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NOTE ON THE DISCRIMINANT OF A BINARY QUANTIC.

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It is well known that the discriminant of a binary quantic $(a, b, c, d, \ldots \between t, 1)^n$ is of the form

$$Ma + Nb^2,$$

but it is further to be remarked that if $b = 0$, then the form is

$$a\,(Ma + Nc^3),$$

if $b = 0$, $c = 0$, the form is

$$a^2\,(Ma + Nd^4),$$

and so on, until only the lowest two coefficients are not put $= 0$. Or, what is the same thing, if in the discriminant of the original function we put $a = 0$, then the discriminant divides by b^2; if $b = 0$, the discriminant divides by a, and, omitting this factor, if we then write $a = 0$, it divides by c^3; if $b = 0$, $c = 0$, the discriminant divides by a^2, and omitting this factor, if we then write $a = 0$, it divides by d^4; and so on, until as before.

Thus if $b = 0$, the discriminant of $(a, 0, c, d, e \between t, 1)^4$, divides by a, and omitting this factor it is

$$\begin{aligned} &a^2e^3 \\ &- 18\,ac^2e^2 \\ &+ 54\,acd^2e \\ &- 27\,ad^4 \\ &+ 81\,c^4e \\ &- 54\,c^3d^2 \end{aligned}$$

which for $a = 0$ has the factor c^3; if $b = 0$, $c = 0$, the discriminant of $(a, 0, 0, d, e \between t, 1)^4$ has the factor a^2, and omitting this factor it is

$$\begin{aligned} &ae^3 \\ &- 27\,d^4, \end{aligned}$$

which for $a = 0$ has the factor d^4; the series of theorems here terminates, since the lowest two coefficients d, e are not to be put $= 0$.