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Steady state fluid flow in viscoelastic tubes. Application to blood flow in human arteries

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A METHOD is outlined for determining the strain-rate dependent viscoelastic properties of the great vessels. The method is based on in vitro measurements of the axial distributions of either cross-sectional area or intraluminal pressure upon passage of a shock wave into the vessel. The corresponding analysis uses a quasi one-dimensional model of steady state wave propagation in a non-linear viscoelastic fluid-filled tube in which axial and bending stresses have been neglected. An approximate analytic expression is derived for the thickness of the shock transition in terms of the viscoelastic parameters, the shock velocity and the pressure difference across the shock. It is predicted on theoretical grounds that the entry length and shock thickness will be impractically large for fluids such as blood or water having densities of order one. In order to overcome this difficulty, mercury is proposed as the filling fluid. Calculations for mercury show that in this unique case, meaningful experiments are feasible using human aortic segments of lengths of the order of 15cm.

Przedstawiono metodę określania zależnych od prędkości deformacji lepkosprężystych własności dużych naczyń krwionośnych. Metoda oparta jest na pomiarach przeprowadzonych poza organizmem i dotyczących bądź przekroju poprzecznego, bądź też ciśnienia wewnętrznego w kanale przy przejściu fali uderzeniowej. Analiza oparta jest na quasi-jednowymiarowym modelu ustalonej propagacji fali w nieliniowo lepkosprężystej rurze wypełnionej cieczą z pominięciem naprężeń osiowych oraz zginania. Wyprowadzono przybliżony wzór na grubość fali uderzeniowej w zależności od parametrów lepkosprężystych, prędkość jej propagacji oraz różnicę ciśnień po obu stronach fali. Z teoretycznych przesłanek wynika, że grubość ta wypada zbyt wielka dla celów praktycznych w takich cieczach jak woda lub krew, które mają gęstość rzędu jedności. Dla pokonania tej trudności zaproponowano użycie rtęci jako cieczy wypełniającej przewód. W tym szczególnym przypadku wykazano, że można przeprowadzić rozsądne doświadczenia na odcinkach aorty o długości około 15cm.

Представлен метод определения зависящих от скорости деформаций, вязкоупругих свойств больших кровеносных сосудов. Метод опирается на измерениях произведенных вне организма и касающихся или поперечного сечения, или же внутреннего давления в канале при переходе ударной волны. Анализ опирается на квази-одномерной модели установившегося распространения волны в нелинейной вязкоупругой трубе заполненной жидкостью при пренебрежении осевыми напряжениями, а также изгибом. Выведена приближенная формула для ширины ударной волны в зависимости от вязкоупругих параметров, скорости ее распространения и разницы давлений по обоих сторонах волны. Из теоретических предпосылок следует, что эта ширина получается слишком большой для практических целей для таких жидкостей как вода или кровь, которые имеют плотность порядка единицы. Для преодоления этой трудности предложено использование ртути как жидкости заполняющей трубу. В этом частном случае доказано, что можно проводить разумные эксперименты на отрезках сосуда с длиной примерно в 15 см.

1. Introduction

IT IS KNOWN that the large vessels, such as the aorta, although elastic at low rates of extension, behave as viscoelastic materials of ever-increasing stiffness as the rate of strain to which they are subjected increases FUNG (1970), COLLINS and HU (1972). Meaningful calculations of the stresses to which these vessels are exposed depend most critically on these dynamic stress-strain-strain rate relationships. Results of acceleration experiments performed over a number of years by STAPP (1970) on young military volunteers indicate the surprisingly wide range of forces and accelerations to which the subject may be exposed during falls, vehicular collisions, vibrations, impacts and ejection from aircraft, in addition to the yet less well documented spectrum of forces encountered during space travel. For example, rates of decelerations exceeding 1000 g per second at levels higher than 25 g become progressively more difficult to withstand, even in the backwardfacing position, or with pelvic and shoulder girdle restraints, eliciting transient musculo-skeletal or visceral pain, changes in vision and perturbations of the cardiac and respiratory functions.

It is clear that theoretical and experimental investigations of accelerative phenomena will require accurate estimates of the dynamic material properties of biological tissues in these wide force regimes. Presently, such data are rare or non-existent. For the great vessels, material properties have been determined from "jerk-type" experiments, in which the vessel is abruptly elongated from its position at rest. Large strains and strain-rates occur only after significant stretching of the complete vessel segment, often to levels which are physiologically unrealistic.

