ 0·23	+ 0·63	+ 0·22	— 0·05	+ 0·50	— 0·17	4·754
Aug. 17	+ 0·21	+ 0·84	+ 0·19	+ 0·14	+ 0·41	+ 0·24	5·006
Sept. 6	+ 0·17	+ 1·01	+ 0·14	+ 0·28	+ 0·28	+ 0·52	5·226

	$d\delta\pi$	$'f$	$\lambda d\delta n$	$'f$	$''f$	P	$'f$
	"	"	"	"	"	"	"
1946 Oct. 7	+ 0·96	+ 3·01	− 0·003	+ 0·128	− 0·368	− 2·07	+ 2·83
Oct. 27	+ 0·85	+ 3·86	− 0·012	+ 0·116	− 0·240	− 1·89	+ 0·94
Nov. 16	+ 0·62	+ 4·48	− 0·021	+ 0·095	− 0·124	− 1·47	− 0·53
Dec. 6	+ 0·27	+ 4·75	− 0·028	+ 0·067	− 0·029	− 0·82	− 1·35
Dec. 26	− 0·10	+ 4·65	− 0·032	+ 0·035	+ 0·038	− 0·09	− 1·44
1947 Jan. 15	− 0·52	+ 4·13	− 0·033	+ 0·002	+ 0·073	+ 0·73	− 0·71
Feb. 4	− 0·85	+ 3·28	− 0·030	− 0·028	+ 0·075	+ 1·43	+ 0·72
Feb. 24	− 1·06	+ 2·22	− 0·024	− 0·052	+ 0·047	+ 1·94	+ 2·66
March 16	− 1·14	+ 1·08	− 0·016	− 0·068	− 0·005	+ 2·21	+ 4·87
Apr. 5	− 1·05	+ 0·03	− 0·007	− 0·075	− 0·071	+ 2·22	+ 7·09
Apr. 25	− 0·84	− 0·81	+ 0·003	− 0·072	− 0·146	+ 1·98	+ 9·07
May 15	− 0·53	− 1·34	+ 0·012	− 0·060	− 0·218	+ 1·52	+ 10.59
June 4	− 0·15	− 1·49	+ 0·020	− 0·040	− 0·278	+ 0·91	+ 11·50
June 24	+ 0·28	− 1·21	+ 0·026	− 0·014	− 0·318	+ 0·20	+ 11·70
July 14	+ 0·63	− 0·58	+ 0·031	+ 0·017	− 0·332	− 0·48	+ 11·22
Aug. 3	+ 0·95	+ 0·37	+ 0·032	+ 0·049	− 0·315	− 1·12	+ 10·10
Aug. 23	+ 1·17	+ 1·54	+ 0·031	+ 0·080	− 0·266	− 1·66	+ 8·44
Sept. 12	+ 1·26	+ 2·80	+ 0·027	+ 0·107	− 0·186	− 2·01	+ 6·43
Oct. 2	+ 1·19	+ 3·99	+ 0·021	+ 0·128	− 0·079	− 2·14	+ 4·29
Oct. 22	+ 0·96	+ 4·95	+ 0·012	+ 0·140	+ 0·049	− 1·80	+ 2·49
Nov. 11	+ 0·59	+ 5·54	+ 0·001	+ 0·141	+ 0·189	− 1·63	+ 0·86
Dec. 1	+ 0·11	+ 5·65	− 0·011	+ 0·130	+ 0·330	− 1·02	− 0·16
Dec. 21	− 0·42	+ 5·23	− 0·022	+ 0·108	+ 0·460	− 0·26	− 0·42
1948 Jan. 10	− 0·91	+ 4·32	− 0·031	+ 0·077	+ 0·568	+ 0·55	+ 0·13
Jan. 30	− 1·31	+ 3·01	− 0·037	+ 0·040	+ 0·645	+ 1·31	+ 1·44
Feb. 19	− 1·55	+ 1·46	− 0·039	+ 0·001	+ 0·685	+ 1·90	+ 3·34
March 10	− 1·61	0·15	− 0·037	− 0·036	+ 0·686	+ 2·25	+ 5·59
March 30	− 1·47	− 1·62	− 0·032	− 0·068	+ 0·650	+ 2·34	+ 7·93
Apr. 19	− 1·18	− 2·80	− 0·024	− 0·092	+ 0·582	+ 2·18	+ 10·11
May 9	− 0·75	− 3·55	− 0·013	− 0·105	+ 0·490	+ 1·79	+ 11·90
May 29	− 0·24	− 3·79	− 0·002	− 0·107	+ 0·385	+ 1·22	+ 13·12
June 18	+ 0·30	− 3·49	+ 0·010	− 0·097	+ 0·278	+ 0·54	+ 13·66
July 8	+ 0·82	− 2·67	+ 0·021	− 0·076	+ 0·181	− 0·20	+ 13·46
July 28	+ 1·29	− 1·38	+ 0·031	− 0·045	+ 0·105	− 0·91	+ 12·55
Aug. 17	+ 1·56	+ 0·18	+ 0·038	− 0·007	+ 0·060	− 1·46	+ 11·09
Sept. 6	+ 1·75	+ 1·93	+ 0·042	+ 0·035	+ 0·053	− 1·94	+ 9·15

	$d\delta\Omega$	'f	$d\delta i$	'f	$d\delta\varphi$	'f	Δ
	″	″	″	″	″	″	
1948 Sept. 6	+ 0·17	+ 1·01	+ 0·14	+ 0·28	+ 0·28	+ 0·52	5·226
Sept. 26	+ 0·10	+ 1·11	+ 0·08	+ 0·36	+ 0·12	+ 0·64	5·400
Oct. 16	+ 0·01	+ 1·12	+ 0·01	+ 0·37	− 0·05	+ 0·59	5·507
Nov. 5	− 0·09	+ 1·03	− 0·06	+ 0·31	− 0·21	+ 0·38	5·535
Nov. 25	− 0·19	+ 0·84	− 0·12	+ 0·19	− 0·34	+ 0·04	5·483
Dec. 15	− 0·28	+ 0·56	− 0·16	+ 0·03	− 0·41	− 0·37	5·351
1949 Jan. 4	− 0·33	+ 0·23	− 0·18	− 0·15	− 0·43	− 0·80	5·143
Jan. 24	− 0·33	− 0·10	− 0·17	− 0·32	− 0·38	− 1·18	4·869
Feb. 13	− 0·30	− 0·40	− 0·14	− 0·46	− 0·27	− 1·45	4·548
March 5	− 0·22	− 0·62	− 0·10	− 0·56	− 0·13	− 1·58	4·199
March 25	− 0·12	− 0·74	− 0·05	− 0·61	+ 0·03	− 1·55	3·846
Apr. 14	0·00	− 0·74	0·00	− 0·61	+ 0·18	− 1·37	3·519
May 4	+ 0·11	− 0·63	+ 0·04	− 0·57	+ 0·31	− 1·06	3·247
May 24	+ 0·22	− 0·41	+ 0·07	− 0·50	+ 0·39	− 0·67	3·057
June 13	+ 0·29	− 0·12	+ 0·09	− 0·41	+ 0·43	− 0·24	2·968
July 3	+ 0·34	+ 0·22	+ 0·09	− 0·32	+ 0·40	+ 0·16	2·977
July 23	+ 0·35	+ 0·57	+ 0·09	− 0·23	+ 0·33	+ 0·49	3·073
Aug. 12	+ 0·33	+ 0·90	+ 0·07	− 0·16	+ 0·22	+ 0·71	3·226
Sept. 1	+ 0·26	+ 1·16	+ 0·05	− 0·11	+ 0·07	+ 0·78	3·407
Sept. 21	+ 0·16	+ 1·32	+ 0·03	− 0·08	− 0·09	+ 0·69	3·593
Oct. 11	+ 0·04	+ 1·36	+ 0·01	− 0·07	− 0·25	+ 0·44	3·764
Oct. 31	− 0·09	+ 1·27	− 0·01	− 0·08	− 0·38	+ 0·06	3·901
Nov. 20	− 0·21	+ 1·06	− 0·02	− 0·10	− 0·47	− 0·41	3·996
Dec. 10	− 0·30	+ 0·76	− 0·02	− 0·12	− 0·49	− 0·90	4·042
Dec. 30	− 0·36	+ 0·40	− 0·01	− 0·13	− 0·44	− 1·34	4·038
1950 Jan. 19	− 0·36	+ 0·04	0·00	− 0·13	− 0·32	− 1·66	3·982
Feb. 8	− 0·32	− 0·28	+ 0·01	− 0·12	− 0·15	− 1·81	3·879
Feb. 28	− 0·24	− 0·52	+ 0·02	− 0·10	+ 0·05	− 1·76	3·732
March 20	− 0·14	− 0·66	+ 0·01	− 0·09	+ 0·26	− 1·50	3·550
Apr. 9	− 0·03	− 0·69	0·00	− 0·09	+ 0·44	− 1·06	3·335
Apr. 29	+ 0·07	− 0·62	− 0·01	− 0·10	+ 0·59	− 0·47	3·096
May 19	+ 0·14	− 0·48	− 0·03	− 0·13	+ 0·68	+ 0·21	2·841
June 8	+ 0·19	− 0·29	− 0·05	− 0·18	+ 0·71	+ 0·92	2·577
June 28	+ 0·20	− 0·09	− 0·07	− 0·25	+ 0·67	+ 1·59	2·313
July 18	+ 0·19	+ 0·10	− 0·08	− 0·33	+ 0·58	+ 2·17	2·058
Aug. 7	+ 0·15	+ 0·25	− 0·08	− 0·41	+ 0·45	+ 2·62	1·832

	$d\delta\pi'$	'f	$\lambda d\delta n$	'f	"f	P	'f
	"	"	"	"	"	"	"
1948 Sept. 6	+ 1·75	+ 1·93	+ 0·042	+ 0·035	+ 0·053	— 1·94	+ 9·15
Sept. 26	+ 1·75	+ 3·68	+ 0·043	+ 0·078	+ 0·088	— 2·20	+ 6·95
Oct. 16	+ 1·53	+ 5·21	+ 0·040	+ 0·118	+ 0·166	— 2·20	+ 4·75
Nov. 5	+ 1·12	+ 6·33	+ 0·030	+ 0·148	+ 0·284	— 1·94	+ 2·81
Nov. 25	+ 0·55	+ 6·88	+ 0·018	+ 0·166	+ 0·432	— 1·44	+ 1·37
Dec. 15	— 0·12	+ 6·76	+ 0·003	+ 0·169	+ 0·598	— 0·74	+ 0·63
1949 Jan. 4	— 0·80	+ 5·96	— 0·013	+ 0·156	+ 0·767	+ 0·06	+ 0·69
Jan. 24	— 1·38	+ 4·58	— 0·028	+ 0·128	+ 0·923	+ 0·84	+ 1·53
Feb. 13	— 1·81	+ 2·77	— 0·041	+ 0·087	+ 1·051	+ 1·52	+ 3·05
March 5	— 2·02	+ 0·75	— 0·050	+ 0·037	+ 1·138	+ 2·01	+ 5·06
March 25	— 2·01	— 1·26	— 0·054	— 0·017	+ 1·175	+ 2·26	+ 7·32
Apr. 14	— 1·79	— 3·05	— 0·053	— 0·070	+ 1·158	+ 2·26	+ 9·58
May 4	— 1·39	— 4·44	— 0·047	— 0·117	+ 1·088	+ 2·05	+ 11·63
May 24	— 0·87	— 5·31	— 0·037	— 0·154	+ 0·971	+ 1·65	+ 13·28
June 13	— 0·31	— 5·62	— 0·024	— 0·178	+ 0·817	+ 1·12	+ 14·40
July 3	+ 0·32	— 5·30	— 0·009	— 0·187	+ 0·639	+ 0·50	+ 14·90
July 23	+ 0·89	— 4·41	+ 0·008	— 0·179	+ 0·452	— 0·14	+ 14·76
Aug. 12	+ 1·38	— 3·03	+ 0·024	— 0·155	+ 0·273	— 0·75	+ 14·01
Sept. 1	+ 1·75	— 1·28	+ 0·039	— 0·116	+ 0·118	— 1·28	+ 12·73
Sept. 21	+ 1·95	+ 0·67	+ 0·052	— 0·064	+ 0·002	— 1·67	+ 11·06
Oct. 11	+ 1·95	+ 2·62	+ 0·060	— 0·004	— 0·062	— 1·87	+ 9·19
Oct. 31	+ 1·74	+ 4·36	+ 0·063	+ 0·059	— 0·066	— 1·87	+ 7·32
Nov. 20	+ 1·33	+ 5·69	+ 0·059	+ 0·118	— 0·007	— 1·66	+ 5·66
Dec. 10	+ 0·78	+ 6·47	+ 0·049	+ 0·167	+ 0·111	— 1·26	+ 4·40
Dec. 30	+ 0·16	+ 6·63	+ 0·034	+ 0·201	+ 0·278	— 0·74	+ 3·66
1950 Jan. 19	— 0·45	+ 6·18	+ 0·014	+ 0·215	+ 0·479	— 0·18	+ 3·48
Feb. 8	— 0·98	+ 5·20	— 0·007	+ 0·208	+ 0·694	+ 0·36	+ 3·84
Feb. 28	— 1·35	+ 3·85	— 0·028	+ 0·180	+ 0·902	+ 0·79	+ 4·63
March 20	— 1·52	+ 2·33	— 0·046	+ 0·134	+ 1·082	+ 1·08	+ 5·71
Apr. 9	— 1·51	+ 0·82	— 0·059	+ 0·075	+ 1·216	+ 1·21	+ 6·92
Apr. 29	— 1·32	— 0·50	— 0·068	+ 0·007	+ 1·291	+ 1·18	+ 8·10
May 19	— 0·98	— 1·48	— 0·071	— 0·064	+ 1·298	+ 1·01	+ 9·11
June 8	— 0·57	— 2·05	— 0·068	— 0·132	+ 1·234	+ 0·76	+ 9·87
June 28	— 0·10	— 2·15	— 0·061	— 0·193	+ 1·102	+ 0·47	+ 10·34
July 18	+ 0·37	— 1·78	— 0·049	— 0·242	+ 0·909	+ 0·16	+ 10·50
Aug. 7	+ 0·80	— 0·98	— 0·035	— 0·277	+ 0·667	— 0·14	+ 10·36

	$d\delta\Omega$	$'f$	$d\delta i$	$'f$	$d\delta\varphi$	$'f$	Δ
	"	"	"	"	"	"	
1950 Aug. 7	+0.15	+0.25	−0.08	−0.41	+0.45	+2.62	1.832
Aug. 27	+0.10	+0.35	−0.06	−0.47	+0.28	+2.90	1.654
Sept. 16	+0.05	+0.40	−0.04	−0.51	+0.10	+3.00	1.555
Oct. 6	+0.02	+0.42	−0.02	−0.53	−0.10	+2.90	1.556
Oct. 26	−0.01	+0.41	+0.01	−0.52	−0.30	+2.60	1.661
Nov. 15	−0.03		+0.05		−0.51		1.840

MARS

1942 June 10.0 — 1950 Oct. 6.0

	$d\delta\Omega$	$'f$	$d\delta i$	$'f$	$d\delta\varphi$	$'f$	Δ
	"	"	"	"	"	"	
1942 May 1	−0.013	+0.004	+0.008	−0.004	−0.042	+0.018	3.71
June 10	−0.008	−0.004	+0.008	+0.004	−0.035	−0.017	3.80
July 20	−0.003	−0.007	+0.007	+0.011	−0.028	−0.045	3.90
Aug. 29	0.000	−0.007	+0.005	+0.016	−0.018	−0.063	4.00
Oct. 8	+0.001	−0.006	+0.002	+0.018	−0.008	−0.071	4.10
Nov. 17	−0.001	−0.007	−0.001	+0.017	+0.004	−0.067	4.18
Dec. 27	−0.007	−0.014	−0.005	+0.012	+0.017	−0.050	4.26
1943 Feb. 5	−0.016	−0.030	−0.008	+0.004	+0.031	−0.019	4.31
March 17	−0.026	−0.056	−0.010	−0.006	+0.042	+0.023	4.32
Apr. 26	−0.034	−0.090	−0.010	−0.016	+0.046	+0.069	4.27
June 5	−0.035	−0.125	−0.007	−0.023	+0.038	+0.107	4.16
July 15	−0.025	−0.150	−0.004	−0.027	+0.021	+0.128	4.00
Aug. 24	−0.009	−0.159	−0.001	−0.028	−0.001	+0.127	3.80
Oct. 3	+0.009	−0.150	0.000	−0.028	−0.021	+0.106	3.57
Nov. 12	+0.023	−0.127	0.000	−0.028	−0.034	+0.072	3.34
Dec. 22	+0.032	−0.095	−0.001	−0.029	−0.040	+0.032	3.15
1944 Jan. 31	+0.035	−0.060	−0.002	−0.031	−0.040	−0.008	3.03
March 11	+0.034	−0.026	−0.003	−0.034	−0.036	−0.044	3.00
Apr. 20	+0.032	+0.006	−0.004	−0.038	−0.029	−0.073	3.09
May 30	+0.028	+0.034	−0.004	−0.042	−0.018	−0.091	3.29
July 9	+0.020	+0.054	−0.004	−0.046	−0.006	−0.097	3.61
Aug. 18	+0.011	+0.065	−0.002	−0.048	+0.009	−0.088	4.01
Sept. 27	−0.002	+0.063	0.000	−0.048	+0.025	−0.063	4.45

	$d\delta\pi$	$'f$	$\lambda d\delta n$	$'f$	$''f$	P	$'f$
	″	″	″	″	″	″	″
1950 Aug. 7	+ 0·80	— 0·98	— 0·035	— 0·277	+ 0·667	— 0·14	+ 10·36
Aug. 27	+ 1·15	+ 0·17	— 0·020	— 0·297	+ 0·390	— 0·38	+ 9·98
Sept. 16	+ 1·37	+ 1·54	— 0·005	— 0·302	+ 0·093	— 0·53	+ 9·45
Oct. 6	+ 1·42	+ 2·96	+ 0·011	— 0·291	— 0·209	— 0·57	+ 8·88
Oct. 26	+ 1·24	+ 4·20	+ 0·028	— 0·263	— 0·500	— 0·48	+ 8·40
Nov. 15	+ 0·99		+ 0·046		— 0·763	— 0·34	

MARS

1942 June 10.0 — 1950 Oct. 6.0

	$d\delta\pi$	f'	$\lambda d\delta n$	$'f$	$''f$	P	$'f$
	″	″	″	″	″	″	″
1942 May 1	+ 0·010	+ 0·014	+ 0·0069	— 0·0032	+ 0·0027	— 0·028	— 0·003
June 10	— 0 034	— 0 020	+ 0 0063	+ 0 0031	— 0 0005	+ 0 011	+ 0 008
July 20	— 0 055	— 0 075	+ 0 0055	+ 0 0086	+ 0 0026	+ 0 025	+ 0 033
Aug. 29	— 0 068	— 0 143	+ 0 0043	+ 0 0129	+ 0 0112	+ 0 034	+ 0 067
Oct. 8	— 0 077	— 0 220	+ 0 0030	+ 0 0159	+ 0 0241	+ 0 037	+ 0 104
Nov. 17	— 0 076	— 0 296	+ 0 0013	+ 0 0172	+ 0 0400	+ 0 032	+ 0 136
Dec. 27	— 0 064	— 0 360	— 0 0008	+ 0 0164	+ 0 0572	+ 0 015	+ 0 151
1943 Feb. 5	— 0 034	— 0 394	— 0 0036	+ 0 0128	+ 0 0736	— 0 018	+ 0 133
March 17	+ 0 016	— 0 378	— 0 0068	+ 0 0060	+ 0 0864	— 0 066	+ 0 067
Apr. 26	+ 0 082	— 0 296	— 0 0100	— 0 0040	+ 0 0924	— 0 121	— 0 054
June 5	+ 0 149	— 0 147	— 0 0121	— 0 0161	+ 0 0884	— 0 170	— 0 224
July 15	+ 0 194	+ 0 047	— 0 0122	— 0 0283	+ 0 0723	— 0 189	— 0 413
Aug. 24	+ 0 200	+ 0 247	— 0 0104	— 0 0387	+ 0 0440	— 0 168	— 0 581
Oct. 3	+ 0 171	+ 0 418	— 0 0074	— 0 0461	+ 0 0053	— 0 116	— 0 697
Nov. 12	+ 0 120	+ 0 538	— 0 0040	— 0 0501	— 0 0408	— 0 050	— 0 747
Dec. 22	+ 0 064	+ 0 602	— 0 0009	— 0 0510	— 0 0909	+ 0 016	— 0 731
1944 Jan. 31	+ 0 013	+ 0 615	+ 0 0015	— 0 0495	— 0 1419	+ 0 073	— 0 658
March 11	— 0 031	+ 0 584	+ 0 0032	— 0 0463	— 0 1914	+ 0 116	— 0 542
Apr. 20	— 0 065	+ 0 519	+ 0 0045	— 0 0418	— 0 2377	+ 0 147	— 0 395
May 30	— 0 095	+ 0 424	+ 0 0055	— 0 0363	— 0 2795	+ 0 171	— 0 224
July 9	— 0 120	+ 0 304	+ 0 0064	— 0 0299	— 0 3158	+ 0 185	— 0 039
Aug. 18	— 0 139	+ 0 165	+ 0 0070	— 0 0229	— 0 3457	+ 0 191	+ 0 152
Sept. 27	— 0 150	+ 0 015	+ 0 0074	— 0 0155	— 0 3686	+ 0 184	+ 0 336

