425.

NOTE ON THE GEODESIC LINES ON AN ELLIPSOID.

[From the *Philosophical Magazine*, vol. XLI. (1871), pp. 534, 535.]

THE general configuration of the geodesic lines on an ellipsoid is established by means of the known theorem (an immediate consequence of Jacobi's fundamental formulæ, but which was first given by Mr Michael Roberts, *Comptes Rendus*, vol. XXI. p. 1470, Dec. 1845) that every geodesic line touches a curve of curvature; that is, attending to the two opposite ovals which constitute the curve of curvature, the geodesic line is in general an infinite curve undulating between these opposite ovals, and so touching each of them an infinite number of times (but possibly in particular cases it is a reentrant curve touching each oval a finite number of times). The geodesic lines thus divide themselves into two kinds, accordingly as they touch a curve of curvature of the one or the other kind; and there is besides a third limiting kind, the lines which pass through an umbilicus: any such geodesic line passes through the opposite umbilicus, and is in general an infinite curve passing an infinite number of times alternately through the two umbilici; but possibly it is in particular cases a reentrant curve passing a finite number of times through the two umbilici. I annex a figure giving a general idea of the configuration of the geodesic lines drawn in different directions from a given point P on the surface of the ellipsoid: this is drawn (as it were) on the plane of the greatest and least axes; but it is not a perspective or geometrical representation of any kind, but a mere diagram for the purpose in question. We have A, A, B, C, C the extremities of the axes; U_1, U_2, U_3, U_4 the umbilici; P the point on the surface; $1P2$ and $1P4$ the curves of curvature through P, viz. these are ovals

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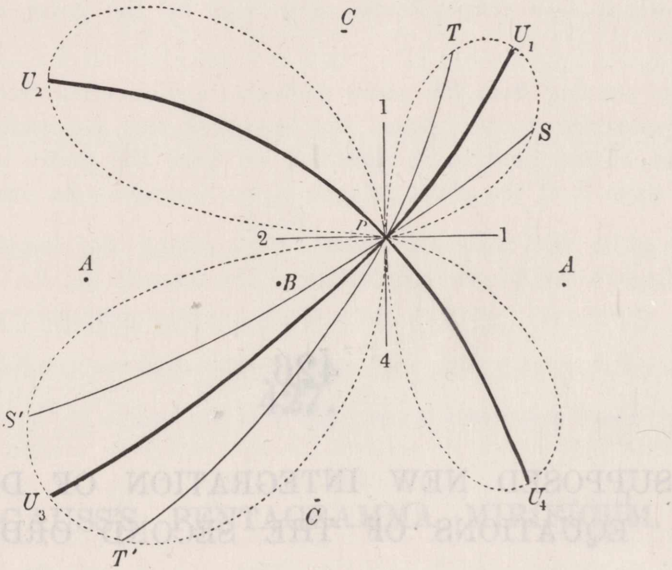
containing the umbilici U_1, U_2 and U_1, U_4 respectively. Then U_1PU_3 and U_2PU_4 are the limiting geodesics passing through the umbilici; the line TPT' represents a



geodesic line of the one kind, viz. this at T touches an oval (curve of curvature) U_1U_4, and at T' the conjugate oval U_2U_3. Similarly SPS' is a geodesic line of the other kind, viz. this at S touches an oval (curve of curvature) U_1U_2, and at S' the conjugate oval U_3U_4; the dotted figure-of-eight curves are the loci of the points of contact T, T', S, S'.

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