In this work, a more practical approach is suggested, based upon the passage of a shock front through a liquid-filled vessel. The specimen is strained significantly and rapidly only in the immediate vicinity of the propagating front. In the following sections, a mathematical model is formulated for shock propagation in a fluid-filled distensible tube, and resulting changes in intraluminal pressure and cross-sectional area are related directly, in closed analytical form, to the material properties of the tube wall. Shock experiments are then proposed which would yield the viscoelastic properties of arteries and veins on the basis of this analysis.

2. Shock waves in the greater vessels

2.1. Basic physical assumptions

The formation and propagation of shock waves in fluid-filled distensible tubes have recently been considered by RUDINGER (1970), BEAM (1968), LAMBERT (1958) and ANLIKER *et al.* (1971). The formation of a shock wave is a result of the steepening of a finite amplitude continuous wave, in an elastic tube having a non-linear pressure-area relation. When no mechanism is provided in the mathematical model to smooth rapid changes on the flow, the wave steepens to the level at which a discontinuity eventually forms.

In the proposed experiments, the tube segment must be sufficiently long for formation of a shock front. This length is related to the mechanical properties of the tube, the fluid density, and the initial rise-time of the wave (see RUDINGER (1970) Eq. (12)). The dispersive effects of fluid viscosity and wall viscoelasticity in fact "smear" the mathematical shock discontinuity over a finite thickness, the precise form of which then depends upon these parameters. For the larger vessels, however, the dispersion depends most critically upon the viscoelastic properties of the tube wall rather than of the fluid which is therefore taken as inviscid in this study. It will subsequently be shown, however, that the fluid density plays an important role in determining the entrance length and shock thickness and that these can be shortened significantly for heavy fluids.

In order to simplify the mathematical problem somewhat, it is assumed that the shock wave propagates in a tube with uniform mechanical properties and cross-sectional area and that a steady state is achieved, i.e., that the flow-field becomes stationary for an observer moving with the shock. Under these assumptions, the shock structure is governed by an ordinary differential equation, which allows one to study the basic properties of the shock transition fairly easily.

2.2. Mathematical model

We adopt here a quasi one-dimensional model for the flow of an inviscid incompressible fluid in a viscoelastic tube, based upon the assumptions that (a) the wave length is long compared to tube diameter and (b) that the tube is constrained from longitudinal motions. Under these assumptions the governing equations of motion may be expressed as

$$A_t+(Au)_x = 0 \quad \text{(continuity)}, \tag{2.1}$$

$$u_t+uu_x+\frac{p_x}{\varrho} = 0 \quad \text{(conservation of momentum)}. \tag{2.2}$$

Here t denotes the time and x is the distance along the axis of the tube, measured in the flow direction, u the fluid velocity (averaged over the cross-section), A the cross-sectional area, p the pressure and ϱ the constant density of the fluid.

To these equations must be added a relation between the pressure and the cross-sectional area. For a viscoelastic tube, the pressure depends on the cross-sectional area and its time rate of change, $\eta = \partial A/\partial t$. The strain-rate dependent pressure-area relation may be written as:

$$p = f(A)+g(A, \eta); \quad \eta = \frac{\partial A}{\partial t}, \tag{2.3}$$

where the function (A) corresponds to static loading, and the g-term accounts for the viscoelastic properties of the tube wall. g is a monotonically increasing function of η with

$$g(A, 0) = 0. \tag{2.4}$$

2.3. Stationary flow

For the analysis of the stationary shock-wave, it is convenient to write the differential equations in a coordinate system moving with the shock.

Assuming that the shock wave is moving to the right with a constant velocity U, we employ the transformation

$$z = x-Ut, \quad \tau = t. \tag{2.5}$$

The differential equations (2.1) and (2.2) become

$$A_\tau + (vA)_z = 0, \tag{2.6}$$

$$v_\tau + \left(\frac{v^2}{2} + \frac{p}{\varrho}\right)_z = 0, \tag{2.7}$$

where

$$v = u - U. \tag{2.8}$$

The functional relation (2.3) remains unchanged, but now η is given by

$$\eta = \frac{\partial A}{\partial \tau} - U\frac{\partial A}{\partial z}. \tag{2.9}$$