	$d\delta\Omega$	$'f$	$d\delta i$	$'f$	$d\delta\varphi$	$'f$	Δ
	"	"	"	"	"	"	
1944 Sept. 27	−0·002	+0·063	0·000	−0·048	+0·025	−0·063	4·45
Nov. 6	−0·017	+0·046	+0·005	−0·043	+0·041	−0·022	4·95
Dec. 16	−0·032	+0·014	+0·010	−0·033	+0·053	+0·031	5·45
1945 Jan. 25	−0·046	−0·032	+0·016	−0·017	+0·060	+0·091	5·90
March 6	−0·052	−0·084	+0·020	+0·003	+0·055	+0·146	6·28
Apr. 15	−0·048	−0·132	+0·021	+0·024	+0·036	+0·182	6·55
May 25	−0·033	−0·165	+0·016	+0·040	+0·007	+0·189	6·70
July 4	−0·013	−0·178	+0·007	+0·047	−0·024	+0·165	6·71
Aug. 13	+0·006	−0·172	−0·004	+0·043	−0·049	+0·116	6·56
Sept. 22	+0·020	−0·152	−0·012	+0·031	−0·062	+0·054	6·28
Nov. 1	+0·027	−0·125	−0·018	+0·013	−0·066	−0·012	5·94
Dec. 11	+0·029	−0·096	−0·021	−0·008	−0·061	−0·073	5·55
1946 Jan. 20	+0·027	−0·069	−0·021	−0·029	−0·052	−0·125	5·12
March 1	+0·023	−0·046	−0·020	−0·049	−0·040	−0·165	4·72
Apr. 10	+0·018	−0·028	−0·017	−0·066	−0·027	−0·192	4·41
May 20	+0·012	−0·016	−0·013	−0·079	−0·011	−0·203	4·20
June 29	+0·006	−0·010	−0·007	−0·086	+0·005	−0·198	4·15
Aug. 8	0·000	−0·010	0·000	−0·086	+0·023	−0·175	4·28
Sept. 17	−0·006	−0·016	+0·008	−0·078	+0·043	−0·132	4·59
Oct. 27	−0·010	−0·026	+0·018	−0·060	+0·061	−0·071	5·03
Dec. 6	−0·013	−0·039	+0·027	−0·033	+0·075	+0·004	5·52
1947 Jan. 15	−0·013	−0·052	+0·033	0·000	+0·078	+0·082	6·03
Feb. 24	−0·010	−0·062	+0·033	+0·033	+0·066	+0·148	6·47
Apr. 5	−0·005	−0·067	+0·025	+0·058	+0·041	+0·189	6·80
May 15	−0·001	−0·068	+0·012	+0·070	+0·008	+0·197	7·00
June 24	0·000	−0·068	−0·002	+0·068	−0·022	+0·175	7·03
Aug. 3	+0·001	−0·067	−0·014	+0·054	−0·043	+0·132	6·89
Sept. 12	−0·003	−0·070	−0·021	+0·033	−0·053	+0·079	6·64
Oct. 22	−0·006	−0·076	−0·024	+0·009	−0·055	+0·024	6·26
Dec. 1	−0·008	−0·084	−0·024	−0·015	−0·051	−0·027	5·85
1948 Jan. 10	−0·009	−0·093	−0·021	−0·036	−0·042	−0·069	5·36
Feb. 19	−0·010	−0·103	−0·018	−0·054	−0·032	−0·101	4·84
March 30	−0·008	−0·111	−0·013	−0·067	−0·019	−0·120	4·36
May 9	−0·006	−0·117	−0·007	−0·074	−0·005	−0·125	3·90
June 18	−0·001	−0·118	−0·001	−0·075	+0·010	−0·115	3·56
July 28	+0·005	−0·113	−0·005	−0·070	+0·024	−0·091	3·36

		$d\delta\pi$	$'f$	$\lambda d\delta n$	$'f$	$''f$	P	$'f$
		"	"	"	"	"	"	"
1944 Sept.	27	−0.150	+0.015	+0.0074	−0.0155	−0.3686	+0.184	+0.336
Nov.	6	−0.148	−0.133	+0.0075	−0.0080	−0.3841	+0.159	+0.495
Dec.	16	−0.128	−0.261	+0.0068	−0.0012	−0.3921	+0.111	+0.606
1945 Jan.	25	−0.088	−0.349	+0.0054	+0.0042	−0.3933	+0.038	+0.644
March	6	−0.026	−0.375	+0.0029	+0.0071	−0.3891	−0.056	+0.588
Apr.	15	+0.044	−0.331	−0.0002	+0.0069	−0.3820	−0.148	+0.440
May	25	+0.104	−0.227	−0.0033	+0.0036	−0.3751	−0.212	+0.228
July	4	+0.133	−0.094	−0.0056	−0.0020	−0.3715	−0.225	+0.003
Aug.	13	+0.130	+0.036	−0.0067	−0.0087	−0.3735	−0.192	−0.189
Sept.	22	+0.103	+0.139	−0.0066	−0.0153	−0.3822	−0.130	−0.319
Nov.	1	+0.062	+0.201	−0.0059	−0.0212	−0.3975	−0.056	−0.375
Dec.	11	+0.025	+0.226	−0.0048	−0.0260	−0.4187	+0.009	−0.366
1946 Jan.	20	−0.011	+0.215	−0.0037	−0.0297	−0.4447	+0.068	−0.298
March	1	−0.040	+0.175	−0.0025	−0.0322	−0.4744	+0.115	−0.183
Apr.	10	−0.065	+0.110	−0.0014	−0.0336	−0.5066	+0.153	−0.030
May	20	−0.083	+0.027	−0.0003	−0.0339	−0.5402	+0.179	+0.149
June	29	−0.091	−0.064	+0.0008	−0.0331	−0.5741	+0.194	+0.343
Aug.	8	−0.091	−0.155	+0.0020	−0.0311	−0.6072	+0.192	+0.535
Sept.	17	−0.078	−0.233	+0.0033	−0.0278	−0.6383	+0.169	+0.704
Oct.	27	−0.052	−0.285	+0.0046	−0.0232	−0.6661	+0.123	+0.827
Dec.	6	−0.012	−0.297	+0.0058	−0.0174	−0.6893	+0.052	+0.879
1947 Jan.	15	+0.038	−0.259	+0.0065	−0.0109	−0.7067	−0.040	+0.839
Feb.	24	+0.086	−0.173	+0.0063	−0.0046	−0.7176	−0.136	+0.703
Apr.	5	+0.115	−0.058	+0.0050	+0.0004	−0.7222	−0.204	+0.499
May	15	+0.114	+0.056	+0.0029	+0.0033	−0.7218	−0.225	+0.274
June	24	+0.086	+0.142	+0.0005	+0.0038	−0.7185	−0.195	+0.079
Aug.	3	+0.043	+0.185	−0.0016	+0.0022	−0.7147	−0.136	−0.057
Sept.	12	−0.003	+0.182	−0.0032	−0.0010	−0.7125	−0.065	−0.122
Oct.	22	−0.043	+0.139	−0.0043	−0.0053	−0.7135	+0.003	−0.119
Dec.	1	−0.076	+0.063	−0.0051	−0.0104	−0.7188	+0.061	−0.058
1948 Jan.	10	−0.100	−0.037	−0.0055	−0.0159	−0.7292	+0.110	+0.052
Feb.	19	−0.117	−0.154	−0.0057	−0.0216	−0.7451	+0.148	+0.200
March	30	−0.126	−0.280	−0.0058	−0.0274	−0.7667	+0.177	+0.377
May	9	−0.127	−0.407	−0.0056	−0.0330	−0.7941	+0.196	+0.573
June	18	−0.119	−0.526	−0.0053	−0.0383	−0.8271	+0.205	+0.778
July	28	−0.098	−0.624	−0.0045	−0.0428	−0.8654	+0.197	+0.975

	$d\delta\Omega$	$'f$	$d\delta i$	$'f$	$d\delta\varphi$	f'	Δ
	″	″	″	″	″	″	
1948 July 28	+0·005	−0·113	+0·005	−0·070	+0·024	−0·091	3·36
Sept. 6	+0 014	−0 099	+0 011	−0 059	+0 038	−0 053	3 33
Oct. 16	+0 023	−0 076	+0 017	−0 042	+0 048	−0 005	3 52
Nov. 25	+0 032	−0 044	+0 020	−0 022	+0 050	+0 045	3 83
1949 Jan. 4	+0 035	−0 009	+0 020	−0 002	+0 044	+0 089	4 19
Feb. 13	+0 026	+0 017	+0 014	+0 012	+0 027	+0 116	4 56
March 25	+0 016	+0 033	+0 007	+0 019	+0 004	+0 120	4 85
May 4	0 000	+0 033	0 000	+0 019	−0 016	+0 104	5 05
June 13	−0 015	+0 018	−0 005	+0 014	−0 028	+0 076	5 14
July 23	−0 025	−0 007	−0 006	+0 008	−0 033	+0 043	5 14
Sept. 1	−0 031	−0 038	−0 005	+0 003	−0 030	+0 013	5 05
Oct. 11	−0 032	−0 070	−0 004	−0 001	−0 024	−0 011	4 90
Nov. 20	−0 029	−0 099	−0 002	−0 003	−0 014	−0 025	4 71
Dec. 30	−0 024	−0 123	−0 001	−0 004	−0 004	−0 029	4 49
1950 Feb. 8	−0 018	−0 141	+0 001	−0 003	+0 009	−0 020	4 26
March 20	−0 010	−0 151	+0 001	−0 002	+0 022	+0 002	4 02
Apr. 29	−0 003	−0 154	+0 001	−0 001	+0 036	+0 038	3 77
June 8	+0 005	−0 149	−0 001	−0 002	+0 050	+0 088	3 52
July 18	+0 010	−0 139	−0 004	−0 006	+0 062	+0 150	3 26
Aug. 27	+0 012	−0 127	−0 008	−0 014	+0 069	+0 219	2 98
Oct. 6	+0 011	−0 116	−0 011	−0 025	+0 068	+0 287	2 67
Nov. 15	+0 006	−0 110	−0 012	−0 037	+0 055	+0 342	2 33
Dec. 25	0 000		−0 009		+0 028		2 00

	$d\delta\pi$	$'f$	$\lambda d\delta n$	$'f$	$''f$	P	$'f$
	″	″	″	″	″	″	″
1948 July 28	−0·098	−0·624	−0·0045	−0·0428	−0·8654	+0·197	+0·975
Sept. 6	−0·059	−0·683	−0·0034	−0·0462	−0·9082	+0·164	+1·139
Oct. 16	0·000	−0·683	−0·0006	−0·0468	−0·9544	+0·100	+1·239
Nov. 25	+0·075	−0·608	+0·0026	−0·0442	−1·0012	+0·010	+1·249
1949 Jan. 4	+0·150	−0·458	+0·0064	−0·0378	−1·0454	−0·093	+1·156
Feb. 13	+0·202	−0·256	+0·0094	−0·0284	−1·0832	−0·178	+0·978
March 25	+0·212	−0·044	+0·0109	−0·0175	−1·1116	−0·222	+0·756
May 4	+0·181	+0·137	+0·0106	−0·0069	−1·1291	−0·217	+0·539
June 13	+0·127	+0·264	+0·0088	+0·0019	−1·1360	−0·178	+0·361
July 23	+0·066	+0·330	+0·0063	+0·0082	−1·1341	−0·121	+0·240
Sept. 1	+0·011	+0·341	+0·0037	+0·0119	−1·1259	−0·065	+0·175
Oct. 11	−0·034	+0·307	+0·0011	+0·0130	−1·1140	−0·016	+0·159
Nov. 20	−0·068	+0·239	−0·0012	+0·0118	−1·1010	+0·023	+0·182
Dec. 30	−0·091	+0·148	−0·0033	+0·0085	−1·0892	+0·052	+0·234
1950 Feb. 8	−0·105	+0·043	−0·0053	+0·0032	−1·0807	+0·071	+0·305
March 20	−0·111	−0·068	−0·0072	−0·0040	−1·0775	+0·083	+0·388
Apr. 29	−0·106	−0·174	−0·0090	−0·0130	−1·0815	+0·084	+0·472
June 8	−0·092	−0·266	−0·0108	−0·0238	−1·0945	+0·078	+0·550
July 18	−0·064	−0·330	−0·0121	−0·0359	−1·1183	+0·060	+0·610
Aug. 27	−0·023	−0·353	−0·0129	−0·0488	−1·1542	+0·031	+0·641
Oct. 6	+0·032	−0·321	−0·0126	−0·0614	−1·2030	−0·005	+0·636
Nov. 15	+0·090	−0·231	−0·0106	−0·0720	−1·2644	−0·042	+0·594
Dec. 25	+0·137		−0·0066		−1·3364	−0·065	

URANUS

1942 June 10.0 — 1950 Oct. 6.0

	$d\delta\Omega$	$'f$	$d\delta i$	$'f$	$d\delta\varphi$	$'f$	Δ
	″	″	″	″	″	″	
1942 May 1	+0·001	0·000	—0·001	0·000	+0·008	—0·005	18·7
June 10	+0 001	+0 001	—0 001	—0 001	+0 011	+0 006	18 2
July 20	+0 001	+0 002	—0 002	—0 003	+0 013	+0 019	17 8
Aug. 29	0 000	+0 002	—0 002	—0 005	+0 014	+0 033	17 5
Oct. 8	—0 001	+0 001	—0 003	—0 008	+0 013	+0 046	17 2
Nov. 17	—0 003	—0 002	—0 003	—0 011	+0 011	+0 057	17 0
Dec. 27	—0 005	—0 007	—0 003	—0 014	+0 009	+0 066	16 8
1943 Feb. 5	—0 006	—0 013	—0 003	—0 017	+0 007	+0 073	16 7
March 17	—0 008	—0 021	—0 003	—0 020	+0 005	+0 078	16 7
Apr. 26	—0 009	—0 030	—0 002	—0 022	+0 003	+0 081	16 7
June 5	—0 010	—0 040	—0 002	—0 024	+0 002	+0 083	16 7
July 15	—0 010	—0 050	—0 001	—0 025	+0 001	+0 084	16 8
Aug. 24	—0 010	—0 060	—0 001	—0 026	+0 001	+0 085	16 9
Oct. 3	—0 010	—0 070	—0 001	—0 027	+0 001	+0 086	17 1
Nov. 12	—0 010	—0 080	0 000	—0 027	+0 001	+0 087	17 2
Dec. 22	—0 009	—0 089	0 000	—0 027	+0 001	+0 088	17 4
1944 Jan. 31	—0 008	—0 097	+0 001	—0 026	+0 001	+0 089	17 6
March 11	—0 008	—0 105	+0 001	—0 025	+0 001	+0 090	17 8
Apr. 20	—0 007	—0 112	+0 001	—0 024	0 000	+0 090	18 0
May 30	—0 005	—0 117	+0 001	—0 023	0 000	+0 090	18 2
July 9	—0 004	—0 121	+0 001	—0 022	—0 001	+0 089	18 4
Aug. 18	—0 003	—0 124	+0 001	—0 021	—0 002	+0 087	18 6
Sept. 27	—0 002	—0 126	+0 001	—0 020	—0 003	+0 084	18 8
Nov. 6	—0 001	—0 127	0 000	—0 020	—0 004	+0 080	19 0
Dec. 16	0 000	—0 127	0 000	—0 020	—0 005	+0 075	19 2
1945 Jan. 25	0 000	—0 127	0 000	—0 020	—0 006	+0 069	19 4
March 6	+0 001	—0 126	—0 001	—0 021	—0 007	+0 062	19 6
Apr. 15	+0 002	—0 124	—0 001	—0 022	—0 008	+0 054	19 8
May 25	+0 003	—0 121	—0 001	—0 023	—0 009	+0 045	20 0
July 4	+0 003	—0 118	—0 002	—0 025	—0 010	+0 035	20 1
Aug. 13	+0 004	—0 114	—0 002	—0 027	—0 011	+0 024	20 3
Sept. 22	+0 004	—0 110	—0 002	—0 029	—0 012	+0 012	20 5
Nov. 1	+0 004	—0 106	—0 003	—0 032	—0 014	—0 002	20 7
Dec. 11	+0 005	—0 101	—0 003	—0 035	—0 015	—0 017	20 8

URANUS

1942 June 10.0 — 1950 Oct. 6.0

	$d\delta\pi$	$'f$	$\lambda d\delta n$	$'f$	$''f$	P	$'f$
	″	″	″	″	″	″	″
1942 May 1	+0·003	—0·001	—0·0015	+0·0011	—0·0009	+0·001	0·000
June 10	+0 002	+0 001	—0 0022	—0 0011	+0 0002	0 000	0 000
July 20	+0 001	+0 002	—0 0026	—0 0037	—0 0009	—0 002	—0 002
Aug. 29	0 000	+0 002	—0 0026	—0 0063	—0 0046	—0 005	—0 007
Oct. 8	—0 002	0 000	—0 0024	—0 0087	—0 0109	—0 007	—0 014
Nov. 17	—0 006	—0 006	—0 0018	—0 0105	—0 0196	—0 006	—0 020
Dec. 27	—0 013	—0 019	—0 0011	—0 0116	—0 0301	—0 003	—0 023
1943 Feb. 5	—0 020	—0 039	—0 0003	—0 0119	—0 0417	+0 003	—0 020
March 17	—0 028	—0 067	+0 0005	—0 0114	—0 0536	+0 010	—0 010
Apr. 26	—0 034	—0 101	+0 0011	—0 0103	—0 0650	+0 017	+0 007
June 5	—0 040	—0 141	+0 0017	—0 0086	—0 0753	+0 025	+0 032
July 15	—0 045	—0 186	+0 0021	—0 0065	—0 0839	+0 032	+0 064
Aug. 24	—0 048	—0 234	+0 0023	—0 0042	—0 0904	+0 037	+0 101
Oct. 3	—0 048	—0 282	+0 0025	—0 0017	—0 0946	+0 042	+0 143
Nov. 12	—0 047	—0 329	+0 0025	+0 0008	—0 0963	+0 045	+0 188
Dec. 22	—0 046	—0 375	+0 0025	+0 0033	—0 0955	+0 047	+0 235
1944 Jan. 31	—0 045	—0 420	+0 0024	+0 0057	—0 0922	+0 048	+0 283
March 11	—0 042	—0 462	+0 0022	+0 0079	—0 0865	+0 049	+0 332
Apr. 20	—0 039	—0 501	+0 0020	+0 0099	—0 0786	+0 047	+0 379
May 30	—0 035	—0 536	+0 0018	+0 0117	—0 0687	+0 047	+0 426
July 9	—0 031	—0 567	+0 0016	+0 0133	—0 0570	+0 045	+0 471
Aug. 18	—0 027	—0 594	+0 0013	+0 0146	—0 0437	+0 044	+0 515
Sept. 27	—0 024	—0 618	+0 0011	+0 0157	—0 0291	+0 042	+0 557
Nov. 6	—0 021	—0 639	+0 0009	+0 0166	—0 0134	+0 040	+0 597
Dec. 16	—0 018	—0 657	+0 0007	+0 0173	+0 0032	+0 038	+0 635
1945 Jan. 25	—0 015	—0 672	+0 0004	+0 0177	+0 0205	+0 036	+0 671
March 6	—0 012	—0 684	+0 0002	+0 0179	+0 0382	+0 034	+0 705
Apr. 15	—0 010	—0 694	0 0000	+0 0179	+0 0561	+0 032	+0 737
May 25	—0 009	—0 703	—0 0002	+0 0177	+0 0740	+0 031	+0 768
July 4	—0 007	—0 710	—0 0003	+0 0174	+0 0917	+0 029	+0 797
Aug. 13	—0 006	—0 716	—0 0005	+0 0169	+0 1091	+0 027	+0 824
Sept. 22	—0 005	—0 721	—0 0007	+0 0162	+0 1260	+0 026	+0 850
Nov. 1	—0 005	—0 726	—0 0008	+0 0154	+0 1422	+0 025	+0 875
Dec. 11	—0 004	—0 730	—0 0009	+0 0145	+0 1576	+0 023	+0 898

