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ADDITION TO MR GENESE'S NOTE ON THE THEORY OF ENVELOPES.

[From the *Messenger of Mathematics*, vol. VII. (1878), pp. 62, 63.]

THE example, although simple, is an instructive one. Introducing z, μ for homogeneity, the equation is

$$\lambda^2 y(y-bz)+2\lambda\mu xy+\mu^2 x(x-az)=0,$$

giving the envelope

$$xy[(x-az)(y-bz)-xy]=0;$$

that is,

$$xy(bx+ay-abz)z=0;$$

viz. we have thus the four lines

$$x=0,\quad y=0,\quad \frac{x}{a}+\frac{y}{b}-z=0,\quad z=0.$$

Writing these values successively in the equation of the curve, we find respectively

$$\lambda^2 y(y-bz)=0,$$

$$\mu^2 x(x-az)=0,$$

$$(b\lambda-a\mu)^2\frac{xy}{ab}=0,$$

$$(\lambda y+\mu x)^2=0;$$

viz. in each case the equation in λ, μ has (as it should have) two equal roots; but in the first three cases the values are *constant;* viz. we find $\lambda=0$, $\mu=0$, $b\lambda-a\mu=0$, respectively; and the curves $x=0$, $y=0$, $\frac{x}{a}+\frac{y}{b}-z=0$, are for this reason not proper envelopes.

It is to be remarked that writing in the equation of the parabola these values $\lambda=0$, $\mu=0$, $b\lambda-a\mu=0$ successively, we find respectively

$$x(x-az)=0,$$

$$y(y-bz)=0,$$

$$(bx+ay)(bx+ay-abz)=0;$$

viz. in each case the parabola reduces itself to a pair of lines, one of the given lines and a line parallel thereto through the intersection of the other two lines; the parabola thus becomes a curve having a dp on the line at infinity.

In the fourth case $z=0$, the equation in λ, μ is $(\lambda y+\mu x)^2=0$, giving a variable value $\lambda\div\mu=-x\div y$; hence $z=0$, the line at infinity is a proper envelope.

The true geometrical result is that the envelope consists of the three points A, B, C, and the line at infinity; a point *quâ* curve of the order 0 and class 1 is not representable by a single equation in point-coordinates, and hence the peculiarity in the form of the analytical result.