The steady state equations are obtained by putting $\partial/\partial\tau = 0$ in the differential equations (2.6) and (2.7). They then become ordinary differential equations, which after one integration with respect to z give

$$vA = m, \tag{2.10}$$

$$\frac{v^2}{2} + \frac{p}{\varrho} = F. \tag{2.11}$$

The pressure-area relation becomes

$$p = f(A) + g\left(A,\ -U\frac{dA}{dz}\right). \tag{2.12}$$

The shock jump conditions can now be obtained by considering the flow conditions far ahead of the shock and far behind the shock. Denoting these states by subscripts 1 and 2 respectively, one has

$$v_1 A_1 = v_2 A_2 = m, \tag{2.13}$$

$$\frac{v_1^2}{2} + \frac{p_1}{\varrho} = \frac{v_2^2}{2} + \frac{p_2}{\varrho} = F, \tag{2.14}$$

where $p_{1,2} = f(A_{1,2})$ since dA/dz vanishes at both ends and Eq. (2.4) is noted. The relations for the shock velocity U and the change in the fluid velocity $(u_2 - u_1)$ follow from Eqs. (2.13), (2.14) and (2.8):

$$\varrho(U-u_1)^2 = \frac{f(A_2) - f(A_1)}{\frac{1}{2}[1-(A_1/A_2)^2]}, \tag{2.15}$$

$$\frac{u_2 - u_1}{U - u_1} = 1 - \frac{A_1}{A_2}. \tag{2.16}$$

The set of Eqs. (2.10), (2.11) and (2.12) can be reduced to a single equation by eliminating p and v from (2.11) using (2.10) and (2.12). The resulting differential equation for A is

$$g\left(A,\ -U\frac{dA}{dz}\right) = \varrho\left(F - \frac{1}{2}\frac{m^2}{A^2}\right) - f(A). \tag{2.17}$$

If the constants m and F from Eqs. (2.10) and (2.11) are expressed in terms of the flow variables in states 1 and 2, Eq. (2.17) becomes

$$(2.18)\qquad -g\left(A,\ -U\frac{dA}{dz}\right) = f(A)-f(A_1)-\frac{f(A_2)-f(A_1)}{(1-A_1^2/A_2^2)}\Bigg)(1-A_1^2/A^2).$$

Once the functions g and f are known, Eq. (2.18) constitutes a first-order differential equation for $A(z)$ which may be integrated by simple quadrature. Alternatively, if $A(z)$ is measured, Eq. (2.18) provides a relation between f and g. If, in addition, the function f is known, then the function g may be determined directly from Eq. (2.18). It is, in fact, this approach which forms the basis of the presently proposed method.

The function $f(A)$, corresponding to the zero strain-rate response, may be measured accurately by static loading experiments. The function will have the following property. For physically realistic behaviour, the cross-sectional area A increases with pressure p, i.e. $df/dA > 0$. If in addition, a continuous wave travelling in the tube is to steepen into a shock front, one also requires that (BEAM, 1968)

$$(2.19)\qquad \frac{d^2\bar{f}}{d\alpha^2} > 0,$$

where

$$(2.20)\qquad \alpha \equiv \frac{1}{2}(1-A_0^2/A^2) \quad \text{and} \quad \bar{f}(\alpha) \equiv f(A)$$

and A_0 is a reference cross-sectional area.

This condition can be shown equivalent to the fact that sound speed c in a flexible tube generally increases with intraluminal pressure, or more rigorously, $(1+\varrho c dc/dp) > 0$, as has been verified by the experiments of KING (1947) which were carried out over the physiological range of pressure. In fact, recent measurements (COLLINS and HU, 1972) have corroborated the property (2.19) at higher intraluminal pressures.

2.4. Particular solutions

The general relation (2.18) may be solved for specific functional forms of g and f. From the experiments of COLLINS and HU (1972) for the aorta, the resulting stress-strain relation would indicate that the function $g(A, \eta)$ is linear in η, so that it may be written as

$$(2.21)\qquad g(A, \eta) = G(A)\eta,$$

where $G(A) > 0$ since the stiffness increases with increasing strain-rate. The differential equation for the shock structure now becomes

$$(2.22)\qquad dz = U(A_1, A_2)\frac{G(A)dA}{H(A)},$$

where $H(A)$ is given by the right-hand side of Eq. (2.18), that is,

$$H(A) = f(A) - f(A_1) - \frac{f(A_2) - f(A_1)}{1 - A_1^2/A_2^2}\left(1 - \frac{A_1^2}{A^2}\right) \tag{2.23}$$

and $f(A)$ is known from static measurements.