	$d\delta\Omega$	$'f$	$d\delta i$	$'f$	$d\delta\varphi$	$'f$	Δ
	"	"	"	"	"	"	
1945 Dec. 11	+0·005	—0·101	—0·003	—0·035	—0·015	—0·017	20·8
1946 Jan. 20	+0·005	—0·096	—0·004	—0·039	—0·016	—0·033	21·0
March 1	+0·005	—0·091	—0·004	—0·043	—0·016	—0·049	21·2
Apr. 10	+0·005	—0·086	—0·005	—0·048	—0·017	—0·066	21·3
May 20	+0·005	—0·081	—0·005	—0·053	—0·018	—0·084	21·4
June 29	+0·005	—0·076	—0·005	—0·058	—0·018	—0·102	21·5
Aug. 8	+0·004	—0·072	—0·006	—0·064	—0·019	—0·121	21·7
Sept. 17	+0·004	—0·068	—0·006	—0·070	—0·019	—0·140	21·8
Oct. 27	+0·004	—0·064	—0·006	—0·076	—0·019	—0·159	21·9
Dec. 6	+0·003	—0·061	—0·007	—0·083	—0·019	—0·178	22·0
1947 Jan. 15	+0·003	—0·058	—0·007	—0·090	—0·020	—0·198	22·1
Feb. 24	+0·002	—0·056	—0·007	—0·097	—0·019	—0·217	22·2
Apr. 5	+0·002	—0·054	—0·007	—0·104	—0·019	—0·236	22·3
May 15	+0·001	—0·053	—0·008	—0·112	—0·019	—0·255	22·4
June 24	0·000	—0·053	—0·008	—0·120	—0·019	—0·274	22·5
Aug. 3	0·000	—0·053	—0·008	—0·128	—0·019	—0·293	22·6
Sept. 12	—0·001	—0·054	—0·008	—0·136	—0·018	—0·311	22·6
Oct. 22	—0·002	—0·056	—0·008	—0·144	—0·018	—0·329	22·7
Dec. 1	—0·003	—0·059	—0·008	—0·152	—0·017	—0·346	22·8
1948 Jan. 10	—0·003	—0·062	—0·008	—0·160	—0·017	—0·363	22·8
Feb. 19	—0·004	—0·066	—0·008	—0·168	—0·016	—0·379	22·8
March 30	—0·005	—0·071	—0·008	—0·176	—0·015	—0·394	22·9
May 9	—0·006	—0·077	—0·008	—0·184	—0·015	—0·409	22·9
June 18	—0·007	—0·084	—0·007	—0·191	—0·014	—0·423	23·0
July 28	—0·008	—0·092	—0·007	—0·198	—0·013	—0·436	23·0
Sept. 6	—0·008	—0·100	—0·007	—0·205	—0·012	—0·448	23·0
Oct. 16	—0·009	—0·109	—0·006	—0·211	—0·012	—0·460	22·9
Nov. 25	—0·009	—0·118	—0·006	—0·217	—0·011	—0·471	22·9
1949 Jan. 4	—0·010	—0·128	—0·006	—0·223	—0·010	—0·481	22·8
Feb. 13	—0·010	—0·138	—0·005	—0·228	—0·010	—0·491	22·8
March 25	—0·011	—0·149	—0·005	—0·233	—0·009	—0·500	22·7
May 4	—0·011	—0·160	—0·004	—0·237	—0·009	—0·509	22·7
June 13	—0·011	—0·171	—0·003	—0·240	—0·008	—0·517	22·6
July 23	—0·011	—0·182	—0·003	—0·243	—0·008	—0·525	22·4
Sept. 1	—0·011	—0·193	—0·002	—0·245	—0·008	—0·533	22·3
Oct. 11	—0·010	—0·203	—0·001	—0·246	—0·008	—0·541	22·1

	dδπ	'f	λdδn	'f	''f	P	'f
	″	″	″	″	″	″	″
1945 Dec. 11	— 0·004	— 0·730	— 0·0009	+ 0·0145	+ 0·1576	+ 0·023	+ 0·898
1946 Jan. 20	— 0 004	— 0 734	— 0 0011	+ 0 0134	+ 0 1721	+ 0 022	+ 0 920
March 1	— 0 004	— 0 738	— 0 0012	+ 0 0122	+ 0 1855	+ 0 021	+ 0 941
Apr. 10	— 0 005	— 0 743	— 0 0013	+ 0 0109	+ 0 1977	+ 0 020	+ 0 961
May 20	— 0 006	— 0 749	— 0 0014	+ 0 0095	+ 0 2086	+ 0 019	+ 0 980
June 29	— 0 006	— 0 755	— 0 0014	+ 0 0081	+ 0 2181	+ 0 018	+ 0 998
Aug. 8	— 0 007	— 0 762	— 0 0015	+ 0 0066	+ 0 2262	+ 0 018	+ 1 016
Sept. 17	— 0 008	— 0 770	— 0 0016	+ 0 0050	+ 0 2328	+ 0 017	+ 1 033
Oct. 27	— 0 010	— 0 780	— 0 0016	+ 0 0034	+ 0 2378	+ 0 017	+ 1 050
Dec. 6	— 0 011	— 0 791	— 0 0016	+ 0 0018	+ 0 2412	+ 0 016	+ 1 066
1947 Jan. 15	— 0 012	— 0 803	— 0 0017	+ 0 0001	+ 0 2430	+ 0 016	+ 1 082
Feb. 24	— 0 013	— 0 816	— 0 0017	— 0 0016	+ 0 2431	+ 0 015	+ 1 097
Apr. 5	— 0 014	— 0 830	— 0 0017	— 0 0033	+ 0 2415	+ 0 015	+ 1 112
May 15	— 0 016	— 0 846	— 0 0017	— 0 0050	+ 0 2382	+ 0 014	+ 1 126
June 24	— 0 017	— 0 863	— 0 0017	— 0 0067	+ 0 2332	+ 0 014	+ 1 140
Aug. 3	— 0 018	— 0 881	— 0 0017	— 0 0084	+ 0 2265	+ 0 013	+ 1 153
Sept. 12	— 0 019	— 0 900	— 0 0016	— 0 0100	+ 0 2181	+ 0 012	+ 1 165
Oct. 22	— 0 020	— 0 920	— 0 0016	— 0 0116	+ 0 2081	+ 0 011	+ 1 176
Dec. 1	— 0 020	— 0 940	— 0 0015	— 0 0131	+ 0 1965	+ 0 011	+ 1 187
1948 Jan. 10	— 0 021	— 0 961	— 0 0015	— 0 0146	+ 0 1834	+ 0 010	+ 1 197
Feb. 19	— 0 021	— 0 982	— 0 0014	— 0 0160	+ 0 1688	+ 0 009	+ 1 206
March 30	— 0 021	— 1 003	— 0 0013	— 0 0173	+ 0 1528	+ 0 008	+ 1 214
May 9	— 0 021	— 1 024	— 0 0012	— 0 0185	+ 0 1355	+ 0 006	+ 1 220
June 18	— 0 021	— 1 045	— 0 0011	— 0 0196	+ 0 1170	+ 0 005	+ 1 225
July 28	— 0 020	— 1 065	— 0 0010	— 0 0206	+ 0 0974	+ 0 003	+ 1 228
Sept. 6	— 0 019	— 1 084	— 0 0009	— 0 0215	+ 0 0768	+ 0 001	+ 1 229
Oct. 16	— 0 018	— 1 102	— 0 0007	— 0 0222	+ 0 0553	0 000	+ 1 229
Nov. 25	— 0 017	— 1 119	— 0 0006	— 0 0228	+ 0 0331	— 0 002	+ 1 227
1949 Jan. 4	— 0 015	— 1 134	— 0 0004	— 0 0232	+ 0 0103	— 0 004	+ 1 223
Feb. 13	— 0 013	— 1 147	— 0 0002	— 0 0234	— 0 0129	— 0 006	+ 1 217
March 25	— 0 010	— 1 157	0 0000	— 0 0234	— 0 0363	— 0 008	+ 1 209
May 4	— 0 007	— 1 164	+ 0 0002	— 0 0232	— 0 0597	— 0 011	+ 1 198
June 13	— 0 004	— 1 168	+ 0 0005	— 0 0227	— 0 0829	— 0 013	+ 1 185
July 23	— 0 001	— 1 169	+ 0 0007	— 0 0220	— 0 1056	— 0 015	+ 1 170
Sept. 1	+ 0 002	— 1 167	+ 0 0010	— 0 0210	— 0 1276	— 0 017	+ 1 153
Oct. 11	+ 0 005	— 1 162	+ 0 0012	— 0 0198	— 0 1486	— 0 018	+ 1 135

		$d\delta\Omega$	$'f$	$d\delta i$	$'f$	$d\delta\varphi$	$'f$	Δ
		″	″	″	″	″	″	
1949 Oct.	11	— 0·010	— 0·203	— 0·001	— 0·246	— 0·008	— 0·541	22·1
Nov.	20	— 0 010	— 0 213	— 0 001	— 0 247	— 0 008	— 0 549	21 9
Dec.	30	— 0 009	— 0 222	0 000	— 0 247	— 0 009	— 0 558	21 7
1950 Feb.	8	— 0 008	— 0 230	0 000	— 0 247	— 0 009	— 0 567	21 4
March	20	— 0 007	— 0 237	+ 0 001	— 0 246	— 0 009	— 0 576	21 1
Apr.	29	— 0 006	— 0 243	+ 0 001	— 0 245	— 0 010	— 0 586	20 8
June	8	— 0 004	— 0 247	+ 0 001	— 0 244	— 0 009	— 0 595	20 5
July	18	— 0 003	— 0 250	+ 0 001	— 0 243	— 0 008	— 0 603	20 1
Aug.	27	— 0 001	— 0 251	+ 0 001	— 0 242	— 0 006	— 0 609	19 7
Oct.	6	0 000	— 0 251	+ 0 001	— 0 241	— 0 003	— 0 612	19 2
Nov.	15	0 000	— 0 251	0 000	— 0 241	0 000	— 0 612	18 8
Dec.	25	0 000		— 0 001		+ 0 004		18 4

	$d\delta\pi$	'f	$\lambda d\delta n$	'f	''f	P	'f
1949 Oct. 11	+0.005″	−1.162″	+0.0012″	−0.0198″	−0.1486″	−0.018″	+1.135″
Nov. 20	+0.008	−1.154	+0.0014	−0.0184	−0.1684	−0.019	+1.116
Dec. 30	+0.011	−1.143	+0.0017	−0.0167	−0.1868	−0.020	+1.096
1950 Feb. 8	+0.012	−1.131	+0.0019	−0.0148	−0.2035	−0.019	+1.077
March 20	+0.014	−1.117	+0.0020	−0.0128	−0.2183	−0.018	+1.059
Apr. 29	+0.014	−1.103	+0.0020	−0.0108	−0.2311	−0.016	+1.043
June 8	+0.013	−1.090	+0.0019	−0.0089	−0.2419	−0.013	+1.030
July 18	+0.012	−1.078	+0.0016	−0.0073	−0.2508	−0.010	+1.020
Aug. 27	+0.011	−1.067	+0.0012	−0.0061	−0.2581	−0.006	+1.014
Oct. 6	+0.010	−1.057	+0.0006	−0.0055	−0.2642	−0.004	+1.010
Nov. 15	+0.011	−1.046	−0.0001	−0.0056	−0.2697	−0.004	+1.006
Dec. 25	+0.013		−0.0009		−0.2753	−0.006	

Wacław Sierpiński

O pewnych wnioskach z twierdzenia p. Kondô dotyczącego uniformizacji dopełnień analitycznych

Sur quelques conséquences du théorème de M. Kondô concernant l'uniformisation des complémentaires analytiques

Note présentée à la séance du 7 Mai 1951.

M. Motokiti Kondô a démontré [1]) que tout complémentaire analytique plan peut être uniformisé au moyen d'un complémentaire analytique [2]) et il en a déduit que tout ensemble $PC(A)$ plan peut être uniformisé au moyen d'un ensemble $PC(A)$ [3]). Le but de cette Note est de tirer quelques conséquences, d'ailleurs faciles, de ce théorème remarquable de M. Kondô.

1. Théorèmes de réduction généralisés pour les ensembles $C(A)$ et pour les ensembles $PC(A)$.

On dit, d'après M. Kuratowski [4]) qu'une famille F d'ensembles satisfait au *théorème de réduction généralisé* si, étant donnée une suite infinie d'ensembles $U^1, U^2, \ldots$ appartenant à la famille F, il existe dans cette famille une suite d'ensembles disjoints $V^1, V^2, \ldots$, telle que

(1) $$V^n \subset U^n \quad \text{pour} \quad n=1,2,\ldots$$

et

(2) $$\sum_{n=1}^{\infty} V^n = \sum_{n=1}^{\infty} U^n.$$

Théorème. *La famille d'ensembles linéaires $C(A)$ ainsi que la famille d'ensembles linéaires $PC(A)$ satisfont au théorème de réduction généralisé* [5]).

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[1]) Sur l'uniformisation des complémentaires analytiques et les ensembles projectifs de la seconde classe. *Jap. J. Math.* **15** (1939), p. 197—230.

[2]) l. c., p. 198.

[3]) l. c., p. 229 (Théorème 6).

[4]) *Fund. Math.* **26**, p. 184.

[5]) Ce théorème a été démontré par une autre voie par M. Kuratowski, l. c. p. 186 et 187,

Démonstration. Soit $U^1, U^2, \ldots$ une suite infinie d'ensembles $C(A)$ (resp. $PC(A)$) linéaires.

Posons, pour n naturels

$$(3) \qquad Q^n = \underset{x,\,y}{E}\,[x \in U^n,\ y = n].$$

Les ensembles (3) sont évidemment des ensembles $C(A)$ (resp. $PC(A)$) plans, ainsi que leur somme

$$(4) \qquad Q = Q^1 + Q^2 + \ldots.$$

D'après le théorème de M. Kondô, l'ensemble Q peut être uniformisé au moyen d'un ensemble $C(A)$ (resp. $PC(A)$), soit H. Posons

$$(5) \qquad V^n = P(HQ^n) \quad \text{pour} \quad n = 1, 2, \ldots$$

(où PE désigne la projection de l'ensemble plan E sur l'axe OX).

Le produit de deux ensembles $C(A)$ (resp. $PC(A)$) étant un ensemble $C(A)$ (resp. $PC(A)$), les ensembles HQ^n sont des $C(A)$ (resp. $PC(A)$) situés, d'après (3), sur la droite $y = n$, et il en résulte tout de suite que les ensembles (5) sont des $C(A)$ (resp. $PC(A)$).

D'après (3) on a $PQ^n = U^n$ pour $n = 1, 2, \ldots$, donc, vu que $P(HQ^n) \subset PQ^n$, la formule (5) donne la formule (1).

Je dis que les ensembles (5) sont disjoints. En effet, admettons que ce n'est pas le cas. Il existe donc deux nombres naturels distincts, k et l, et un élément x_0, tels que $x_0 \in V^k V^l$. D'après (5) il existe donc des nombres réels y_1 et y_2, tels que

$$(6) \qquad (x_0, y_1) \in HQ^k \quad \text{et} \quad (x_0, y_2) \in HQ^l$$

et, d'après (3), on trouve $y_1 = k$, $y_2 = l$, donc $y_1 \neq y_2$.

D'après (6), (x_0, y_1) et (x_0, y_2) seraient donc deux points différents de l'ensemble H situés sur la droite $x = x_0$, ce qui est impossible, H étant un uniformisateur de Q.

Je dis maintenant que

$$(7) \qquad \sum_{n=1}^{\infty} U^n \subset \sum_{n=1}^{\infty} V^n.$$

Soit, en effet, $x_0 \epsilon \sum_{n=1}^{\infty} U^n$: il existe donc un nombre naturel k, tel que $x_0 \epsilon U^k$. D'après (3) on a donc $(x_0, k) \epsilon Q^k$, donc, d'après (4), $(x_0, k) \epsilon Q$, et, l'ensemble Q étant uniformisé au moyen de l'ensemble H, il existe un nombre réel y_0, tel que $(x_0, y_0) \epsilon H$. D'après $H \subset Q$ et (4) il existe donc un nombre naturel m, tel que $(x_0, y_0) \epsilon Q^m$. On a donc $(x_0, y_0) \epsilon HQ^m$, d'où, d'aprés (5), $x_0 \epsilon V^m$. La formule (7) est ainsi établie. Or, les formules (1) et (7) donnent l'égalité (2).

Notre théorème est ainsi démontré.

2. Soit E un ensemble plan, tel que toute droite parallèle à l'axe d'ordonnées a précisément deux points communs avec E. Un tel ensemble se décompose en deux ensembles, $\varphi(E)$ et $\psi(E)$, définis comme il suit: pour toute droite $x = a$ qui rencontre E en deux points (a, y_1) et (a, y_2), où $y_1 < y_2$, le point (a, y_1) appartient à $\varphi(E)$ et le point (a, y_2) appartient a $\psi(E)$. En 1930 j'ai posé le problème suivant [1]). *Si E est un ensemble $CPC(A)$, les ensembles $\varphi(E)$ et $\psi(E)$ sont-ils nécessairement aussi des $CPC(A)$?*

Le théorème de M. Kondô permet de démontrer que *la réponse à ce problème est négative.*

Soit, en effet, H un ensemble $PC(A)$ situé sur l'axe OX qui n'est pas un $CPC(A)$. L'ensemble H étant un $PC(A)$, il résulte du théorème de M. Kondô qu'il existe un ensemble $C(A)$ plan, soit E_1, tel que H est une projection biunivoque de l'ensemble E_1 sur l'axe OX et nous pouvons évidemment supposer que l'ensemble E_1 est situé au dessus de la droite $y = 0$. Soit E_2 l'ensemble de tous les points de la droite $y = 0$ et soit E_3 l'ensemble de tous les points (x, y) du plan, tels que $x \text{ non} \epsilon H$ et $y = -1$, et posons $E = E_1 + E_2 + E_3$: ce sera évidemment un ensemble $CPC(A)$ plan.

Comme on voit sans peine, $\varphi(E) = H + E_3$ et l'intersection de $\varphi(E)$ par la droite $y = 0$ est l'ensemble H qui n'est pas un $CPC(A)$, d'où il résulte que $\varphi(E)$ n'est pas un ensemble $CPC(A)$.

3. Comme l'a démontré N. Lusin [2]), si E est un ensemble (A) plan, tel que toute droite parallèle à l'axe OY

[1]) *Mathematica* **4**, p. 181.

[2]) *C. R. Paris* t. **180**, p. 81.

rencontre E au plus dans un point, il existe un ensemble plan (A), $Q \subset E$, tel que toute droite parallèle à l'axe OY rencontre Q précisément dans un point [1]. Or, le théorème de M. Kondô permet de démontrer qu'on n'y peut pas remplacer les ensembles (A) par les ensembles $C(A)$. De plus, il permet de démontrer qu'*il existe un ensemble $C(A)$ plan, E, tel que toute droite parallèle à l'axe OY le rencontre au plus dans un point et qu'il n'existe aucun ensemble plan $PC(A)$, $Q \supset E$, tel que toute droite parallèle à l'axe OY le rencontre en un et un seul point* [2].

Soit, en effet, H un ensemble $PC(A)$ situé sur l'axe OX qui n'est pas un $CPC(A)$. D'après le théorème de M. Kondô, H est une projection biunivoque d'un ensemble plan $C(A)$, soit E. Admettons maintenant qu'il existe un ensemble $PC(A)$ plan, $Q \supset E$, tel que toute droite parallèle à l'axe OY a un et un seul point commun avec Q. L'ensemble E étant un $C(A)$, l'ensemble $Q-E$ est un $PC(A)$, ainsi que sa projection $P(Q-E)$. Or, on voit sans peine que $CH = P(Q-E)$: l'ensemble CH est donc un $PC(A)$, contrairement à l'hypothèse sur l'ensemble H. Notre assertion est ainsi démontrée.

4. Il résulte du théorème de M. Kondô que *le problème, si tout ensemble $PC(A)$ linéaire indénombrable contient un sous-ensemble parfait, équivaut au problème analogue concernant les ensembles $C(A)$.*

En effet, admettons que tout ensemble $C(A)$ linéaire indénombrable contient un sous-ensemble parfait et soit H un ensemble $PC(A)$ linéaire indénombrable. D'après le théorème de M. Kondô H est une projection biunivoque sur l'axe OX d'un ensemble plan $C(A)$, soit E, et, comme on le voit sans peine, on peut supposer que les ordonnées des points de E sont toutes irrationnelles. L'ensemble H étant indénombrable, l'ensemble E est évidemment aussi indénombrable ainsi que l'ensemble E_1 de tous les points de E dont les coordonnées sont irrationnelles. Or, l'ensemble de tous les points du plan

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[1]) Un tel ensemble Q est nécessairement mesurable B. Voir W. Sierpiński, *Fund. Math.* 2, p. 78 et 79 et N. Lusin, *Mathematica* 10, p. 71.

[2]) Cela résout un des problèmes posés par N. Lusin dans son livre *Leçons sur les ensembles analytiques et leurs applications* Paris 1930, p. 274.

aux coordonnées irrationnelles est, comme on sait, homéomorphie à l'ensemble de tous les nombres irrationnels et, dans cette homéomorphie l'ensemble E_1, en tant que $C(A)$ indénombrable, est transformé en un ensemble E_2 linéaire qui est encore un $C(A)$ indénombrable, donc, d'après notre hypothèse, contenant un sous-ensemble parfait E_3 que nous pouvons évidemment supposer borné. Or, l'ensemble E_1 est une image homéomorphe de E_2, soit $E_1 = \psi(E_2)$.

L'ensemble $P\psi(E_3)$ est donc une image continue et biunivoque de l'ensemble parfait et borné E_3, donc un ensemble parfait. Or, on a évidemment $P\psi(E_3) \subset P\psi(E_2) = PE_1 \subset PE = H$: l'ensemble H contient donc un sous-ensemble parfait et notre assertion est démontrée.

5. Une autre conséquence facile du théorème de M. Kondô est que *le problème d'existence d'un ensemble $C(A)$ linéaire de puissance $\aleph_1$ équivaut au problème analogue concernant les ensembles $PC(A)$.*

Il suffit évidemment de démontrer que s'il existe un ensemble $PC(A)$ linéaire de puissance $\aleph_1$, il existe aussi un ensemble $C(A)$ linéaire de puissance $\aleph_1$.

Soit donc H un ensemble linéaire $PC(A)$ de puissance $\aleph_1$: d'après le théorème de M. Kondô H est une projection biunivoque d'un ensemble $C(A)$ plan, dont les points ont des ordonnées irrationnelles, soit E, qui est évidemment également de puissance $\aleph_1$. L'ensemble E_1 de tous les points de E aux coordonnées irrationnelles est donc aussi un ensemble $C(A)$ de puissance $\aleph_1$. Or, comme dans le n^0 4, nous trouvons que l'ensemble E_1 est homéomorphe à un ensemble $C(A)$ linèaire. Ce dernier est donc de puissance $\aleph_1$ et notre assertion est démontrée.

6. N. Lusin a posé le problème si toute fonction $f(x)$ d'une variable réelle, dont l'image géométrique $I(f) = \underset{x,y}{E}\,[y = f(x)]$ est un complémentaire analytique (plan), est mesurable. Le théorème de M. Kondô permet de démontrer que ce problème est équivalent au problème si tout ensemble linéaire B_2, c. à d. qui est à la fois $PC(A)$ et $CPC(A)$, est mesurable L.

Supposons, en effet, que tout ensemble B_2 linéaire est mesurable L, et soit $f(x)$ une fonction d'une variable réelle pour laquelle l'ensemble $I(f)$ est un $C(A)$. Les ensembles

$\underset{x}{E}\,[f(x) \geqslant a]$ sont, comme on voit sans peine, pour tout nombre réel a donné, des ensembles B_2, puisqu'on a évidemment

$$\underset{x}{E}\,[f(x) \geqslant a] = P\left\{I(f) \cdot \underset{x,\,y}{E}\,[y \geqslant a]\right\} \tag{8}$$

et

$$C\underset{x}{E}\,[f(x) \geqslant a] = \underset{x}{E}\,[f(x) < a] = P\left\{I(f) \cdot \underset{x,\,y}{E}\,[y < a]\right\}, \tag{9}$$

et les ensembles à droite sont des projections des complémentaires analytiques. Donc, les ensembles linéaires B_2 étant, d'après l'hypothèse, mesurables L, les ensembles $\underset{x}{E}\,[f(x) \geqslant a]$ le sont également (pour tout a réel donné), ce qui prouve que la fonction $f(x)$ est mesurable.

Supposons, d'autre part, que toute fonction $f(x)$, dont l'image $I(f)$ est un ensemble $C(A)$, est mesurable, et soit H un ensemble linéaire B_2 que nous supposerons situé sur l'axe OX.

Les ensembles H et CH (le complémentaire de H par rapport à l'axe OX) sont donc des $PC(A)$ et il existe des ensembles $C(A)$ plans, Q_1 et Q_2, tels que $PQ_1 = H$ et $PQ_2 = CH$ et nous pouvons évidemment supposer que l'ensemble Q_1 est situé au dessus et l'ensemble Q_2 au dessous de l'axe OX. Or, d'après le théorème de M. Kondô les ensembles Q_1 et Q_2 sont uniformisables au moyen des complémentaires analytiques E_1 et E_2.

L'ensemble $E = E_1 + E_2$ est, comme on voit sans peine, coupé par toute droite parallèle à l'axe OY dans un point et un seul: il est donc une image d'une fonction d'une variable réelle, soit $f(x)$. Comme $I(f) = E$ et E est un $C(A)$, d'après notre hypothèse, $f(x)$ est une fonction mesurable. Or, on a évidemment $H = \underset{x}{E}\,[f(x) > 0]$: l'ensemble H est donc mesurable L.

L'équivalence en question est ainsi démontrée.

En s'appuyant sur cette équivalence, je démontrerai que *le problème de N. Lusin équivaut au problème si toute fonction $f(x)$ d'une variable réelle dont l'image géométrique est un ensemble $PC(A)$ est mesurable.*

Supposons, en effet, que la réponse au problème de M. Lusin est positive et soit $f(x)$ une fonction d'une variable réelle, telle que l'ensemble $I(f)$ est un $PC(A)$. Soit a un nombre réel donné. Il résulte tout de suite des formules (8) et (9) que l'ensemble $\underset{x}{E}\,[f(x) \geqslant a]$ est un B_2.

Or, d'après notre hypothèse et d'après l'équivalence démontrée, il est (en tant que B_2) mesurable L. La fonction $f(x)$ est donc mesurable et notre proposition se trouve, comme on voit sans peine, démontrée.

Wacław Sierpiński

O homeomorfizmie klasy 1,1 między odcinkiem prostej a kwadratem

Sur une homéomorphie de classe 1,1 entre un segment de droite et un carré

Note présentée à la séance du 7 Mai 1951.

Une transformation biunivoque $y=f(x)$ entre deux ensembles de points X et Y est dite *homéomorphie de classe* α, β, lorsque la fonction $f(x)$ est de classe α dans X et la fonction inverse $f^{-1}(y)$ est de classe β dans Y. M. Kuratowski a démontré [1]) que X et Y étant deux espaces complets séparables de la même puissance, il existe entre eux une homéomorphie de classe 1,1. Donc, en particulier, *il existe une homéomorphie de classe* 1,1 *entre le segment* $S=[-1 \leqslant t \leqslant 1]$ *et le carré* $Q=[0 \leqslant x \leqslant 1,\ 0 \leqslant y \leqslant 1]$.

En 1927 j'ai démontré qu'une transformation biunivoque entre les points du segment S et du carré Q peut être obtenue à l'aide de deux fonctions partout continues du côté gauche dans S [2]). Le but de cette Note est de démontrer que cette transformation est une homéomorphie de classe 1,1.

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[1]) *Fund. Math.* 22 (34), p. 212.

[2]) *Revista Matem. Hispano-Americana* 1927, p. 193—197. La connaissance de ce travail n'est pas nécessaire pour comprendre cette Note.

t étant un nombre donné de l'ensemble $T=[0<t\leqslant 1]$, soit

(1) $$t=2^{-\nu_1(t)}+2^{-\nu_1(t)-\nu_2(t)}+2^{-\nu_1(t)-\nu_2(t)-\nu_3(t)}+\ldots,$$

son développement en fraction dyadique infinie.

Les nombres naturels

(2) $$\nu_k(t)\quad (k=1,2,\ldots)$$

sont, comme on sait, bien déterminés par le nombre t (pour tout $t\in T$).

Posons maintenant

(3) $$\begin{cases} \varphi(t)=0,\ \psi(t)=-2t-1, & \text{pour } -1\leqslant t\leqslant -\tfrac{1}{2}, \\ \varphi(t)=2t+1,\ \psi(t)=0, & \text{pour } -\tfrac{1}{2}<t\leqslant 0, \end{cases}$$

et

(4) $$\begin{cases} \varphi(t)=2^{-\nu_1(t)}+2^{-\nu_1(t)-\nu_3(t)}+2^{-\nu_1(t)-\nu_3(t)-\nu_5(t)}+\ldots, \\ \psi(t)=2^{-\nu_2(t)}+2^{-\nu_2(t)-\nu_4(t)}+2^{-\nu_2(t)-\nu_4(t)-\nu_6(t)}+\ldots, \\ \text{pour } 0<t\leqslant 1. \end{cases}$$

Les fonctions $\varphi(t)$ et $\psi(t)$ sont ainsi définies pour $t\in S$. Nous démontrerons que les formules

(5) $$x=\varphi(t),\ y=\psi(t)\quad (-1\leqslant t\leqslant 1)$$

établissent une homéomorphie de classe 1,1 entre S et Q.

A ce but nous prouverons d'abord que les fonctions (2) sont partout continues du côté gauche dans T.

Soit donc k un nombre naturel donné et t_0 un nombre donné de T. Pour

$$2^{-\nu_1(t_0)}+2^{-\nu_1(t_0)-\nu_2(t_0)}+\ldots+2^{-\nu_1(t_0)-\nu_2(t_0)-\ldots-\nu_k(t_0)}<t<t_0$$

on a, comme on voit sans peine:

$$\nu_i(t)=\nu_i(t_0)\quad \text{pour}\quad i=1,2,\ldots,k,$$

donc, en particulier

$$\nu_k(t)=\nu_k(t_0),$$

ce qui prouve que la fonction $\nu_k(t)$ est pour $t=t_0$ continue du côté gauche.

Il résulte de (4) et de la propriété démontrée des fonctions (2) que $\varphi(t)$ et $\psi(t)$ sont pour $0 < t \leqslant 1$ sommes des séries uniformément convergentes de fonctions partout continues du côté gauche, donc des fonctions de même nature pour $0 < t \leqslant 1$ et, d'après (3), aussi pour $-1 \leqslant t \leqslant 1$.

Posons maintenant

$$(6) \qquad \pi(t) = (\varphi(t), \psi(t)) \quad \text{pour} \quad -1 \leqslant t \leqslant 1,$$

c. à d. désignons par $\pi(t)$ le point du plan aux coordonnées $\varphi(t)$ et $\psi(t)$. La fonction $\pi(t)$ sera évidemment de classe $\leqslant 1$ dans S et on déduit sans peine de (3) et (4) que

$$(7) \qquad \pi(t) \neq \pi(t') \quad \text{pour} \quad -1 \leqslant t < t' \leqslant 1.$$

Posons

$$\nu_k(0) = 1 \quad \text{pour} \quad k = 1, 2, \ldots :$$

les fonctions (2) étant partout continues du côté gauche dans T, elles le seront évidemment encore dans S.

p étant un point du carré Q, désignons par $\xi(p)$ et $\eta(p)$ l'abscisse et l'ordonnée de p. Les fonctions $\xi(p)$ et $\eta(p)$ sont continues dans Q.

Posons, pour $p \in Q$:

$$(8) \qquad \vartheta(p) = 2^{-\nu_1(\xi(p))} + 2^{-\nu_1(\xi(p)) - \nu_1(\eta(p))} + $$
$$+ 2^{-\nu_1(\xi(p)) - \nu_1(\eta(p)) - \nu_2(\xi(p))} +$$
$$+ 2^{-\nu_1(\xi(p)) - \nu_1(\eta(p)) - \nu_2(\xi(p)) - \nu_2(\eta(p))} + \ldots$$

et

$$(9) \qquad \begin{cases} \tau(p) = \vartheta(p) & \text{si} \quad \xi(p)\eta(p) \neq 0 \\ \tau(p) = -\frac{1}{2}\eta(p) - \frac{1}{2}, & \text{si} \quad \xi(p) = 0, \\ \tau(p) = \frac{1}{2}\xi(p) - \frac{1}{2}, & \text{si} \quad \xi(p) \neq 0, \quad \eta(p) = 0. \end{cases}$$

D'après (3) et (4) on a

$$0 \leqslant \varphi(t) \leqslant 1 \quad \text{et} \quad 0 \leqslant \psi(t) \leqslant 1 \quad \text{pour} \quad t \in S,$$

donc, d'après (6) et la définition de Q:

$$(10) \qquad \pi(t) \in Q \quad \text{pour} \quad t \in S.$$

Or, d'après (8) et (9) on a

(11) $$-1 \leqslant \tau(p) \leqslant 1 \quad \text{pour} \quad p \epsilon Q.$$

D'après (3), (4), (8), (9) et (1) on vérifie sans peine que

$$\varphi(\tau(p)) = \xi(p), \ \psi(\tau(p)) = \eta(p) \quad \text{pour} \quad p \epsilon Q,$$

d'où, d'après (6):

(12) $$\pi(\tau(p)) = p \quad \text{pour} \quad p \epsilon Q.$$

D'après (10), (7) et (12) on voit que $\pi(t)$ est une transformation biunivoque du segment S en carré Q et, d'après (12) il en résulte que $\tau(p)$ est la transformation inverse pour $\pi(p)$ (du carré Q en segment S).

Les fonctions (2) étant de classe $\leqslant 1$ pour $0 \leqslant t \leqslant 1$ et les fonctions $\xi(p)$ et $\eta(p)$ étant continues dans Q, il résulte sans peine de (8) que la fonction $\vartheta(p)$ est de classe $\leqslant 1$ dans Q.

Soit $\rho(t)$ la fonction égale à 1 pour $0 < t \leqslant 1$ et à 0 pour $t = 0$: c'est évidemment une fonction de classe 1 pour $0 \leqslant t \leqslant 1$. D'après (9) on a

(13) $$\tau(p) = \vartheta(p)\rho(\xi(p)\eta(p)) - \tfrac{1}{2}[\eta(p)+1][1-\rho(\xi(p))] + $$
$$+ \tfrac{1}{2}[\xi(p)-1]\rho(\xi(p))[1-\rho(\eta(p))] \quad \text{pour} \quad p \epsilon Q.$$

Les fonctions $\xi(p)$ et $\eta(p)$ étant continues dans Q, les fonctions $\rho(\xi(p)\eta(p))$, $\rho(\xi(p))$ et $\rho(\eta(p))$ sont de classe $\leqslant 1$ dans Q: le même étant pour la fonction $\vartheta(p)$, il résulte de (13) que $\tau(p)$ est une fonction de classe $\leqslant 1$ dans Q. Or, $\tau(p)$ étant une transformation biunivoque du carré Q en segment S, $\tau(p)$ ne peut pas être continue dans Q [1]: c'est donc une fonction de classe 1 dans Q. Pareillement $\pi(t)$, en tant qu'une transformation biunivoque du segment S en carré Q, ne peut pas être continue et par suite est de classe 1. Il est ainsi démontré que les formules (5) établissent une homéomorphie de classe 1,1 entre S et Q.

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[1]) Voir p. e. mon livre *Leçons sur les nombres transfinis*, Paris 1950, p. 73—74.

P o s i e d z e n i e

z dnia 13 października 1951 r.

Michał Kamieński

Researches on the origin of the Comet P/Wolf I
Part XI

The motion of the Comet under the influence of Jupiter and Saturn during the period 1768 Apr. 1.5 — 1760 Jan. 14.5

Mémoire présenté à la séance du 13 octobre 1951.

1. The period 1768—1760 is the last — relatively quiet — space of time in the life of the Comet. It wandered then sufficiently far from Jupiter, so that perturbations of its motion were comparatively small. On the contrary, during the previous period 1760 — 1750 it approached Jupiter so closely that its minimum distance Δ from this planet amounted only to $\Delta_{min} = 0.08$ A. U. It is the greatect approach which took place during the 200-years of the investigated period in the life of the Comet from 1750 up to 1950.

The author considers nearly as aimless the continuation of his investigations of the Comet's motion prior to 1750, owing to the uncertainty of the systems of its elements for those times. They were deduced only by the calculation for the 134-year period of its invisibility from 1884 backwards.

The summary of the results of all the authors investigations concerning the motion of the Comet from 1950 backward up to 1750 will be published in his subsequent paper „The Comet P/Wolf I 1750—1950". This paper is under preparation.

2. The system P_{-20} of elements of the Comet P/Wolf I for 1768 Apr. 1.5 Gr. M. T. published in the X Part of his „Researches on the origin of the Comet P/Wolf I" was taken as a basis for the computations given below:

1768 April 1.5 Gr. M. T.

$$P_{-20} \ldots \begin{array}{ll} M = 359^0 10' 39''.3 & \Omega = 213^0 40' 44''.6 \\ \varphi = \ 24^0 19' \ 9''.6 & \pi = \ 12^0 55' 14''.7 \\ n = 427''.6613 & i = \ 26^0 41' 32''.9 \end{array} \Bigg\} \ 1950.0$$

Applying the method of Variation of Arbitrary Constants and changing permanently the systems of elements every $\lambda = 50$ days, the author computed the differentials of perturbations given below in the adiacent Tables. This intervall is quite sufficient for our purposes, owing to the comparatively great distances Δ of the Comet from Jupiter during the period in question. It was $4.97 < \Delta < 9.35$.

For the masses of Jupiter and Saturn the values 1/1047.40 resp. 1/3501.6 were used throughout. After the integration of the differentials the author obtained:

1768 Apr. 1.5 — 1760 Jan. 14.5			
	Jupiter	Saturn	Total
δM	$-3284''.9$	$+150''.5$	$-3134''.4$
$\delta\varphi$	$+350''.5$	$+35''.4$	$+385''.9$
$\delta\Omega$	$+108''.8$	$+10''.4$	$+119''.2$
$\delta\pi$	$-197''.0$	$-70''.6$	$-267''.6$
δi	$+128''.8$	$+2''.0$	$+130''.8$
δn	$+2''.4891$	$+0''.0147$	$+2''.5038$

Adding the perturbations given above to the system P_{-20}, the author derived a new perturbed system P_{-21} of elements for 1760 Jan. 14.5:

1760 Jan. 14.5 Gr. M. T.

$$P_{-21} \ldots \begin{array}{ll} M = \ 1^0 55' 21''.0 & \Omega = 213^0 42' 43''.8 \\ \varphi = 24^0 25' 35''.5 & \pi = \ 12^0 50' 47''.1 \\ n = 430''.1651 & i = \ 26^0 43' 43''.7 \end{array} \Bigg\} \ 1950.0$$

The system P_{-21} was used as a basis for the backward computations of perturbations during the last period 1760—1750 of the Comet's motion.