For the discussion of the solution of (2.22), it is advantageous to replace A by the Beam parameter α [Eq. (2.20)]. Equation (2.22) becomes

$$dz = U(\alpha_1, \alpha_2)\bar{G}(\alpha)\frac{d\alpha}{\xi(\alpha)}, \tag{2.24}$$

where

$$\xi(\alpha) = \bar{f}(\alpha) - \bar{f}(\alpha_1) - \frac{\bar{f}(\alpha_2) - \bar{f}(\alpha_1)}{\alpha_2 - \alpha_1}(\alpha - \alpha_1), \tag{2.25}$$

$$\bar{G}(\alpha) = G(A(\alpha))\frac{dA(\alpha)}{d\alpha} \tag{2.26}$$

and U is given from Eq. (2.15) as

$$\varrho[U(\alpha_1, \alpha_2) - u_1]^2 = \left(\frac{A_0}{A_1}\right)^2 \frac{\bar{f}(\alpha_2) - \bar{f}(\alpha_1)}{\alpha_2 - \alpha_1}. \tag{2.27}$$

It is clear that Eq. (2.24) possesses singularities at the ends of the interval $\alpha_1 \leqslant \alpha \leqslant \alpha_2$ and that a special treatment is required there before numerical integration can be attempted.

Since ξ vanishes at $\alpha = \alpha_{1,2}$ and also $\xi''(\alpha) > 0$ in $\alpha_1 \leqslant \alpha \leqslant \alpha_2$, it follows that

$$\xi(\alpha) \neq 0 \quad \text{for} \quad \alpha_1 < \alpha < \alpha_2.$$

Near the ends ($\alpha = \alpha_{1,2}$), $\xi(\alpha)$ approaches zero as $B_{1,2}(\alpha - \alpha_{1,2})$, where from Eq. (2.25)

$$B_{1,2} = \left.\frac{d\bar{f}}{d\alpha}\right|_{(\alpha_{1,2})} - \frac{\bar{f}(\alpha_2) - \bar{f}(\alpha_1)}{\alpha_2 - \alpha_1}. \tag{2.28}$$

The solution of (2.24) near α_1 and α_2 is therefore

$$z + \text{const} = \frac{U\bar{G}(\alpha_{1,2})}{B_{1,2}} \ln|\alpha - \alpha_{1,2}|, \quad \text{for} \quad \alpha \to \alpha_{1,2}. \tag{2.29}$$

Since the function $\xi < 0$ for all α, and behaves as $B_{1,2}(\alpha - \alpha_{1,2})$ near $\alpha = \alpha_{1,2}$, it follows that $B_1 < 0$ and $B_2 > 0$. Then $z \to \infty$ when $\alpha \to \alpha_1$ and $z \to -\infty$ when $\alpha \to \alpha_2$, which implies that the mathematical shock transition extends from minus infinity to plus infinity. However, it will be shown that the principal variations in the flow field occur over a finite extent, and this may be considered for all practical purposes as the shock "thickness".

The solution of (2.24) for general functions $\bar{G}(\alpha)$ and $\bar{f}(\alpha)$ can be obtained by quadrature with the help of the limiting forms (2.29).

In a similar way, limiting forms may be derived for the solution of the general case embodied in Eq. (2.18).

Solutions of Eq. (2.24) may now be derived for polynomial representations of $\bar{G}$ and $\bar{f}$. The simplest closed-form solution is obtained for

$$\bar{f}(\alpha) = B(\alpha+\Gamma\alpha^2), \tag{2.30}$$

$$\bar{G}(\alpha) = \text{const} = \mu. \tag{2.31}$$

The parameters B and Γ in the function $\bar{f}$ can be selected so that $\bar{f}(\alpha)$ closely follows the experimentally measured curves. However, the experimental results are usually expressed in the form of a relation between the speed of sound c and the pressure p. From the Moens-Korteweg relation for the speed of sound in an elastic tube

$$c^2 = \frac{A}{\varrho}\frac{df}{dA}, \tag{2.32}$$

which for the form of $\bar{f}(\alpha)$ given by (2.30), yields

$$c_0 \equiv c|_{p=p_0} = \sqrt{\frac{B}{\varrho}} \tag{2.33}$$

and

$$\varrho c_0 \left.\frac{dc}{dp}\right|_{p=p_0} = \Gamma - 1. \tag{2.34}$$

Relations (2.33) and (2.34) serve to determine B and Γ in the function $\bar{f}(\alpha)$ of relation (2.30) in terms of the measured c_0 and dc/dp at $p = p_0$.