JUPITER

1768 Apr. 1.5 — 1760 Jan. 14.5

	dδΩ	'f	dδi	'f	dδφ	'f	Δ
	″	″	″	″	″	″	
1759 Oct. 6.5	−5·0	+110·7	+2·1	+127·3	−10·7	+358·4	4·222
Nov. 25.5	−1·7	+109·0	+1·3	+128·6	−6·4	+352·0	4·590
1760 Jan. 14.5	−0·2	+108·8	+0·3	+128·9	−2·3	+349·7	4·968
March 4.5	0·0	+108·8	−0·6	+128·3	+1·2	+350·9	5·348
Apr. 23.5	−0·7	+108·1	−1·2	+127·1	+3·8	+354·7	5·720
June 12.5	−1·9	+106·2	−1·6	+125·5	+5·2	+359·9	6·072
Aug. 1.5	−3·3	+102·9	−1·8	+123·7	+5·8	+365·7	6·400
Sept. 20.5	−4·7	+98·2	−1·7	+122·0	+5·8	+371·5	6·705
Nov. 9.5	−5·9	+92·3	−1·5	+120·5	+5·3	+376·8	6·989
Dec. 29.5	−6·8	+85·5	−1·2	+119·3	+4·6	+381·4	7·254
1761 Feb. 17.5	−7·5	+78·0	−0·8	+118·5	+3·5	+384·9	7·503
Apr. 8.5	−7·9	+70·1	−0·4	+118·1	+2·5	+387·4	7·734
May 28.5	−8·0	+62·1	0·0	+118·1	+1·2	+388·6	7·949
July 17.5	−7·9	+54·2	+0·3	+118·4	−0·1	+388·5	8·147
Sept. 5.5	−7·5	+46·7	+0·6	+119·0	−1·4	+387·1	8·329
Oct. 25.5	−6·9	+39·8	+0·8	+119·8	−2·9	+384·2	8·496
Dec. 14.5	−6·1	+33·7	+1·0	+120·8	−4·3	+379·9	8·648
1762 Feb. 2.5	−5·2	+28·5	+1·0	+121·8	−5·8	+374·1	8·786
March 24.5	−4·2	+24·3	+1·0	+122·8	−7·2	+366·9	8·906
May 13.5	−3·1	+21·2	+0·8	+123·6	−8·7	+358·2	9·010
July 2.5	−2·0	+19·2	+0·6	+124·2	−10·1	+348·1	9·103
Aug. 21.5	−0·9	+18·3	+0·3	+124·5	−11·4	+336·7	9·181
Oct. 10.5	+0·2	+18·5	−0·1	+124·4	−12·7	+324·0	9·243
Nov. 29.5	+1·1	+19·6	−0·5	+123·9	−13·8	+310·2	9·292
1763 Jan. 18.5	+2·0	+21·6	−1·0	+122·9	−14·9	+295·3	9·326
Mar. 9.5	+2·8	+24·4	−1·5	+121·4	−15·8	+297·5	9·348
Apr. 28.5	+3·4	+27·8	−2·1	+119·3	−16·7	+262·8	9·353
June 17.5	+3·9	+31·7	−2·6	+116·7	−17·3	+245·5	9·348
Aug. 6.5	+4·3	+36·0	−3·2	+113·5	−17·9	+227·6	9·322
Sept. 25.5	+4·5	+40·5	−3·7	+109·8	−18·3	+209·3	9·288
Nov. 14.5	+4·6	+45·1	−4·2	+105·6	−18·5	+190·8	9·243
1764 Jan. 3.5	+4·5	+49·6	−4·7	+100·9	−18·7	+172·1	9·185
Feb. 22.5	+4·2	+53·8	−5·1	+95·8	−18·6	+153·5	9·116
Apr. 12.5	+3·9	+57·7	−5·5	+90·3	−18·5	+135·0	9·037

JUPITER

1768 Apr. 1.5 — 1760 Jan. 14.5

	$d\delta\pi$	$'f$	$\lambda d\delta n$	$'f$	$''f$	P	$'f$
	"	"	"	"	"	"	"
1759 Oct. 6.5	+21·6	−220·1	+2·978	+122·420	−3849·566	−13·8	+328·1
Nov. 25.5	+16·2	−203·9	+1·694	+124·114	−3727·146	−7·4	+320·7
1760 Jan. 14.5	+13·7	−190·2	+0·497	+124·611	−3603·032	−5·0	+315·7
March 4.5	+14·0	−176·2	−0·524	+124·087	−3478·421	−5·9	+309·8
Apr. 23.5	+16·2	−160·0	−1·316	+122·771	−3354·334	−9·2	+300·6
June 12.5	+19·3	−140·7	−1·872	+120·899	−3231·563	−13·7	+286·9
Aug. 1.5	+22·6	−118·1	−2·243	+118·656	−3110·664	−18·5	+268·4
Sept. 20.5	+25·8	−92·3	−2·477	+116·179	−2992·008	−23·1	+245·3
Nov. 9.5	+28·6	−63·7	−2·621	+113·558	−2875·829	−27·5	+217·8
Dec. 29.5	+31·1	−32·6	−2·704	+110·854	−2762·271	−31·4	+186·4
1761 Feb. 17.5	+33·3	+0·7	−2·746	+108·108	−2651.417	−35·0	+151·4
Apr. 8.5	+35·2	+35·9	−2·765	+105·343	−2543·309	−38·1	+113·3
May 28.5	+36·7	+72·6	−2·764	+102·579	−2437·966	−40·9	+72·4
July 17.5	+37·9	+110·5	−2·747	+99·832	−2335·387	−43·3	+29·1
Sept. 5.5	+38·8	+149·3	−2·718	+97·114	−2235·555	−45·1	−16·0
Oct. 25.5	+39·4	+188·7	−2·687	+94·427	−2138·441	−46·6	−62·6
Dec. 14.5	+39·7	+228·4	−2·651	+91·776	−2044·014	−47·6	−110·2
1762 Feb. 2.5	+39·7	+268·1	−2·608	+89·168	−1952·238	−48·2	−158·4
March 24.5	+39·4	+307·5	−2·563	+86·605	−1863·070	−48·3	−206·7
May 13.5	+38·6	+346·1	−2·512	+84·093	−1776·465	−48·0	−254·7
July 2.5	+37·6	+383·7	−2·462	+81·631	−1692·372	−47·1	−301·8
Aug. 21.5	+36·2	+419·9	−2·408	+79·223	−1610·741	−45·8	−347·6
Oct. 10.5	+34·4	+454·3	−2·353	+76·870	−1531·518	−44·1	−391·7
Nov. 29.5	+32·5	+486·8	−2·296	+74·574	−1454·648	−42·1	−433·8
1763 Jan. 18.5	+30·0	+516·8	−2·239	+72·335	−1380·074	−39·3	−473·1
Mar. 9.5	+27·4	+544·2	−2·183	+70·152	−1307·739	−36·3	−509·4
Apr. 28.5	+24·5	+568·7	−2·130	+68·022	−1237·587	−33·0	−542·4
June 17.5	+21·4	+590·1	−2·079	+65·943	−1169·565	−29·4	−571·8
Aug. 6.5	+18·2	+608·3	−2·029	+63·914	−1103·622	−25·5	−597·3
Sept. 25.5	+14·8	+623·1	−1·982	+61·932	−1039·708	−21·4	−618·7
Nov. 14.5	+11·2	+634·3	−1·941	+59·991	−977·776	−17·1	−635·8
1764 Jan. 3.5	+7·7	+642·0	−1·902	+58·089	−917·785	−12·8	−648·6
Feb. 22.5	+4·1	+646·1	−1·870	+56·219	−859·696	−8·3	−656·9
Apr. 12.5	+0·5	+646·6	−1·844	+54·375	−803·477	−3·9	−660·8

		$d\delta\Omega$	$'f$	$d\delta i$	$'f$	$d\delta\varphi$	$'f$	Δ
		″	″	″	″	″	″	
1764 Apr.	12.5	+3·9	+57·7	−5·5	+90·3	−18·5	+135·0	9·037
June	1.5	+3·4	+61·1	−5·8	+84·5	−18·1	+116·9	8·947
July	21.5	+2·9	+64·0	−6·1	+78·4	−17·7	+99·2	8·849
Sept.	9.5	+2·3	+66·3	−6·2	+72·2	−17·1	+82·1	8·744
Oct.	29.5	+1·6	+67·9	−6·3	+65·9	−16·3	+65·8	8·630
Dec.	18.5	+0·9	+68·8	−6·3	+59·6	−15·5	+50·3	8·511
1765 Feb.	6.5	+0·2	+69·0	−6·3	+53·3	−14·6	+35·7	8·389
Mar.	28.5	−0·5	+68·5	−6·1	+47·2	−13·5	+22·2	8·262
May	17.5	−1·2	+67·3	−5·9	+41·3	−12·4	+9·8	8·134
July	6.5	−1·9	+65·4	−5·6	+35·7	−11·1	−1·3	8·004
Aug.	25.5	−2·5	+62·9	−5·3	+30·4	−9·9	−11·2	7·876
Oct.	14.5	−3·0	+59·9	−5·0	+25·4	−8·6	−19·8	7·750
Dec.	3.5	−3·4	+56·5	−4·6	+20·8	−7·2	−27·0	7·630
1766 Jan.	22.5	−3·8	+52·7	−4·1	+16·7	−5·9	−32·9	7·516
Mar.	13.5	−4·1	+48·6	−3·7	+13·0	−4·6	−37·5	7·415
May	2.5	−4·3	+44·3	−3·2	+9·8	−3·2	−40·7	7·325
June	21.5	−4·4	+39·9	−2·8	+7·0	−2·0	−42·7	7·248
Aug.	10.5	−4·5	+35·4	−2·4	+4·6	−0·7	−43·4	7·186
Sept.	29.5	−4·4	+31·0	−2·0	+2·6	+0·4	−43·0	7·143
Nov.	18.5	−4·4	+26·6	−1·6	+1·0	+1·4	−41·6	7·120
1767 Jan.	7.5	−4·3	+22·3	−1·2	−0·2	+2·4	−39·2	7·118
Feb.	26.5	−4·1	+18·2	−0·9	−1·1	+3·2	−36·0	7·138
Apr.	17.5	−3·8	+14·4	−0·6	−1·7	+4·0	−32·0	7·180
June	6.5	−3·5	+10·9	−0·3	−2·0	+4·7	−27·3	7·244
July	26.5	−3·1	+7·8	0·0	−2·0	+5·3	−22·0	7·328
Sept	14.5	−2·7	+5·1	+0·2	−1·8	+5·6	−16·4	7·428
Nov.	3.5	−2·2	+2·9	+0·4	−1·4	+5·6	−10·8	7·535
Dec.	23.5	−1·6	+1·3	+0·5	−0·9	+5·1	−5·7	7·642
1768 Feb.	11.5	−1·0	+0·3	+0·6	−0·3	+4·2	−1·5	7·742
Apr.	1.5	−0·5	−0·2	+0·6	+0·3	+2·7	+1·2	7·822
May	21.5	−0·1	−0·3	+0·5	+0·8	+1·2	+2·4	7·878
July	10.5	+0·1		+0·3		−0·3		7·903

	$d\delta\pi$	$'f$	$\lambda d\delta n$	$'f$	$''f$	P	$'f$
	"	"	"	"	"	"	"
1764 Apr. 12.5	+ 0·5	+ 646·6	− 1·844	+ 54·375	− 803·477	− 3·9	− 660·8
June 1.5	− 3·0	+ 643·6	− 1·822	+ 52·553	− 749·102	+ 0·5	− 660·3
July 21.5	− 6·5	+ 637·1	− 1·806	+ 50·747	− 696·549	+ 4·8	− 655·5
Sept. 9.5	− 9·8	+ 627·3	− 1·795	+ 48·952	− 645·802	+ 9·0	− 646·5
Oct. 29.5	− 13·0	+ 614.3	− 1·790	+ 47·162	− 596·850	+ 13·1	− 633·4
Dec. 18.5	− 16·1	+ 598·2	− 1·790	+ 45·372	− 549·688	+ 16·9	− 616·5
1765 Feb. 6.5	− 18·9	+ 579·3	− 1·796	+ 43·576	− 504·316	+ 20·5	− 596·0
Mar. 28.5	− 21·5	+ 557·8	− 1·806	+ 41·770	− 460·740	+ 23·8	− 572·2
May 17.5	− 23·8	+ 534·0	− 1·821	+ 39·949	− 418·970	+ 26·7	− 545·5
July 6.5	− 25·9	+ 508·1	− 1·841	+ 38·108	− 379·021	+ 29·3	− 516·2
Aug. 25.5	− 27·6	+ 480·5	− 1·863	+ 36·245	− 340·913	+ 31·6	− 484·6
Oct. 14.5	− 29·0	+ 451·5	− 1·891	+ 34·354	− 304·668	+ 33·3	− 451·3
Dec. 3.5	− 30·1	+ 421·4	− 1·920	+ 32·434	− 270·314	+ 34·7	− 416·6
1766 Jan. 22.5	− 30·9	+ 390·5	− 1·953	+ 30·481	− 237·880	+ 35·5	− 381·1
Mar. 13.5	− 31·3	+ 359·2	− 1·985	+ 28·496	− 207·399	+ 35·9	− 345·2
May 2.5	− 31·5	+ 327·7	− 2·019	+ 26·477	− 178·903	+ 35·9	− 309·3
June 21.5	− 31·3	+ 296·4	− 2·054	+ 24·423	− 152·426	+ 35·5	− 273·8
Aug. 10.5	− 30·9	+ 265·5	− 2·086	+ 22·337	− 128·003	+ 34·5	− 239·3
Sept. 29.5	− 30·3	+ 235·2	− 2·122	+ 20·215	− 105·666	+ 33·3	− 206·0
Nov. 18.5	− 29·4	+ 205·8	− 2·154	+ 18·061	− 85·451	+ 31·7	− 174·3
1767 Jan. 7.5	− 28·6	+ 177·2	− 2·182	+ 15·879	− 67·390	+ 29·9	− 144·4
Feb. 26.5	− 27·3	+ 149·9	− 2·207	+ 13·672	− 51·511	+ 27·9	− 116·5
Apr. 17.5	− 26·2	+ 123·7	− 2·219	+ 11·453	− 37·839	+ 25·5	− 91·0
June 6.5	− 24·8	+ 98·9	− 2·208	+ 9·245	− 26·386	+ 22·8	− 68·2
July 26.5	− 23·0	+ 75·9	− 2·164	+ 7·081	− 17·141	+ 19·8	− 48·4
Sept 14.5	− 20·9	+ 55·0	− 2·060	+ 5·021	− 10·060	+ 16·5	− 31·9
Nov. 3.5	− 18·5	+ 36·5	− 1·873	+ 3·148	− 5·039	+ 13·0	− 18·9
Dec. 23.5	− 16·1	+ 20·4	− 1·584	+ 1·564	− 1·891	+ 9·6	− 9·3
1768 Feb. 11.5	− 14·0	+ 6·4	− 1·180	+ 0·384	− 0·327	+ 6·7	− 2·6
Apr. 1.5	− 12·9	− 6·5	− 0·681	− 0·297	+ 0·057	+ 5·0	+ 2·4
May 21.5	− 12·8	− 19·3	− 0·138	− 0·435	− 0·240	+ 4·7	+ 7·1
July 10.5	− 14·4		+ 0·392		− 0·675	+ 6·0	

SATURN

1768 Apr. 1.5 — 1760 Jan. 14.5

	$d\delta\Omega$	$'f$	$d\delta i$	$'f$	$d\delta\varphi$	$'f$	Δ
	″	″	″	″	″	″	
1759 Oct. 6.5	−0·5	+10·8	+0·2	+1·7	−0·1	+36·3	7·41
Nov. 25.5	−0·3	+10·5	+0·2	+1·9	−0·5	+35·8	7·48
1760 Jan. 14.5	−0·1	+10·4	+0·2	+2·1	−0·8	+35·0	7·69
Mar. 4.5	0·0	+10·4	+0·2	+2·3	−0·9	+34·1	8·00
Apr. 23.5	+0·1	+10·5	+0·1	+2·4	−0·8	+33·3	8·37
June 12.5	+0·1	+10·6	+0·1	+2·5	−0·6	+32·7	8·80
Aug. 1.5	0·0	+10·6	0·0	+2·5	−0·5	+32·2	9·23
Sept 20.5	0·0	+10·6	0·0	+2·5	−0·3	+31·9	9·66
Nov. 9.5	−0·1	+10·5	0·0	+2·5	−0·2	+31·7	10·09
Dec. 29.5	−0·1	+10·4	0·0	+2·5	−0·1	+31·6	10·52
1761 Feb. 17.5	−0·2	+10·2	0·0	+2·5	−0·1	+31·5	10·90
Apr. 8.5	−0·3	+9·9	0·0	+2·5	−0·1	+31·4	11·22
May 28.5	−0·3	+9·6	0·0	+2·5	−0·2	+31·2	11·55
July 17.5	−0·3	+9·3	0·0	+2·5	−0·2	+31·0	11·88
Sept. 5.5	−0·3	+9·0	0·0	+2·5	−0·3	+30·7	12·20
Oct. 25.5	−0·3	+8·7	0·0	+2·5	−0·3	+30·4	12·45
Dec. 14.5	−0·3	+8·4	+0·1	+2·6	−0·4	+30·0	12·68
1762 Feb. 2.5	−0·3	+8·1	+0·1	+2·7	−0·4	+29·6	12·91
Mar. 24.5	−0·3	+7·8	+0·1	+2·8	−0·5	+29·1	13·12
May 13.5	−0·3	+7·5	+0·1	+2·9	−0·5	+28·6	13·30
July 2.5	−0·3	+7·2	+0·1	+3·0	−0·6	+28·0	13·47
Aug. 21.5	−0·3	+6·9	+0·1	+3·1	−0·6	+27·4	13·65
Oct. 10.5	−0·2	+6·7	+0·1	+3·2	−0·7	+26·7	13·81
Nov. 29.5	−0·2	+6·5	+0·1	+3·3	−0·7	+26·0	13·93
1763 Jan. 18.5	−0·2	+6·3	+0·1	+3·4	−0·7	+25·3	14·04
Mar. 9.5	−0·1	+6·2	+0·1	+3·5	−0·8	+24·5	14·14
Apr. 28.5	−0·1	+6·1	+0·1	+3·6	−0·8	+23·7	14·24
June 17.5	−0·1	+6·0	0·0	+3·6	−0·8	+22·9	14·31
Aug. 6.5	−0·1	+5·9	0·0	+3·6	−0·8	+22·1	14·38
Sept. 25.5	0·0	+5·9	0·0	+3·6	−0·8	+21·3	14·45
Nov. 14.5	0·0	+5·9	0·0	+3·6	−0·8	+20·5	14·53
1764 Jan. 3.5	0·0	+5·9	0·0	+3·6	−0·9	+19·6	14·60
Feb. 22.5	0·0	+5·9	0·0	+3·6	−0·9	+18·7	14·60
Apr. 12.5	0·0	+5·9	−0·1	+3·5	−0·9	+17·8	14·60

SATURN

1768 Apr. 1.5 — 1760 Jan. 14.5

	dδπ	'f	λdδn	'f	''f	P	'f
	"	"	"	"	"	"	"
1759 Oct. 6.5	−1·4	−69·0	+0·003	+0·530	+31·454	+0·5	+117·4
Nov. 25.5	−1·1	−70·1	+0·114	+0·644	+31·984	+0·3	+117·7
1760 Jan. 14.5	−1·0	−71·1	+0·195	+0·839	+32·628	+0·4	+118·1
Mar. 4.5	−0·9	−72·0	+0·220	+1·059	+33·467	+0·7	+118·8
Apr. 23.5	−0·7	−72·7	+0·200	+1·259	+34·526	+0·8	+119·6
June 12.5	−0·4	−73·1	+0·150	+1·409	+35·785	+0·7	+120·3
Aug. 1.5	−0·1	−73·2	+0·093	+1·502	+37·194	+0·5	+120·8
Sept 20.5	+0·4	−72·8	+0·034	+1·536	+38·696	+0·2	+121·0
Nov. 9.5	+0·8	−72·0	−0·016	+1·520	+40·232	−0·2	+120·8
Dec. 29.5	+1·2	−70·8	−0·057	+1·463	+41·752	−0·7	+120·1
1761 Feb. 17.5	+1·5	−69·3	−0·089	+1·374	+43·215	−1·0	+119·1
Apr. 8.5	+1·8	−67·5	−0·109	+1·265	+44·589	−1·4	+117·7
May 28.5	+2·0	−65·5	−0·124	+1·141	+45·854	−1·7	+116·0
July 17.5	+2·1	−63·4	−0·137	+1·004	+46·995	−2·0	+114·0
Sept. 5.5	+2·2	−61·2	−0·146	+0·858	+47·999	−2·2	+111·8
Oct. 25.5	+2·3	−58·9	−0·151	+0·707	+48·857	−2·4	+109·4
Dec. 14.5	+2·3	−56·6	−0·153	+0·554	+49·564	−2·5	+106·9
1762 Feb. 2.5	+2·3	−54·3	−0·154	+0·400	+50·118	−2·7	+104·2
Mar. 24.5	+2·3	−52·0	−0·153	+0·247	+50·518	−2·8	+101·4
May 13.5	+2·3	−49·7	−0·152	+0·095	+50·765	−2·9	+98·5
July 2.5	+2·3	−47·4	−0·148	−0·053	+50·860	−2·8	+95·7
Aug. 21.5	+2·2	−45·2	−0·144	−0·197	+50·807	−3·0	+92·7
Oct. 10.5	+2·2	−43·0	−0·140	−0·337	+50·610	−3·0	+89·7
Nov. 29.5	+2·1	−40·9	−0·138	−0·475	+50·273	−3·1	+86·6
1763 Jan. 18.5	+2·0	−38·9	−0·133	−0·608	+49·798	−3·1	+83·5
Mar. 9.5	+2·0	−36·9	−0·127	−0·735	+49·190	−3·1	+80·4
Apr. 28.5	+1·9	−35·0	−0·122	−0·857	+48·455	−3·1	+77·3
June 17.5	+1·8	−33·2	−0·117	−0·974	+47·598	−3·1	+74·2
Aug. 6.5	+1·7	−31·5	−0·110	−1·084	+46·624	−3·0	+71·2
Sept. 25.5	+1·6	−29·9	−0·104	−1·188	+45·540	−3·0	+68·2
Nov. 14.5	+1·5	−28·4	−0·098	−1·286	+44·352	−3·0	+65·2
1764 Jan. 3.5	+1·4	−27·0	−0·092	−1·378	+43·066	−3·0	+62·2
Feb. 22.5	+1·3	−25·7	−0·086	−1·464	+41·688	−2·9	+59·3
Apr. 12.5	+1·2	−24·5	−0·079	−1·543	+40·224	−2·9	+56·4

	dδΩ	'f	dδi	'f	dδφ	'f	Δ
	"	"	"	"	"	"	
1764 Apr. 12.5	0·0	+5·9	−0·1	+3·5	−0·9	+17·8	14·60
June 1.5	0·0	+5·9	−0·1	+3·4	−0·9	+16·9	14·60
July 21.5	0·0	+5·9	−0·1	+3·3	−0·8	+16·1	14·58
Sept. 9.5	0·0	+5·9	−0·1	+3·2	−0·8	+15·3	14·57
Oct. 29.5	0·0	+5·9	−0·1	+3·1	−0·8	+14·5	14·56
Dec. 18.5	0·0	+5·9	−0·1	+3·0	−0·8	+13·7	14·53
1765 Feb. 6.5	0·0	+5·9	−0·2	+2·8	−0·8	+12·9	14·50
Mar. 28.5	0·0	+5·9	−0·2	+2·6	−0·8	+12·1	14·40
May 17.5	0·0	+5·9	−0·2	+2·4	−0·7	+11·4	14·30
July 6.5	−0·1	+5·8	−0·2	+2·2	−0·7	+10·7	14·20
Aug. 25.5	−0·1	+5·7	−0·2	+2·0	−0·7	+10·0	14·12
Oct. 14.5	−0·1	+5·6	−0·2	+1·8	−0·7	+9·3	14·05
Dec. 3.5	−0·2	+5·4	−0·2	+1·6	−0·7	+8·6	13·95
1766 Jan. 22.5	−0·2	+5·2	−0·2	+1·4	−0·6	+8·0	13·82
Mar. 13.5	−0·3	+4·9	−0·2	+1·2	−0·6	+7·4	13·69
May 2.5	−0·3	+4·6	−0·2	+1·0	−0·6	+6·8	13·55
June 21.5	−0·3	+4·3	−0·2	+0·8	−0·5	+6·3	13·40
Aug. 10.5	−0·4	+3·9	−0·2	+0·6	−0·5	+5·8	13·20
Sept. 29.5	−0·4	+3·5	−0·2	+0·4	−0·5	+5·3	13·00
Nov. 18.5	−0·4	+3·1	−0·2	+0·2	−0·5	+4·8	12·80
1767 Jan. 7.5	−0·5	+2·6	−0·1	+0·1	−0·5	+4·3	12·55
Feb. 26.5	−0·5	+2·1	−0·1	0·0	−0·5	+3·8	12·30
Apr. 17.5	−0·4	+1·7	−0·1	−0·1	−0·6	+3·2	12·00
June 6.5	−0·4	+1·3	0·0	−0·1	−0·6	+2·6	11·68
July 26.5	−0·4	+0·9	0·0	−0·1	−0·6	+2·0	11·35
Sept. 14.5	−0·3	+0·6	0·0	−0·1	−0·6	+1·4	11·00
Nov. 3.5	−0·3	+0·3	0·0	−0·1	−0·6	+0·8	10·61
Dec. 23.5	−0·2	+0·1	+0·1	0·0	−0·5	+0·3	10·15
1768 Feb. 11.5	−0·1	0·0	0·0	0·0	−0·3	0·0	9·62
Apr. 1.5	0·0	0·0	0·0	0·0	0·0	0·0	9·08
May 21.5	0·0	0·0	0·0	0·0	+0·3	+0·3	8·55
July 10.5	0·0		−0·1		+0·5		8·05

	$d\delta\pi$	$'f$	$\lambda d\delta n$	$'f$	$''f$	P	$'f$
	″	″	″	″	″	″	″
1764 Apr. 12.5	+1·2	−24·5	−0·079	−1·543	+40·224	−2·9	+56·4
June 1.5	+1·2	−23·3	−0·072	−1·615	+38·681	−2·8	+53·6
July 21.5	+1·1	−22·2	−0·064	−1·679	+37·066	−2·7	+50·9
Sept. 9.5	+1·0	−21 2	−0·057	−1·736	+35·387	−2·7	+48·2
Oct. 29.5	+0·9	−20·3	−0·049	−1·785	+33·651	−2·6	+45·6
Dec. 18.5	+0·8	−19·5	−0·041	−1·826	+31·866	−2·6	+43·0
1765 Feb. 6.5	+0·8	−18·7	−0·033	−1·859	+30·040	−2·5	+40·5
Mar. 28.5	+0·7	−18·0	−0·025	−1·884	+28·181	−2·4	+38·1
May 17.5	+0·7	−17·3	−0·016	−1·900	+26·297	−2·4	+35·7
July 6.5	+0·6	−16·7	−0·007	−1·907	+24·397	−2·3	+33·4
Aug. 25.5	+0·6	−16·1	+0·002	−1·905	+22·490	−2·3	+31·1
Oct. 14.5	+0·6	−15·5	+0·012	−1·893	+20·585	−2·2	+28·9
Dec. 3.5	+0 6	−14·9	+0·022	−1·871	+18·692	−2·2	+26·7
1766 Jan. 22.5	+0·6	−14·3	+0·033	−1·838	+16·821	−2·1	+24·6
Mar. 13.5	+0·6	−13·7	+0·045	−1·793	+14·983	−2·1	+22·5
May 2.5	+0·6	−13·1	+0·058	−1·735	+13·190	−2·1	+20·4
June 21.5	+0·7	−12·4	+0·071	−1·664	+11·455	−2·1	+18·3
Aug. 10.5	+0·8	−11·6	+0·084	−1·580	+9·791	−2·0	+16·3
Sept. 29.5	+0·8	−10·8	+0·098	−1·482	+8·211	−2·0	+14·3
Nov. 18.5	+0·9	−9·9	+0·114	−1·368	+6 729	−2·0	+12·3
1767 Jan. 7.5	+1·0	−8·9	+0·129	−1·239	+5·361	−1·9	+10·4
Feb. 26.5	+1·1	−7·8	+0·145	−1·094	+4·122	−1·9	+8·5
Apr. 17.5	+1·2	−6·6	+0·159	−0·935	+3·028	−1·8	+6·7
June 6.5	+1·2	−5·4	+0·172	−0·763	+2·093	−1·7	+5·0
July 26.5	+1·2	−4·2	+0·181	−0·582	+1·330	−1·5	+3·5
Sept. 14.5	+1·2	−3·0	+0·179	−0·403	+0·748	−1·3	+2·2
Nov. 3.5	+1·0	−2·0	+0·167	−0·236	+0·345	−1·0	+1·2
Dec. 23.5	+0·9	−1·1	+0·138	−0·098	+0·109	−0·7	+0·5
1768 Feb. 11.5	+0·8	−0·3	+0·086	−0·012	+0·011	−0·4	+0·1
Apr. 1.5	+0·7	+0·4	+0·010	−0·002	−0·001	−0·3	−0·2
May 21.5	+0·9	+1·3	−0·078	−0·080	−0·003	−0·4	−0·6
July 10.5	+1·2		−0·164		−0·083	−0·8	

Stanisław Mrówka

Sur une propriété des ensembles fermés et bornés

Communication présentée par M. W. Sierpiński
à la séance du 13 octobre 1951.

A la séance de la Société Polonaise de Mathématique, Section de Varsovie, qui a eu lieu le 12 octobre 1951, M. W. Sierpiński a énoncé et démontré le théorème suivant:

Si E est un ensemble plan fermé et borné (non vide), il existe dans le plan $2^{\aleph_0}$ droites distinctes qui ont un et un seul point commun avec l'ensemble E.

Après la communication de M. Sierpiński, M. K. Zarankiewicz a posé le problème si, E étant un ensemble fermé et borné (non vide) situé dans l'espace à 3 dimensions, il existe toujours $2^{\aleph_0}$ plans distincts dont chacun a un et un seul point commun avec l'ensemble E.

Le but de cette communication est de démontrer que la réponse à ce problème est positive. Je démontrerai notamment le théorème suivant:

Théorème. *Si E est un ensemble fermé et borné (non vide) situé dans l'espace euclidien à 3 dimensions, il existe dans cet espace $2^{\aleph_0}$ plans distincts dont chacun a un et un seul point commun avec l'ensemble E.*

Démonstration. Soit E un ensemble fermé et borné (non vide) situé dans l'espace euclidien à 3 dimensions. L'ensemble E étant borné, il existe une droite D dans l'espace qui n'a aucun point commun avec l'ensemble E.

Soit p un point quelconque de la droite D. Soit δ la borne supérieure des nombres $\rho(p,q)$, où $q \in E$ et où $\rho(p,q)$ désigne la distance entre les points p et q. L'ensemble E étant borné, on conclut que δ est un nombre fini et, l'ensemble E étant fermé, il existe au moins un point q_0 de E, tel que $\rho(p,q_0)=\delta$. Soit $P(p)$ le plan possant par q_0 et perpendiculaire à la droite pq_0. Il est évident que le plan $P(p)$ a un seul point commun avec l'ensemble E, à savoir le point q_0.

Ainsi à tout point p de la droite D correspond au moins un plan $P(p)$ contenant un et un seul point de l'ensemble E.

Je démontrerai que les plans $P(p_1)$ et $P(p_2)$ correspondants aux points différents p_1 et p_2 de la droite D sont toujours distincts.

Si les plans $P(p_1)$ et $P(p_2)$ contiennent le même point q de E, ce point n'est pas situé sur la droite D, vu que cette droite ne contient aucun point de E. Les plans $P(p_1)$ et $P(p_2)$ sont donc perpendiculaires aux droites différentes $p_1 q$ et $p_2 q$ respectivement et par suite sont distincts.

Or, si les plans $P(p_1)$ et $P(p_2)$ contiennent des points différents de E, ils sont distincts, vu que chacun de ces plans contient un seul point de E.

Il est ainsi démontré que si $p_1 \epsilon D$, $p_2 \epsilon D$ et $p_1 \neq p_2$, on a $P(p_1) \neq P(p_2)$. Notre théorème est une conséquence immédiate de ce fait.

D'une façon tout à fait analogue on peut démontrer le théorème plus général suivant:

Si E est un ensemble fermé et borné (non vide) situé dans l'espace euclidien R_n à n dimensions (où n est un nombre naturel >1), il existe dans R_n $2^{\aleph_0}$ hyperplans distincts à $n-1$ dimensions dont chacun contient un et un seul point de l'ensemble E.

Wiktor Kemula and Zbigniew R. Grabowski

Latent currents in polarography

Latent diffusion currents of nitrate

Mémoire présenté le 13 octobre 1951.

Introduction

The application of polarography as a method of quantitative chemical analysis of solutions is based on the existence of a *univocal* relation between the concentration of the reducible constituent and the corresponding limiting reduction current.

The erroneous analytical results are due mainly to the following factors, which exert an influence on the magnitude of the limiting current:

1) the great ohmic *potential drop* $i \cdot R$ across the electrolyte causing the increase of the migration of ions (migration current),
2) the *stirring* of the solution surrounding the dropping cathode, which may be either of mechanical or of electrokinetic origin and which supplies the increased amounts of the reducible material to the electrode surface (maxima, „water waves"),
3) the *adsorption* of the capillary-active substances at the mercury-solution interface, hindering often the electrode processes (the blockade of the cathode surface, the adsorption polarisation effects).

All harmful factors mentioned above may be largely eliminated under the usual conditions of the polarographic work, *i. e.*, in the presence of a large excess of indifferent supporting electrolyte and of a suitable amount of maximum suppressor, providing the mean values of the drop time and of the mercury pressure.

The magnitude of the limiting current, corrected for the residual current, ought to be equal to the diffusion current of the component being reduced at the cathode, and therefore ought to be proportional to the concentration of that component. The exceptions to that rule belong mainly to the classes of kinetic or catalytic currents, *i. e.* to those systems in which the diffusion of the reducible component is not the slowest rate-determining process.

Under the optimal conditions of polarographic analysis the presence of other reducible substances in the solution *does not alter* the diffusion current of a given component. Thus the quantitative determination of a component in a mixture of reducible substances is possible, in spite of the experimental difficulties arising when the concentrations of these substances are very different.

The latent limiting currents

There are however certain well known systems in which under conditions securing usually an undisturbed diffusion the limiting current is abnormally low. The magnitude of such a limiting current depends not only on the concentration of a given component, but also on the concentration of the other component reducible at the more positive potentials. In the last case the decrease of the current is caused by a secondary reaction taking place in the diffusion layer between the products of electro-reduction of one of the components and the reducible ions or molecules of the other, diffusing towards the cathode. That anomaly, named by W. Kemula „the effect of *latent* limiting currents"[1]), appears *e. g.* in the behaviour of the hydrogen wave and heavy metals waves in the presence of dissolved oxygen [1—4]).

The recent results of the polarographic studies carried on in this laboratory have shown the possibilities of application of the latent limiting currents to the quantitative polarographic analysis and to the solution of some theoretical questions. The aim of the present paper is to give a deductive description and classification of the latent limiting currents and a consequent review of the experimental facts.

The discussion of the possibility of existence of the latent limiting current in a general case of two reducible substances A and B present in the solution, led us to the distinction between two kinds of the latent limiting currents.

The first kind of latent limiting currents

Let us consider a solution containing two reducible components, A and B, as solutes. A and B, and their reduction products: A^* and B^*, may be each a cation, an anion, or a neutral molecule.

Beginning from the first cathodic depolarisation potential, the component A undergoes a reduction as follows:

$$A + n\bar{e} \rightarrow p A^*. \quad (1)$$

The reduction-product A^*, appearing at the cathode surface is diffusing back to the bulk of solution and may *react* with the other component, B:

$$p A^* + q B \rightarrow A_p^* B_q. \quad (2)$$

The numbers p and q are representing the stoichiometric ratio of reacting substances and may get fractional values as well.

The reduction of the component B starts at the more negative depolarisation potential and is described by:

$$B + m\bar{e} \rightarrow r B^*. \quad (3)$$

As a consequence of the reaction (2), the influx of the component B to the cathode surface *diminishes* or *ceases*, according to the concentrations of A and B. Provided the new compound $A_p^* B_q$ is not reducible in the given potential range, the polarographic wave of B will have an anomalous height or will not appear at all. The limiting reduction current of B measured under those conditions (i_B) *does not represent* the true concentration of B in the bulk of solution (C_B). The difference between the proper limiting current (i_B^0), observed for the same concentration of B in the absence of A, and „anomalous" limiting current i_B, will be called the *latent* limiting current of the *first kind:*

$$\Delta i = i_B^0 - i_B. \quad (4)$$

A simple example of the first kind of the latent limiting current is demonstrated schematically in fig. 1.

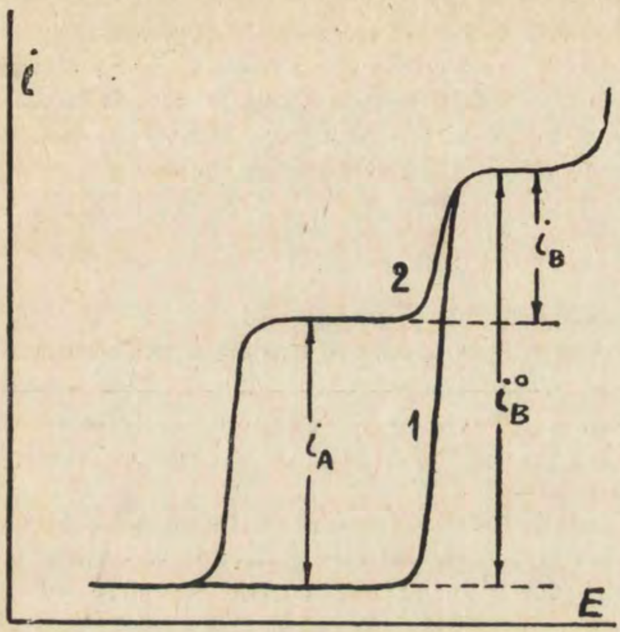

Fig. 1. Latent limiting current of the first kind, when $\frac{q \cdot m}{n} = 1$.
Curve 1 — polarogram of the solution containing *B* in the absence of *A*; 2 — polarogram of the solution of a mixture of *A* and *B* in the first concentration range.

The „equivalence point"

The magnitude of the latent limiting current may be computed easily, provided that the velocity of the reaction (2) is much greater than the rate of diffusion of the components *A* and *B* and that the equilibrium constant is large enough to permit that the reaction (2) may proceed practically to the end. The correction for the influence of the backward diffusion of A^* from the cathode surface to the bulk of solution will be omitted here, for the sake of simplicity.

Under the conditions enabling to apply the *Ilkovič equation*, the average diffusion current is:

$$i_d = k \cdot n \cdot D^{1/2} \cdot C \tag{5}$$

k being constant for the invariables: dropping electrode characteristics and temperature, *n* — number of electrons involved in the particular reduction process, D — polarographic diffusion coefficient of the reducible component, C — its concentration.

At the potentials corresponding to the diffusion current the average influx of the reducible particles to the surface of the dropping cathode is given by the equation:

$$\frac{i_d}{n \cdot F} = \frac{k}{F} \cdot D^{1/2} \cdot C \tag{6}$$

where F is the Faraday number.

The latent limiting current existing under conditions which exclude any noticeable electrical migration of the reducible particles, and any other disturbances of the diffusion with the exception of the secondary reaction (2), will be called a *latent diffusion current.*

At a certain ratio of concentrations of A and B, the influx of these particles to the cathode surface is *equivalent* (stoichiometric) in respect to the reaction (2). Then, according to the equations (1), (2) and (6), we have:

$$D_A^{1/2} \cdot C_A = \frac{1}{q} \cdot D_B^{1/2} \cdot C_B, \quad i.\ e., \quad \frac{C_A}{C_B} = \frac{1}{q} \cdot \left(\frac{D_B}{D_A}\right)^{1/2}. \tag{7}$$

That will be called „equivalent" ratio of concentrations.

Having defined the „equivalence point" we have to distinguish between two different ranges of concentrations.

In the first concentration range

$$\frac{C_A}{C_B} \leq \frac{1}{q} \cdot \left(\frac{D_B}{D_A}\right)^{1/2} \tag{8}$$

the decrease of the influx of the particles B to the dropping electrode surface is determined by the rate of diffusion of the *component* A, and is given by:

$$\frac{\Delta i}{m \cdot F} = q \cdot \frac{k}{F} \cdot D_A^{1/2} \cdot C_A. \tag{9}$$

Hence the latent diffusion current in the first concentration range is:

$$\Delta i = q \cdot m \cdot k \cdot D_A^{1/2} \cdot C_A = \frac{q \cdot m}{n} \cdot i_A. \tag{10}$$

In the second concentration range *i. e.,* when:

$$\frac{C_A}{C_B} \geqslant \frac{1}{q} \cdot \left(\frac{D_B}{D_A}\right)^{\frac{1}{2}} \tag{11}$$

the magnitude of the latent diffusion current is determined in a similar manner by the influx of the *component B*:

$$\Delta i = m \cdot k \cdot D_B^{\frac{1}{2}} \cdot C_B = \dot{i}_B^0 . \tag{12}$$

In the „equivalence point" the latent diffusion current ought to obey *both* relations (10) and (12).

The sum of the wave heights

Substituting the equation (10) into the equation (4) we may distinguish in the *first* concentration range three variants corresponding to the value of the ratio $\frac{q \cdot m}{n}$ which characterizes any system of the particular components A and B.