The differential equation (2.24) becomes

$$dz = \frac{\mu U d\alpha}{\varrho c_0^2 \Gamma(\alpha-\alpha_1)(\alpha-\alpha_2)}$$

whose solution is

$$\ln\frac{\alpha_2-\alpha}{\alpha-\alpha_1} = (\alpha_2-\alpha_1)\frac{\Gamma z}{\nu U}, \tag{2.35}$$

where ν is the ratio of the viscoelastic to elastic moduli at the reference area A_0,

$$\nu = \mu/\varrho c_0^2. \tag{2.36}$$

An alternate form of Eq. (2.35) which is analogous to the classical relation for the structure of weak gas dynamical shocks [see HAYES (1958)] is

$$\alpha - \alpha_1 = \alpha^*\left(1 - \tanh\frac{\alpha^*\Gamma}{\nu U}z\right), \tag{2.37}$$

where

$$\alpha^* \equiv \frac{\alpha_2-\alpha_1}{2}.$$

With this solution for the axial distribution of cross-sectional area through the shock transition, one may now proceed to an estimate of the length of the shock transition, i.e., the shock thickness over which most of the pressure jump occurs.

A general relation between p and A for the stationary flow is given by Eqs. (2.17) and (2.3)

$$p-p_1 = \varrho\left(F-\frac{1}{2}\frac{m^2}{A^2}\right).$$

Inserting the expressions for F and m from (2.13) and (2.14), this becomes

$$p-p_1 = \varrho(U-u_1)^2\frac{1}{2}\left[1-\left(\frac{A_1^2}{A^2}\right)\right]$$

or

$$p-p_1 = \varrho(U-u_1)^2\left(\frac{A_1}{A_0}\right)^2(\alpha-\alpha_1), \tag{2.38}$$

which shows that the pressure drop is proportional to $(\alpha-\alpha_1)$. This fact is used to define the shock wave "thickness" δ as

$$\delta \equiv \frac{p_2-p_1}{\left|\left(\frac{dp}{dz}\right)\right|_{\max}} = \frac{\alpha_2-\alpha_1}{\left|\frac{d\alpha}{dz}\right|_{\max}}. \tag{2.39}$$

Using the solution (2.37) one obtains the shock thickness

$$\delta = \frac{4\nu U}{(\alpha_2-\alpha_1)\Gamma}. \tag{2.40}$$

The thickness δ corresponds to the distance over which three quarters of the total pressure jump takes place.

3. Proposed experimental determination of viscoelastic properties

The function $g(A, \eta)$ characterizing the viscoelastic material behaviour in the pressure-area relation (2.12) may be determined by a series of careful experiments in which either a) the wall profile is measured, giving $A = A(x)$, or b) the axial distribution of pressure $p = p(x)$ is recorded for a given position of the shock front.

A choice between a) and b) might be made purely on the basis of experimental accuracy and facility. For alternative a) one uses the measured distributions $A = A(z)$ over a range of shocks strengths. Numerical differentiation, carried out as accurately as possible, then allows one to evaluate both arguments of $g\left(A, -U\frac{dA}{dz}\right)$ in Eq. (2.18) for a range of values of dA/dz, so that the surface g may be constructed. The right-hand side of (2.18) is evaluated using the function f determined from the usual static tensile tests. If desired, relation (2.12) may then be easily converted to an expression for stress in terms of strain and strain rate.

Method b) requires differentiation of the measured pressure distribution $p(z)$. The pressure and its derivative are calculated in terms of cross-sectional area through relations (2.38) and (2.20), and the procedure for method a) outlined above is followed directly.

The tube diameter, and hence cross-sectional area, may be measured during passage of the uniform shock front by recording, by X-ray cinematography, the motion of two fine metallic wires disposed axially, and diametrically opposite one another, along the outer wall surfaces of the tube. If the fluid which fills the tube is radio-opaque, X-ray ciné will record the instantaneous diameter directly. The pressure may be measured by miniature strain-gauge type semi-conductor transducers mounted in a catheter and oriented along the axis of the lumen.