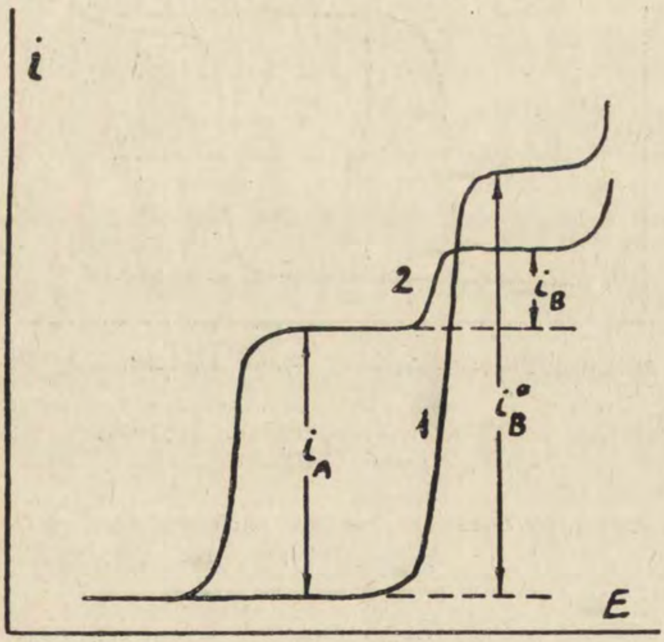

Fig. 2. Latent limiting current of the first kind, when $\frac{q \cdot m}{n} > 1$.
Curve 1 — polarogram of the solution containing B in the absence of A; 2 — polarogram of the solution of a mixture of A and B in the first concentration range.

1) $\frac{q \cdot m}{n} = 1$; the sum of the observed limiting currents is constant:

$$\sum i = i_A + i_B = \overset{0}{i}_B \quad \text{(see fig. 1).} \tag{13}$$

2) $\frac{q \cdot m}{n} > 1$; hence $\sum i < \overset{0}{i}_B$ (fig. 2). (14)

3) $\frac{q \cdot m}{n} < 1$; hence $\sum i > \overset{0}{i}_B$ (fig. 3). (15)

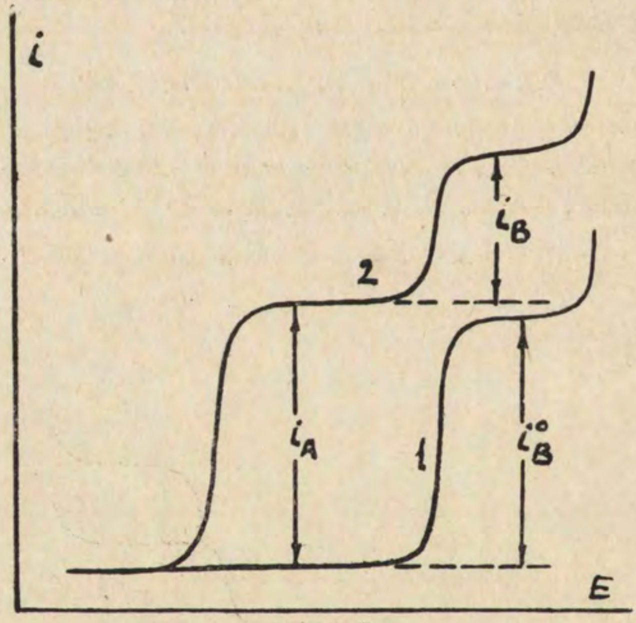

Fig. 3. Latent limiting current of the first kind, when $\frac{q \cdot m}{n} < 1$.
Curve 1 — polarogram of the solution containing *B* in the absence of *A*; 2 — polarogram of the solution of a mixture of *A* and *B* in the first concentration range.

Substituting equation (12) into equation (4) we obtain for all those variants in the *second* range of concentrations the relation:

$$\sum i = i_A. \tag{16}$$

The functions: $\sum i = f(C_A)$ when $C_B = \text{const.}$, and $\sum i = f(C_B)$ when $C_A = \text{const.}$, are represented graphically in the figs. 4 and 5, respectively.

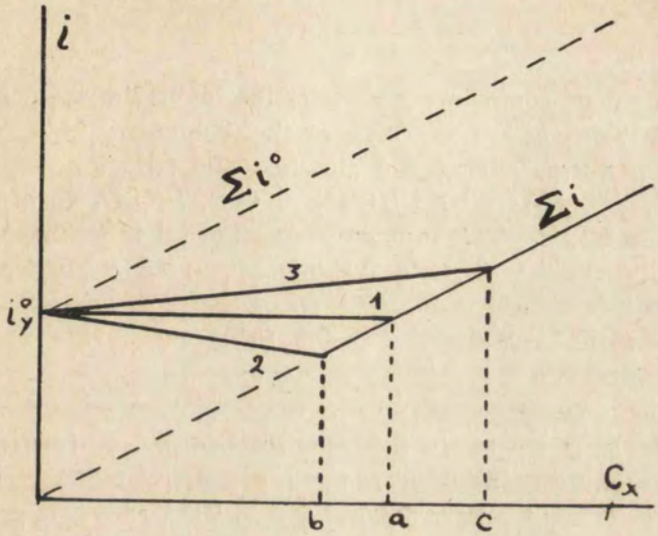

Fig. 4. The sum of the observed limiting currents $\Sigma i = f(C_x)$ when $C_y = \text{const.}$ For the first kind of the latent limiting currents $x = A$, $y = B$. For the second kind $x = B$, $y = A$. Curves 1, 2, 3 correspond to the successive variants, *a*, *b*, *c* being „equivalence points".

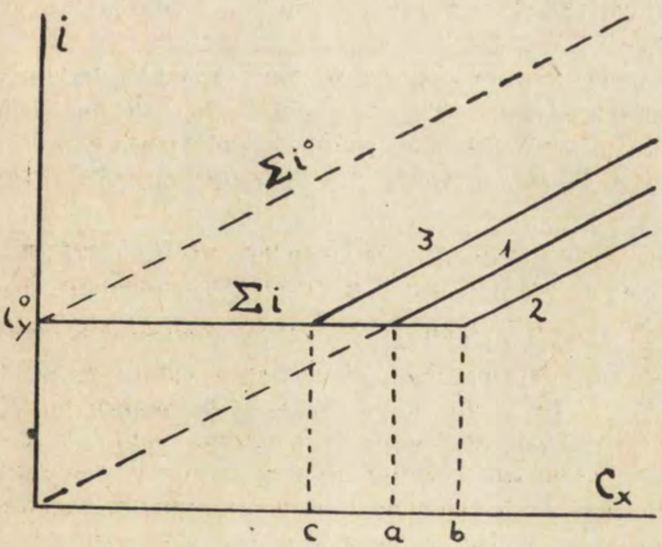

Fig. 5. The sum of the observed limiting currents $\Sigma i = f(C_x)$ when $C_y = \text{const.}$ For the first kind of the latent limiting currents $x = B$, $y = A$. For the second kind $x = A$, $y = B$. Curves 1, 2, 3 correspond to the successive variants, *a*, *b*, *c* being the „equivalence points".

Examples

The most common examples of the latent limiting currents are the cases of the decrease of the height of the observed hydrogen wave, during the simultaneous reduction of O_2 [2]), H_2O_2 [1]), IO_3', BrO_3' [5]), HNO_2 [6]), NO [7]), $H_2C_2O_4$ [8]), aromatic aldehydes [9]), *etc.* The secondary reaction (2) is in these cases either the reaction of neutralization, or a reaction between an intermediate radical *e. g.*, NO' and $H^{\cdot}$-ions or any protogenic particle being responsible for the formation of the so-called *hydrogen* wave.

These examples are partly complicated by the fact that the *rôle* of hydrogen in the true mechanisms of reduction is usually significant, differing from our premise of the independent reduction of both components: A and B. That *rôle* is expressed, *e. g.*, in the pH-dependence of the half-wave potentials, and sometimes in the dependence of the mechanism or even of the appearance of the reduction on the pH value. Nevertheless, the hydrogen-consuming reduction processes are a source of the distinct and best studied examples of the latent limiting currents.

E. F. Orlemann and I. M. Kolthoff [10]), having calculated for the systems: $IO_3' - H^{\cdot}$ and $BrO_3' - H^{\cdot}$ the values of the ratios of concentrations in the „equivalence point", stated a good agreement between the experimental and theoretical values.

For the systems: $O_2 - H^{\cdot}$ and $O_2 - CH_3COOH$, W. Kemula and S. Siekierski [11]) found experimentally the relations: $\sum i = f(C_{H^{\cdot}})$ and $\sum i = f(C_{CH_3COOH})$, which correspond to the curve 1, fig 5. The „equivalence point" found in the system $O_2 - H^{\cdot}$ in the large excess of the supporting electrolyte (1 *MKCl*, *loc. cit.*, table 1) *i. e.*, the value of the ratio $C_{O_2} : C_{H^{\cdot}} = 0{,}436$ agrees within the limits of few per cent with the value 0,473 calculated by introducing into the equation (7) the values: $q = 4$, $D_{O_2} = 2{,}6.10^{-5}$ [3]), $D_{H^{\cdot}} = 9{,}34.10^{-5}\ \mathrm{cm^2\ sec^{-1}}$ (this latter value, calculated from the equivalent ionic conductance at infinite dilution, being probably the cause of the observed discrepancy).

The latent limiting currents of the first kind were found also in the presence of oxygen, on the waves of $Fe^{\cdot\cdot}$, $Mn^{\cdot\cdot}$, $NH_4^{\cdot}$ [1]), $Cd^{\cdot\cdot}$, $Pb^{\cdot\cdot}$ [3]), $Ni^{\cdot\cdot}$, $Zn^{\cdot\cdot}$, in the neutral unbuffered solutions. The secondary reaction (2) leads in those cases to the formation of undissociated molecules or a precipitate of hydroxide *e. g.*:

$$4\,OH' + 2\,Me^{\cdot\cdot} \longrightarrow 2\,Me(OH)_2\,.$$

In the simple polarographic determinations of cations the formation of the harmful latent diffusion currents is omitted by the exclusion of oxygen or by the use a complex-forming supporting electrolyte. The same result may often be obtained by addition of acid [3]) or by suitable buffering of solution, *e. g.*, by the use of ammonium chloride as supporting electrolyte.

The second kind of the latent limiting currents

Consider an analogous system of two solutes, only the cathodic reduction of A:

$$A + n\,e \longrightarrow p\,A^* \tag{1}$$

being not followed by any secondary reaction (2). Instead, beginning with the more negative potential of the reduction of B

$$B + m\,\bar{e} \longrightarrow r\,B^* \tag{3}$$

the product of that reduction B^* may react with the reducible component A:

$$r\,B^* + s\,A \longrightarrow A_s B_r^*. \tag{17}$$

Therefore, the influx of A to the cathode surface decreases or ceases, according to the ratio of concentrations of A and B. The difference between the proper limiting current of B and the observed value:

$$\Delta\,i = i_B^0 - i_B \tag{4}$$

is then called a latent limiting current of the *second kind.*

The essential difference between the two mentioned kinds of latent limiting currents consists in the fact that the magnitude of the *first* one corresponds to the diminution of the real cathodic reduction of the *component B*, while the magnitude of the *second* one corresponds to the diminution of the real cathodic reduction of the *component A*, although in each case only the lowering of the wave of the component B is observed (see fig. 1 and fig. 6).

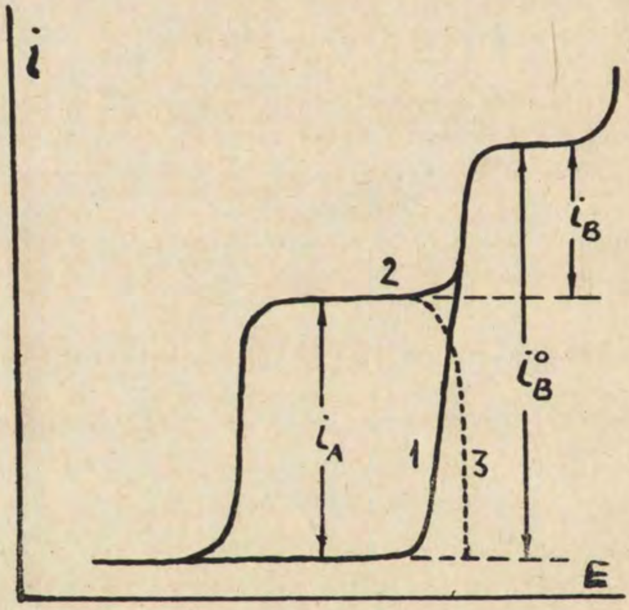

Fig. 6. Latent limiting current of the second kind, when $\frac{s \cdot n}{m} = 1$.
Curve 1 — polarogram of the solution containing B in the absence of A, 2 — polarogram ot the solution of the mixture of A and B in the first concentration range, 3 — hypothetical curve of the *real* cathodic reduction of the component A.

The „equivalence point"

Basing on the premises analogous to those in the case of the first kind of the latent diffusion currents, combining the equations (3), (17), (6), we may obtain for the second kind of the latent diffusion currents the following equation for the „equivalence point":

$$D_A^{1/2} C_A = s \cdot D_B^{1/2} \cdot C_B, \quad i.\ e., \quad \frac{C_A}{C_B} = s \cdot \left(\frac{D_B}{D_A}\right)^{1/2}. \qquad (18)$$

In the first range of concentrations:

$$\frac{C_A}{C_B} \leqslant s \cdot \left(\frac{D_B}{D_A}\right)^{1/2} \tag{19}$$

the component B is in excess, and therefore:

$$\Delta i = n \cdot k \cdot D_A^{1/2} \cdot C_A = i_A . \tag{20}$$

In the second concentration range:

$$\frac{C_A}{C_B} \geqslant s \cdot \left(\frac{D_B}{D_A}\right)^{1/2} \tag{21}$$

the component A is present in excess at the surface of the dropping electrode, and therefore the diminution of the influx of that component is determined by the diffusion of B, as follows:

$$\frac{\Delta i}{n \cdot F} = s \cdot \frac{k}{F} \cdot D_B^{1/2} \cdot C_B . \tag{22}$$

Hence the latent diffusion current is

$$\Delta i = s \cdot n \cdot k \cdot D_B^{1/2} \cdot C_B = \frac{s \cdot n}{m} \cdot \overset{0}{i}_B . \tag{23}$$

In the „equivalence point" both relations (20) and (23) are fulfilled by the latent diffusion current.

The sum of the wave heights

In view of equation (20), in the first concentration range the sum of the heights of the observed waves is

$$\sum i = \overset{0}{i}_B . \tag{24}$$

Provided A and B, every one may form only one polarographic reduction wave, in the first kind of the latent diffusion currents in the first range of concentrations exist always two polarographic waves, whereas in the second range one wave only appears, namely that of A. In the second kind however of the latent diffusion currents the number of waves is dependent on the value of the ratio $\frac{s \cdot n}{m}$ for a particular system. According to that value we may distinguish here three variants, too:

1) $\frac{s \cdot n}{m} = 1$; in the first concentration range two waves exist. In the second range, when according to equation (23) the latent current is:

$$\Delta i = i_B^0 \tag{25}$$

only one wave may be observed, and hence:

$$\sum i = i_A . \tag{26}$$

2) $\frac{s \cdot n}{m} > 1$; in the first range of concentrations two waves appear when $i_A < i_B^0$; when $i_A = i_B^0$ only one wave is visible, at last when $i_A > i_B^0$, *i. e.*, when the ratio of concentrations is:

$$\frac{C_A}{C_B} > \frac{m}{n} \cdot \left(\frac{D_B}{D_A}\right)^{1/2} \tag{27}$$

the limiting current *decreases* starting from the reduction potential of B, forming thus a flat, saddle-shaped *minimum* on the polarographic curve (see fig. 7).

In the second concentration range, according to equation (23), the latent diffusion current is:

$$\Delta i > i_B^0 \tag{28}$$

and hence:

$$\sum i < i_A . \tag{29}$$

The observed *negative* limiting current i_B is visible through the entire second range of concentrations and its magnitude can be computed by means of relations (4) and (23):

$$\sum i - i_A = i_B = i_B^0 - \Delta i = \left(1 - \frac{s \cdot n}{m}\right) \cdot i_B^0 . \tag{30}$$

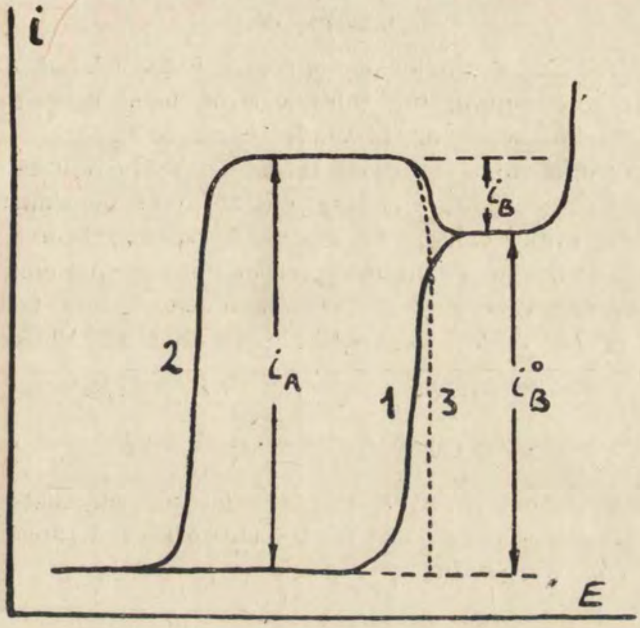

Fig. 7. Latent limiting current of the second kind, when $\frac{s \cdot n}{m} > 1$.
Curve 1 — polarogram of the solution containing B in the absence of A; 2 — polarogram of the solution of a mixture of A and B, obeying relation (27) in the first concentration range; 3 — hypothetical curve of the *real* cathodic reduction of component A.

3) $\frac{s \cdot n}{m} < 1$; in both concentration ranges always two waves appear. In the second range

$$\Delta i < i_B^0 \tag{31}$$

and

$$\sum i > i_A. \tag{32}$$

The remaining in the second concentration range visible wave of B obeys the relation (30), it has however a positive value contrary to the second variant.

The functions: $\sum i = f(C_A)$ when $C_B = \text{const.}$, and $\sum i = f(C_B)$ when $C_A = \text{const.}$, are represented graphically in the figs. 5 and 4, respectively.

Examples

Interesting examples of the second kind of latent diffusion currents, representing the third variant, have been described by J. J. Lingane and L. W. Niedrach [12]).

In the alkaline solutions the wave of the tellurite ion is lowered by the addition of lead, but not more than up to about 2/3 of the initial value. An analogous fact has been observed for the selenite wave in the presence of ammino-cupric ions. The reduction processes of TeO_3'' and SeO_3'' lead to the formation of Te'' or Se'' ions, which react then as follows:

$$Te'' + HPbO_2' + H_2O \longrightarrow PbTe + 3\,OH' \tag{33}$$

or

$$Se'' + Cu(NH_3)_4^{\cdot\cdot} \longrightarrow CuSe + 4\,NH_3\,. \tag{34}$$

Substituting $s=1$, $n=2$, $m=6$, into equation (30) for the „equivalence point" and for the entire second range of concentrations, we obtain:

$$\frac{i_B}{i_B^0} = \frac{2}{3}\,. \tag{35}$$

The value of that ratio has been found experimentally by J. J. Lingane and L. W. Niedrach to be equal 0,66 for system $TeO_3''—HPbO_2'$ and 0,72 for system $SeO_3''— Cu(NH_3)_4^{\cdot\cdot}$. The latter value indicates that the precipitation of $CuSe$ is incomplete under the given conditions.

Among the latent diffusion currents of the second kind we may classify also those of oxygen, *i. e.*, the lowering of the second oxygen wave in the presence of certain cations, such as $Cd^{\cdot\cdot}$ [3]), and those of bromate ion or methylene bromide in the presence of $Ni^{\cdot\cdot}$ in the neutral unbuffered solutions. $Ni^{\cdot\cdot}$ is precipitated as $Ni(OH)_2$ by the hydroxyl ions, formed during the cathodic reduction:

$$BrO_3' + 6\,\overline{e} + 3\,H_2O \longrightarrow Br' + 6\,OH' \tag{36}$$

or

$$CH_2Br_2 + 2\,\overline{e} + H_2O \longrightarrow CH_3Br + Br' + OH'. \tag{37}$$

Another good example is given by the latent limiting currents of nitrate ion which were recently studied by one of the authors (Z. R. Grabowski). That example will be descibed in some details.

Latent limiting currents of nitrate ion

The cathodic reduction of NO_3' at the dropping mercury electrode is an exceedingly complicated process and its mechanism is not yet clear. For the polarographic determination of nitrates two main methods are applied: the reduction in a slightly acid medium in the presence of diluted uranyl salts [13]), or in the neutral solutions of an excess of polyvalent cations [14]). Certain aspects of this latter method will be discussed here.

The determination of nitrate ion is difficult owing to the lack of proportionality between its concentration and limiting current, and owing to the sensitivity of the nitrate wave towards the presence of various cations and anions. Now, it became possible to explain some of the observed anomalies in view of the effect of latent limiting currents.

In a neutral solution of $LaCl_3$ (or of another rare-earth chloride) according to the various authors the total reduction process may be represented by:

$$NO_3' + 8\bar{e} + 6\,H_2O \longrightarrow NH_3 + 9\,OH' \quad (38)$$ [14])

or

$$NO_3' + 6\bar{e} + 5\,H_2O \longrightarrow NH_2OH + 7\,OH' \quad (39)$$ [13])

or by both of these equations [15]). According to L. Meites [16]), NO_3' does not undergo any apparent reduction, being only a catalyst in the catalytic process of the discharge of hydrogen at the dropping cathode:

$$2\,H_2O + 2\bar{e} \longrightarrow H_2 + 2\,OH'. \quad (40)$$

A detailed discussion of the mechanism of the polarographic reduction of NO_3' will be published in another paper. Here we must mention only that every one of those processes leads to the formation of hydroxyl ions during the reduction. The alkalization of the neighbourhood of the cathode during the reduction of NO_3' has in fact been demostrated already by the authors [4]).

The influence of the heavy metals cations

M. Tokuoka and J. Růžička [17]) have observed that in the solutions of $LaCl_3$ or $MgCl_2$ the limiting current of NO_3' has decreased in the presence of the heavy metal cations:

$Zn^{\cdot\cdot}$, $Cu^{\cdot\cdot}$, $Tl^{\cdot}$, $Pb^{\cdot\cdot}$, which are being discharged at more positive potentials than NO_3'. The curves in fig. 8 show that the decrease of the wave height of NO_3' is proportional to the concentration of $Zn^{\cdot\cdot}$.

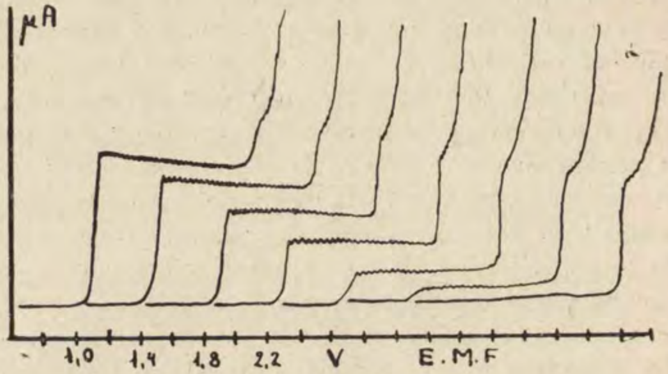

Fig. 8. The influence of $Zn^{\cdot\cdot}$ on the wave of NO_3' in $MgCl_2$, (according to Tokuoka and Růžička, *loc. cit.*, fig. 5). 0,02 N $MgCl_2$ and 0,001 N $LiNO_3$, in the absence of air, with increasing concentrations of $ZnCl_2$ (curves from right to left). All curves begin from 0,6 V.

These authors have attempted to explain this influence by the exaltation of the migration current; the increase of migration in the case of an anion corresponds to the decrease of its cathodic limiting current. „It has however been pointed out that the lowering effect, due to the deposition of cations, on the limiting current of the nitrate ions is much larger than that expected from the increase of the drop of potential across the solution $i \cdot R$" *(loc. cit.)*.

G. Cinquina [18]) described the influence of $Zn^{\cdot\cdot}$, $Pb^{\cdot\cdot}$, $Tl^{\cdot}$, $Cd^{\cdot\cdot}$, $Ni^{\cdot\cdot}$, on the limiting current of NO_3' in a solution of $LaCl_3$.

He has found that in the presence of $Zn^{\cdot\cdot}$ the wave of nitrate is lowered, but the sum of the waves of NO_3' and $Zn^{\cdot\cdot}$ remains constant (see fig. 9) until the diffusion current of $Zn^{\cdot\cdot}$ exceeds the magnitude of the initial wave of nitrate. In that point the concentration ratio $C_{Zn^{\cdot\cdot}} : C_{NO_3'}$ expressed in equivalents was found equal 9.

Mentioning here briefly his experimental results, we do not discuss Cinquina's unfounded hypotheses.

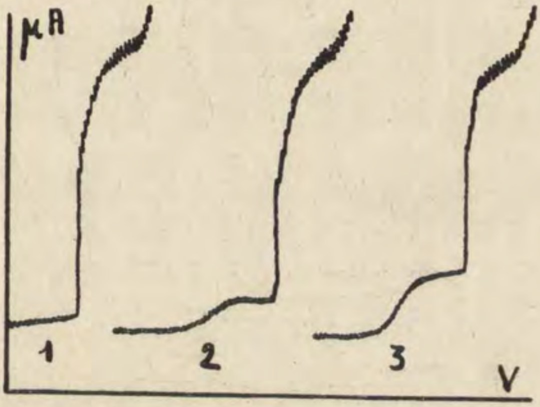

Fig. 9. The influence of $Zn^{\cdot\cdot}$ on the wave of NO_3' in $LaCl_3$ (according to Cinquina, *loc. cit.*, figs. 1—3). Curve 1 — wave of NO_3'; 2 — the same in the presence of an equivalent concentration of $Zn^{\cdot\cdot}$; 3 — the same in the presence of twice equivalent concentration of $Zn^{\cdot\cdot}$. The concentration of NO_3' is kept constant,

The behaviour of the nitrate wave in the presence of $Zn^{\cdot\cdot}$ is comprehensible only in view of one of the reactions: (38), (39) or (40), as a consequence of which the influx of $Zn^{\cdot\cdot}$ to the cathode surface is hindered by the reaction:

$$Zn^{\cdot\cdot} + 2\,OH' \rightarrow Zn(OH)_2. \qquad (41)$$

The numerical results of our experiments are different from those, obtained by Cinquina. The sum of the waves is not constant, increasing with the increasing concentration of $Zn^{\cdot\cdot}$. The real latent limiting current is lower than the calculated one.

Having postulated the reaction (41) being the source of the latent limiting current, we have used a suitably buffered supporting electrolyte, namely NH_4Cl together with $LaCl_3$. In such solution $Zn(OH)_2$ could not be precipitated; the wave of NO_3' starts at a much more negative potential than usually, and the wave height is constant and independent of the concentration of $Zn^{\cdot\cdot}$ (see fig. 10).

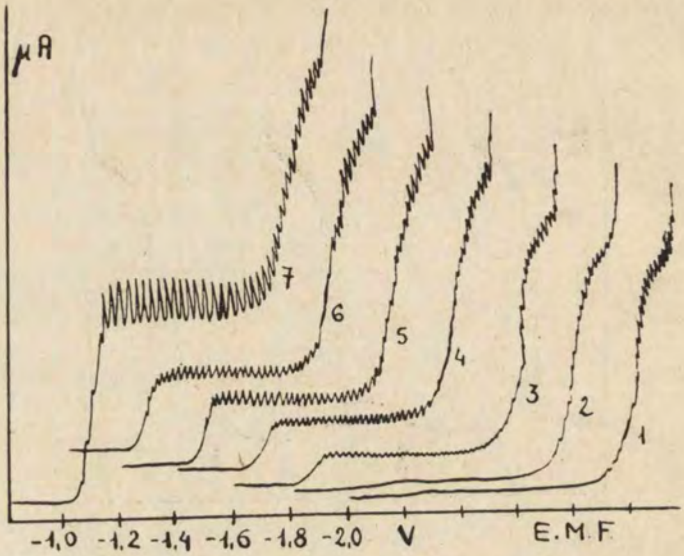

Fig. 10. The wave of NO_3' in the presence of $Zn^{\cdot\cdot}$ in a buffered solution: 1.10^{-3} M NO_3' in 0,017 M $LaCl_3$ and 0,1 M NH_4Cl in the absence of air. Concentration of $Zn^{\cdot\cdot}$: 1) 0; 2) $1{,}5.10^{-4}$; 3) 7.10^{-4}; 4) $1{,}2.10^{-3}$; 5) $1{,}7.10^{-3}$; 6) 2.10^{-3}; 7) 5.10^{-3} M. Curves begin from 0,8 V. Mercury anode.

Reduction of nitrate in the presence of acid

M. Tokuoka and J. Růžička found that „still stranger is the effect of the addition of hydrions. ...When to a nitrate in the presence of $LaCl_3$, acid is added, neither nitrate wave nor that due to the deposition of hydrions appear at — 1,2 V, but only one limiting current suddenly sets in at — 1,4 V, which henceforth remains constant up to the deposition of lanthanum. Only one wave is thus observable on the curve, which must be due the simultaneous reduction of both the hydrions and the nitrate ion. The height of this wave is, however, considerably smaller than the sum of the limiting current due to the reduction of hydrions alone and that of the nitrate ions, before the addition of the acid" [17]). The data, reported by L. Holleck [19]), show also a shift of the reduction potential of NO_3' in $NdCl_3$ to the more negative values after addition of HCl, and confirm the lack of the hydrogen wave.

The shift of the reduction potential in the acid and in the neutral buffered solutions, the abrupt rise of the wave in the neutral solutions, and the normal shape of the wave in the presence of oxygen or in slightly alkaline unbuffered solutions [17]), the difference between the curves of the „normal" and „backward" polarization [13]) — that all may be comprehensible postulating that $La(OH)_3$ (or another basic compound of La) would be a catalyst in the reduction process. The existence of $La(OH)_3$ at the cathode surface is a consequence of the previous reduction of oxygen and facilitates the reduction of NO_3'. In the absence of $La(OH)_3$ in the unbuffered solutions the unusual abruptness of the rise of the wave could be explained by an autocatalytic process; the hydroxyl ions, once formed, react with $La^{\cdot\cdot\cdot}$ precipitating the hydroxide, which is a catalyst in a further reduction and further production of the hydroxide. In the

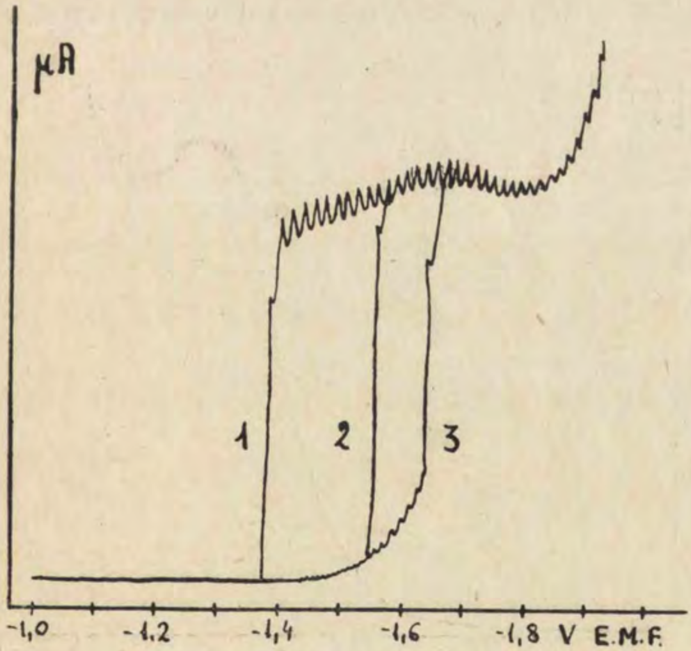

Fig. 11. The influence of the small concentration of HCl on the wave of NO_3' in $LaCl_3$. $9{,}8.10^{-4}$ M NO_3' in 0,017 M $LaCl_3$ and 0,1 M KCl, 0,01% gelatine, in the absence of air. Concentration of HCl: 1) 0; 2) $4{,}7.10^{-4}$; $9{,}4.10^{-4}$ M. Curves begin from 1,0 V applied E. M. F. Mercury anode.

buffered solutions, when the hydroxide could not be precipitated, the reduction potential of NO_3' is much more negative: the catalyst is absent (see fig. 11).

Introducing the proposed equations for the total reduction process into the latent current determining reaction schemes (3) and (17), we could write them as follows:

(3): $$NO_3' + 8\,\bar{e} + 6\,H_2O \rightarrow NH_3 + 9\,OH' \quad (38)$$

(17): $$9\,OH' + 9\,H^{\cdot} \rightarrow 9\,H_2O \quad (42)$$

where $n=1$, $m=8$, $s=9$, or:

(3): $$NO_3' + 6\,\bar{e} + 5\,H_2O \rightarrow NH_2OH + 7\,OH' \quad (39)$$

(17): $$7\,OH' + 7\,H^{\cdot} \rightarrow 7\,H_2O \quad (43)$$

where $n=1$, $m=6$, $s=7$.

According to the previous chapters of this paper, the system $H^{\cdot} - NO_3'$ should represent the second variant of the second

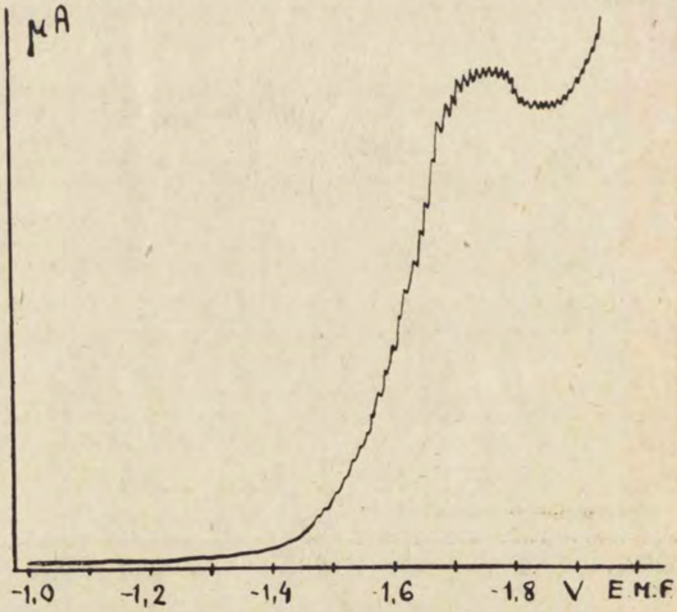

Fig. 12. Example of the wave of NO_3' in an excess of HCl. $9{,}6.10^{-4}\,M$ NO_3' and $4{,}6.10^{-3}\,M\,HCl$ in $0{,}017\,M\,LaCl_3$ and $0{,}1\,M\,KCl$, 0,02% gelatine, in the absence of air. Curve starts from $1{,}0\,V$ applied E. M. F. Mercury anode.

kind of the latent limiting currents. The experiments confirmed that presumption. When the value of the ratio $C_{H^{\cdot}}/C_{NO_3'}$ is sufficiently large, the *negative* limiting current of nitrate appears, an example of the curve being shown in the fig. 12.

It follows from the equation (30) that in the second range of concentrations, in the case of eight-electron reduction process the ratio $-i_{NO_3'}/i^0_{NO_3'}$ should be equal to 0,125, whereas in the case of six-electron process: 0,166. The experimental value has been found 0,11 — 0,12; corrected for the drop time factor $t^{1/6}$ it is: 0,10 — 0,11.

The relation between the current intensity at the potential of the limiting current of NO_3' $(\sum i)$ and the concentration of hydrogen ions is given in the fig. 13. Beginning from the intersecting point of the curves *a* and *e*, two limiting currents appear,

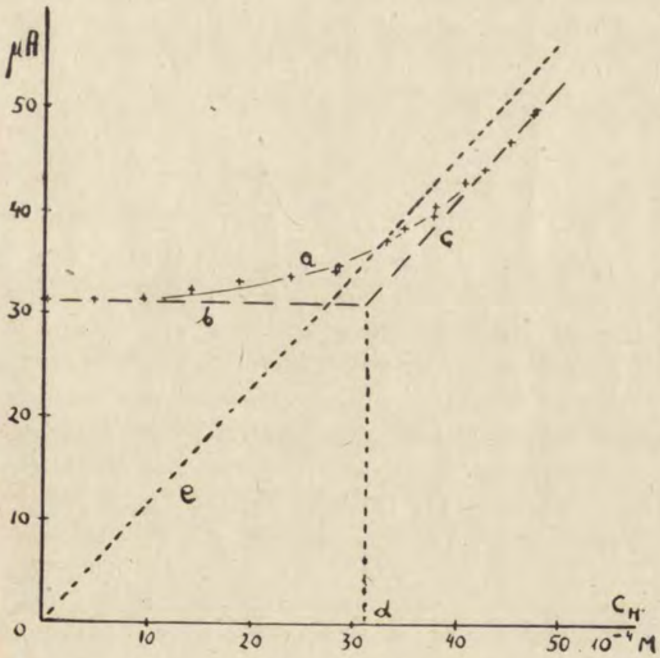

Fig. 13. The sum of the observed limiting currents $\sum i = f(C_{H^{\cdot}})$ when $C_{NO_3'} =$ const. 9,8.10^{-4} M NO_3' in 0,017 M $LaCl_3$ and 0,1 M KCl, 0,01% gelatine, in the absence of air. *a* — experimental curve; *b*, *c* — tangent lines, intersecting in the „equivalence point" *d*; *e* — calibration curve for the hydrogen diffusion current.

the larger one, as in the fig. 12, being the diffusion current of hydrogen ions. The experimental curve *a* shows a marked deviation from the theoretical one near the „equivalent" ratio of concentrations. Introducing into the equation (18) the „equivalent" concentrations and the ionic diffusion coefficients at infinite dilution ($D_{NO_3'} = 1{,}92.10^{-5}$ cm^2 sec^{-1}), we obtain the value $s \cong 7{,}5$.

When basing ourselves on the results of this study of the latent limiting currents of nitrate ion, we are not able to choose which of the proposed reduction processes is the true one, (38) or (39), but those results permit us to state that the number of the hydroxyl ions formed in the reduction, *s*, is larger than the number of electrons accepted, *n*. Therefore we may *reject*, at least in the case of the $LaCl_3$ solutions, Meites' suggestion of the catalytic discharge of hydrogen in the presence of nitrate ions which may be expressed by reaction (40). On the base of Meites' hypothesis one could by no means explain the „odd" behaviour of the nitrate wave in the acid solutions.

General remarks

The simple relations derived in the present paper will be valid only in a border case, according to the conditions named in the chapter on the „eqivalence point". The real latent diffusion currents observed are often *lower* than the calculated ones, *e. g.*, those caused by a precipitation of some heavy metals hydroxides. The precipitation of a hydroxide may be incomplete either owing to an apparent solubility or owing to slowness of the formation of a colloidal precipitate. The influence of the backward diffusion of the particles formed during the cathodic processes was neglected in the foregoing considerations, whereas it can be noticeable especially in the neighbourhood of an „equivalence point". Therefore the equations for the latent diffusion currents discussed above represent only an *upper limit* which cannot be exceeded by a magnitude of a given latent diffusion current.

The shape of the curves: C_x against $\sum i$, which consist of two lines intersecting at the „equivalence point" (figs. 4 and 5), suggests application of the latent diffusion currents to the quantitative analysis. When the formation of a latent diffusion current of hydrogen has been stated for the solutions of acetic acid in the presence of oxygen [20]), W. Kemula and S. Siekierski [11]) already elaborated the first analytical method using deliberately the latent diffision current, namely the method of polarometric (amperometric) titration of the dissolved oxygen with an acetate buffer.

As is shown in the present paper, the study of the latent diffusion currents can contribute to the knowledge of the reduction and secondary processes at the dropping cathode, as well as to a *comparative* determination of the polarographic diffusion coefficients.

SUMMARY

Two kinds of latent limiting currents to be distinguished according to the mechanism of their formation. On the base of the Ilkovič equation the relations between the concentrations of the reducible constituents of the solution and the observed diffusion current have been derived what permits to evaluate the magnitude of the latent diffusion current, provided that certain optimal conditions of its formation exist. The experimental data found in the literature are discussed from the point of view of the new classification of latent currents. The results arrived at in the field of latent limiting currents of nitrate ion are described as a new characteristical example of the effect under consideration.

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TREŚĆ — TABLE DES MATIÈRES

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