Wave propagation experiments in 10 meter-long water-filled rubber and PVC-tubes have been performed by LORENTZ and ZELLER (1972) for weak shocks which did not produce the high rates of strain of interest here.

Errors in determination of the dynamic material properties in this manner can emanate only from inaccuracies in the measurement of diameter (or pressure) and in numerical differentiation of these quantities which should be carried out carefully. It is noted that azimuthal isotropy is implicit in this quasi one-dimensional formulation.

The tube must be sufficiently long for formation of a shock front and for its development into an equilibrium profile. The distances that the wave must travel in order to fulfil this requirement may be estimated as follows:

(i) Distance S_1 for shock formation [RUDINGER, 1970, Eq. (12)]:

$$S_1 = \frac{\varrho c_0^3}{\left.\frac{dp}{dt}\right|_{x=x_0} \left(1+\varrho c_0 \frac{dc}{dp}\right)}, \tag{3.1}$$

where $\left(1+\varrho c_0 \frac{dc}{dp}\right) = \Gamma = 3$ for measurements of COLLINS and HU (1972).

(ii) Distance S_2 for the formed shock to reach an equilibrium state, has been calculated using the general numerical solution of KIVITY and COLLINS (1974) for a blood-filled viscoelastic tube of uniform cross-section with $B = 0.08$ s, $\varrho = 1.05 \text{gm/cm}^3$ and $c_0 = 300 \text{cm/s}$.

The boundary condition at the proximal end was chosen to simulate the flow pattern when the tube is connected to a high pressure reservoir: i.e., the total pressure

$$p_0 = p + \frac{1}{2} p u^2$$

is prescribed as a function of time. The tube is taken sufficiently long to ensure that no signals are transmitted back from the distal end. This calculation predicts a shock thickness of approximately 22cm and an entrance length of 45–55cm (depending on the rise time of the proximal pressure P which varied in the calculations between $2 \sim 20$ms) for a reservoir pressure of 0.66atm.

For these values of the parameters, Eq. (3.1) predicts a distance S_1 for shock formation of less than 1cm, which could be disregarded in estimating the required minimum lengths of test specimens. However, the distance S_2 (entrance length and shock thickness) is clearly too long for testing aortic segments of $10 \sim 15$cm long, corresponding for instance, to the human aortic arch, along which the material properties do not vary significantly. Fortunately, it is possible to reduce the entrance length and shock thickness to acceptable levels by increasing the density of the fluid. This follows from an exact scaling law, directly derivable from the equations of motion (2.1), (2.2), (2.3) and relation (2.32), which shows that distances vary inversely as the square root of the fluid density. In fact, mercury with a density of 13.6gm/cc is ideal for this purpose. The calculated entrance length and shock thickness then become 12–15cm and 6cm, respectively. A shock thickness of 6cm is quite compatible with a test segment of 15cm in length. Shorter shock thicknesses would begin to violate the basic assumption of large wave-length-to-diameter ratio. A furled plastic liner introduced into the aorta (cf. COLLINS and HU (1972) will prevent direct contact between the mercury and the intimal layer of the vessel wall.

4. Conclusions

A general technique has been described for determining the non-linear viscoelastic properties of distensible tubes at high strain-rates. It has been shown possible to generate shocks of acceptable thickness relative to arterial test segments of about 15cm in length by the novel use of mercury as the filling fluid. A series of in vitro tests with different pressure levels performed on the same biological specimen permits one to trace the viscoelastic function g on the strain-strain rate surface.

An internal furled plastic liner can be used to isolate the intimal layer of the wall segment from direct contact with the mercury to prevent contamination during repeated tests. The biological specimen should be preserved in a bath of physiological saline solution maintained at a controlled temperature to ensure that its physical properties do not change significantly after excision.

Separate calculations show that fluid viscosity contributes very little to the stress fields in the vessel wall, and may hence be safely neglected. The time for establishment of a steady state flow régime may be modified somewhat by slight taper of the tube and small material inhomogeneities over a 15cm aortic segment.

Good accuracy in the determination of the viscoelastic material parameters will be achieved if the spatial distributions of pressure and cross-sectional area depend critically upon changes in the viscoelasticity-related function $g(A, \eta)$. Comparisons by KIVITY and COLLINS (1974) of preliminary calculations with experimental test results performed on abruptly decelerated dogs corroborate this high sensitivity.

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